PUBLIC UTILITY REGULATION IN INTERTEMPORAL EQUILIBRIUM

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This paper examines the rate of return which should be allowed on public utilities in intertemporal general equilibrium. The model incorporates four sets of actors: consumers, firms (regulated and unregulated), government and regulators. We assume that consumers maximize their intertemporal utility. Firms maximize the present value of discounted cash flows while the government collects taxes, redistributes income and creates money. Our concept of equilibrium is that of "rational expectations": stock prices, bond prices, goods prices and wage rates are set such that when all parties optimize, all markets clear.

Three types of taxes are collected by the government: corporate income taxes, ordinary income taxes and capital gains taxes. We investigate the effects of each on the output prices which regulators should allow public utilities. The government also creates money and generates an (exogenously determined) inflation rate. The price of the output of public utilities is set by regulators and is outside of the control of the utilities.

We assume that firms are financed by both bonds and equities. Equity holders are compensated either through capital gains or by dividends. Both regulated and unregulated firms maximize their present value in a two-step sequence. First, the firm will choose a method of financing which minimizes its cost of capital. We incorporate the Miller [13] result that at low personal income tax rates, firms will completely bond finance and at high personal rates, they will completely equity finance. Auerbach and King [2] studied the conditions under which firms will specialize in debt financing, equity financing or be indifferent between the two. We do
not incorporate refinements here; we merely assume that the firm makes an
optimal choice of debt equity ratio. Second, we assume the firm chooses
an investment sequence which maximizes its present value, given the cost
of capital determined in step 1.

Section 2 of the paper describes the basic model. Section 3 modifies
the model to incorporate all three types of taxes. Section 4 reports a
"bang-bang" result. We find that when demand is perfectly inelastic and
when utility managers maximize the value of the firm given the rate of
return allowed by regulators, that they will tend to overcapitalize or
undercapitalize the firm dramatically if they are not given the proper
rate of return. This strategic optimization by the utility managers will
lead to intertemporal accumulation or decumulation of the rate base which
can be quite dramatic. In effect, we present a dynamic formulation of the

In Section 4 we derive the unique "bang-bang" regulatory return,
hereafter (BR), which satisfies the knife-edge condition (i.e., presents
either extreme of the "bang-bang"). We examine how this return is
affected by the corporate income tax, the ordinary income tax and the
capital gains tax. We find that in addition to taxes, BR is affected by
macrovariable (discount rates) and firm-specific variables (depreciation
rates). We provide some illustrative numbers of the real return on
capital which regulators should allow using the formula.

A second innovation of the model is the generalization of the cost
This is done by embedding the value maximizing firm's problem into the
intertemporal general equilibrium rational expectations capital asset pricing model of Brock [7].

This procedure relates the value of the firm to the interaction of tastes, technology, uncertainty, and the tax structure. A valuation equation for the firm is derived. This valuation equation must be satisfied in order that there be no intertemporal arbitrage profits from financial re-packaging, marginal investments, or portfolio diversification. Investment policy and financial structure are then chosen to maximize initial firm value generated by the valuation equation.

In the deterministic case the problem of choosing the financial structure as well as the investment policy to maximize firm value breaks into two stages. First the financial package is determined to minimize the cost of capital at each point in time. The cost of capital depends on tastes, technology and the tax structure — personal and corporate. Second the investment path is chosen to maximize the present discounted value of cash flow net of all tax drains at the corporate level. The discount rate is the cost of capital found in stage one.

Thus the separation property holds in an amended form in the uncertainty case as well. The formulae we obtain apply to unregulated firms as well as regulated firms. Our cost of capital formula can be used for capital budgeting decisions as well as rate case calculations.

The advantage of our intertemporal general equilibrium rational expectations approach to this old set of questions over received literature is that we use recent advances in model construction technology in macroeconomics stochastic growth theory, and finance. This enables us to
relate the cost of capital to underlying preference for risk taking and technology, as well as the tax structure.

A sample of implications of our approach that we hope will entice the reader into this article are (1) a procyclical utility should receive a higher "fair" rate of return on its base than a countercyclical utility, (2) allowed rates of return should be higher in periods of high consumer pessimism (as measured by Katona's Index of Consumer Sentiment for example).

(3) The allowed rate of return should be lower the less costly it is for investors to apply the Miller-Scholes [14] shield to dividend income.

(4) The allowed rate of return should be lower if impediments to repurchase of corporate stock are removed. Hence the debate between Feldstein's group (e.g., Feldstein and Greene [11]) and Miller and Scholes [14] on dividends and taxes as well as the effectiveness of legal constraints on share repurchase is directly relevant to a correct calculation of the cost of capital.

(5) Market value is less than book value in a world with positive effective marginal tax rates that are less on capital gains than on ordinary income.

It should be pointed out that what we are doing here is to reorganize and reinterpret regulation and finance literature in a type of recursive equilibrium rational expectations framework much like that used by the modern macroeconomic rational expectations theorists. This exercise yields, for example, the formula (4.20) below. This formula which generates implications (1)-(5) above seems to be more precise than anything available in the current literature on the "correct" rate of return for a regulated firm.
It also should be pointed out that we are showing how the "r" in the standard Averch-Johnson literature (i.e., Takayama [20], Baumol-Klevorick [5], Elhodiri-Takayama [10], Spann [19]) is determined in the presence of taxes at the personal and corporate level, uncertainty, consumer tastes, and systematic risk as opposed to diversifiable risk. Readers of the A.J. literature should identify our "BR" with their "r" (e.g., Baumol-Klevorick [5]) and our "p" with their "s". Our "hairline formula" is their "s = r".

Before we begin we want to be specific about what it is that we think that we have to add to received A.J. literature.

Spann [19] studies a dynamic A.J. problem in the face of taxes. But he does not consider (i) uncertainty and (ii) personal taxes. Furthermore he does not show how the "cost" of capital facing the regulated firm (he calls it "i") is linked to tastes, technology, and the tax environment. We do this below. El-Hodiri-Takayama [10] neatly study a dynamic A.J. problem in the face of adjustment costs. But they do not allow for the presence of taxes and uncertainty. Furthermore they do not show how the cost of capital to the firm is determined by systematic risk of the firm, the tax environment, and the financial packaging by the firm. We do not, however, consider adjustment costs.

A recent thesis by McLeod [12] studies a dynamic A.J. problem under certainty but he does not show how the cost of capital to the firm is determined by tastes and taxes as we do. He does not study uncertainty either.
Furthermore none of the references in the aforementioned papers show how the cost of capital to the regulated firm is determined by rational expectations, investor tastes, technology and the tax environment. The finance literature does not seem to have attended to this task in a manner that integrates the A.J. literature with the recent literature on recursive general equilibrium theory (cf. Prescott-Mehra [16] and references).

Hence we believe that not only have we integrated received cost of capital literature with literature on recursive intertemporal general equilibrium rational expectations models (e.g., Prescott-Mehra [16], Sargent [18]) but we have also significantly extended the "A.J. literature."
2. The Model

In this section we construct a model for the pricing of utility stock. It will be the simplest model that can be constructed in order to balance generality and empirical relevance. We make the following assumptions.

There will be one consumer type and N firm types. Some firms will be competitive and others will be regulated. All will be producing the same product. The model can be easily extended to multiproducts by introducing multigood utility functions for the consumer and general production technologies. Consumers maximize the expected discounted sum of utilities subject to their intertemporal budget constraints; they are price-takers in all markets.

Firms are assumed to act to maximize stock market value. Each firm produces its good with a technology and technologies differ between competitive and regulated industries. Regulated firms face certain rules (e.g., fair rate of return on the rate base) that determine the price they may charge which is related to actions which they have taken at previous dates. For example if a firm builds up a larger rate base, it can subsequently charge a higher price.

Consider first consumers. Consumers maximize

\[
(2.1) \quad E_{\ell} \sum_{t=0}^{t=\infty} B^{t-1} u(c_t, \ell_t, M_t / P_t)
\]

s.t.
\[(2.2) \quad c_{t+1} + \sum_{i=1}^{N} p_{i,t+1}(e_{i,t+1} - e_{i,t}) + q_{t+1} b_{t+1} + (M_{t+1} - M_t)/p_{t+1} \]

\[= \sum_{i=1}^{N} d_{it} e_{it} + w_{t+1} l_{t+1} + (b_t p_t/p_{t+1}) - \tau_h, t+1 + \tau_r, t+1. \]

(All symbols are defined in the list at the end of the paper.) When such a distinction is required, nominal variables are upper case and real variables are lower case letters. Equation (2.1) says that consumers maximize the present value of utility, which increases with consumption and real money holdings and decreases with labor. The discount factor, \( \beta \), is 1 divided by \( \ell \) plus the discount rate. Equation (2.2) is the household intertemporal budget constraint: next period, consumption plus increases in equity, bond and money holdings must equal dividends paid at the end of this period plus next period wages, real bonds maturing and net transfers from the government. In our framework, a bond \( b_{it} \) is purchased by a consumer from firm \( i \) at price \( q_t \) in period \( t \) and it yields \$1 in period \( t+1 \).

The government collects taxes, issues money, and redistributes the proceeds to the public lump-sum. The government budget constraint is given by

\[(2.3) \quad (M^S_t - M^S_{t-1})/p_t + \tau_{ht} + \sum_{i=1}^{N} \tau_{it} = \tau_t. \]

This constraint states that all transfers to the public equal new money creation plus household taxes plus taxes paid by all \( N \) firms.
An equilibrium is a sequence $X$ of stock prices, bond prices, allowed price functions and wage rates such that when both sides of the market optimizes against $X$, all markets clear: i.e., the demand for equities equals the supply of equities; labor demand equals labor supply; the aggregate demand for bonds equals zero; and the demand for goods equals the supply of goods. Our concept of equilibrium is one of "rational expectations."

The marginal necessary conditions for consumer equilibrium are:

\begin{align*}
(2.4) \quad D_{ct} u_{it} &= \beta E_t \{ D_{ct+1} u_{[p_{i,t+1} + d_{it} - D_{eit} \tau_{h,t+1}]} \\
(2.5) \quad D_{ct} u_{q_t} &= \beta E_t \{ D_{ct+1} u_{[(P_t/P_{t+1}) - D_{bt} \tau_{h,t+1}]} \\
(2.6) \quad D_{ct} u(\omega_t - D_{it} \tau_{h,t}) &= -D_{ct} u \\
(2.7) \quad (D_{ct} u - D_{mt} u)/P_t &= \beta E_t \{ D_{et+1} u/P_{t+1} \}.
\end{align*}

Equation (2.4) states that the expected present value of equities held (expected future price plus dividends less the effect of taxes) will equal the marginal utility of present consumption. The next equation states that the expected present value of future utility derived from bonds must equal the marginal utility of bonds this period. The third equation is a labor market clearing condition. The last equation relates money markets and equity markets.

The conditions above look complicated. To illustrate the simplicity of the underlying structure, consider the special case of no taxes, no
nominal balances, no bonds, no regulated firms, and no government. In this case following (2.4) - (2.7), we get:

\[(2.4a) \quad D_{c_t} u_{it} = \beta E_t \{(D_{c_{t+1}} u)(p_{i,t+1} + d_{it})\}\]

\[(2.6a) \quad D_{c_t} u_{wt} = -D_{t} u.\]

The Value of the Firm

In general, bonds plus elementary securities issued by firm \(i\) must be added to "equity value" \(e_{it} p_{it}\) in order to get \(v_{it}\); but for now, we consider only the simplest case of no bonds and all equities. Thus, the value of the firm \(v_{it}\) can be written:

\[(2.8) \quad v_{it} = e_{it} p_{it}.\]

Multiply both sides of (2.4) by \(e_{it}\) to get

\[(2.9) \quad D_{c_t} u v_{it} = \beta E_t \{(D_{c_{t+1}} u)(e_{it} d_{it} + e_{it} p_{i,t+1} - e_{it,t+1} p_{i,t+1} + v_{i,t+1})\}\]

\[(2.10) \quad p_{i,t+1} (e_{i,t+1} - e_{it}) + g_i (x_{it}, \ell_{it}, \eta_{it}) - w_{t+1} \ell_{i,t+1}
= x_{i,t+1} - x_{it} + \xi_{it} x_{it} + e_{it} d_{it}.\]

Equation (2.9) states that the expected marginal utility of the future value of equities equal today's marginal utility of equities. Equation (2.10) is the corporate budget constraint: it states that money inflows from the sale of new securities plus net income this period \((g - w\ell)\) equals net cash outflows. The latter equals gross investment plus dividends paid out. Substitute for \(e_{it} d_{it}\) from (2.10) into (2.9) to get
\[
(2.11) \quad D_{c_t}^u v_{it} = \beta E_t \{ D_{c_{t+1}}^u (v_{i,t+1} + \pi_{i,t+1} - o_{i,t+1}) \}
\]

where \( \pi \) is profits and \( o \) is gross investment

\[
(2.12) \quad \pi_{i,t+1} \equiv g_i - w_{t+1} \ell_{i,t+1}
\]

\[
(2.13) \quad o_{i,t+1} \equiv x_{i,t+1} - x_{i,t} + \xi_i x_{i,t}.
\]

Equation (2.11) parallels equation (5) on page 414 of Modigliani and Miller [15] in which they show that if \( \{\pi_{it}\}, \{x_{it}\} \) are real decisions given independently of financial policy, then (2.11) can be used to demonstrate that firm valuation \( \{v_{it}\} \) is independent of dividend policy. It is important to note that the proof is not valid if financial policy effects \( \{\Gamma_{t, t+1}\} \) defined below or if dividend policy affects the limit defined in (2.18) below.

Set

\[
(2.14) \quad \Gamma_{t, t+1} = \beta D_{c_{t+1}}^u u/D_{c_t}^u u,
\]

which is the present value of the ratio of the marginal utility of consumption next period to the marginal utility of consumption this period. Also set

\[
(2.15) \quad \nu_{i, t+1} = \pi_{i, t+1} - o_{i, t+1}
\]

which is simply the net cash flow for firm \( i \) in period \( t+1 \). Thus,

\[
(2.16) \quad v_{it} = E_t \{ \Gamma_{t, t+1} (v_{i,t+1} + \pi_{i,t+1} - o_{i,t+1}) \}.
\]

Run (2.16) forward to get
\( v_{i1} = \mathbb{E}_1[\Gamma_{12} n_{i12}] + \mathbb{E}_1[\Gamma_{12} \Gamma_{23} n_{i13}] + \ldots + \mathbb{E}_1[\Gamma_{12} \ldots \Gamma_{T-1} T n_{i1T}] + \mathbb{E}_1[\Gamma_{12} \ldots \Gamma_{T-1} T v_{i1T}]. \)

Hence if

\[ (2.18) \quad \mathbb{E}_1[\Gamma_{12} \ldots \Gamma_{T-1}, T v_{i1T}] = 0, \quad \text{T} \to \infty \]

then the valuation formula

\[ (2.19) \quad v_{1t} = \mathbb{E}_1[\Gamma_{12} n_{i12}] + \mathbb{E}_1[\Gamma_{12} \Gamma_{23} n_{i13}] + \ldots \]

results. Thus it is clear that if \( \{\pi_{1t}\} \) satisfies

\[ (2.20) \quad \pi_{1t} = \Gamma_{12} \ldots \Gamma_{t-1}, t. \]

Then value maximization by firms leads to maximization of the expected value of the discounted sum of net cash flow where the discounts are given by (2.14). It is clear also that if firms, given capital stock \( x_{i1} \), in period 1, maximize \( v_{i1} \) over \( \{x_{1t}\}_{t>1} \) where \( v_{i1} \) is given by (2.16) then the maximum amount of utility is produced in equilibrium. This is so because maximization of \( v_{i1} \) where \( v_{i1} \) is given by (2.16) leads to the same necessary conditions as the growth problem analyzed in (2.21) below by Brock (1978):

\[ (2.21) \quad \text{Maximize} \; \mathbb{E}_1 \left\{ \sum_{t=1}^{\infty} \beta^{t-1} u(c_t, \ell_t) \right\} \]

\[ (2.22) \quad c_{t+1} + x_{t+1} - x_t = \sum_{i=1}^{N} (g_i(x_{1t}, \ell_{t+1}, v_{i1}) - \xi_i x_{it}) \]

\[ (2.23) \quad \sum_{i=1}^{N} x_{it} = x_t. \]
Now how is one to use (2.17) in practice? Banz and Miller [4] and Breedan and Litzenberger [6] approximate the analogue of it in their models by considering models in which the state of the world at date \( t \) may be summarized by the value of an aggregate such as the market portfolio at date \( t \). Then they introduce securities that pay unity if a particular range of values is assumed by the market portfolio and zero otherwise. Using option valuation formulae, they value these "elementary" securities and thus, approximate (2.17). If a complete set of elementary securities existed, i.e., one for each state of the world, then the elementary security price must satisfy

\[
(2.24) \quad p_t(s_{t+1})\Gamma_{t,t+1} = q_t(s_{t+1})
\]

so that (2.17) is given by

\[
(2.25) \quad v_{i1} = \sum_{s_2} q_1(s_2) n_{i2}(s_2) + \sum_{s_2, s_3} q_1(s_2) q_2(s_3) n_{i3}(s_2, s_3) + \ldots
\]

Hence a better approximation to (2.17) is given the "finer" the set of elementary securities that is used.

In many applications \( \Gamma_{t,t+1} \) can be written as a function of a low dimensional state variable call it \( y_{t+1} \). If this function is expanded in a Taylor series about its mean then an approximation to \( \Gamma_{t,t+1} \) results. For some applications the mean and covariance of \( \Gamma_{t,t+1} \) with this variable would give a good approximation.

An empirical proxy for the mean of \( \Gamma_{t,t+1} \) could be the price of an index bond that promises to deliver one unit of purchasing power next period independent of the state of the world. The return on such a bond
is defined to be

\begin{equation}
\frac{1}{q_t} = 1 + r_{Ft}
\end{equation}

where \( r_{Ft} \) denotes the risk free rate of return.

An empirical proxy for the covariance of \( \Gamma_{t,t+1} \) with \( y_{t+1} \) could be built up from the price of an index fund and the bond mentioned above provided that \( y_{t+1} \) was the market portfolio. This approach to operations is discussed in Brock [7].

Formulae similar to (2.17) turn up in the case of taxes as we shall see in the next section.
3. The Case of Taxes

We now examine the model when tax distortions are present. Let us use (2.4) and (2.5) to get an expression for the value of firm i. (Since the previous section and the list of symbols at the end of the paper clarifies which variables were firm-specific we drop the firm subscripts in this section.)

\[(3.1) \quad v_t \equiv p_t e_t + q_t b_t.\]

Multiply (2.4) by \(e_t\), (2.5) by \(b_t\) and add to get

\[(3.2) \quad D_{c_t} uv_t = \beta E_t \{D_{c_{t+1}} u(p_{t+1} e_t + e_t d_t - D_{e_t} \tau_{h,t+1} e_t \]
\[\quad \quad \quad + b_t (p_t / p_{t+1}) - D_{b_t} \tau_{h,t+1} b_t)\}
\[\quad = \beta E_t \{D_{c_{t+1}} u(v_{t+1} - p_{t+1} e_{t+1} - b_{t+1} q_{t+1} \]
\[\quad \quad \quad + p_{t+1} e_t + e_t d_t - D_{e_t} \tau_{h,t+1} e_t \]
\[\quad \quad \quad + b_t (p_t / p_{t+1}) - D_{b_t} \tau_{h,t+1} b_t)\}.\]

Substitute the expression for pre-tax income flow to stockholders

\[(3.3) \quad r_{t+1} \equiv e_t d_t + p_{t+1} (e_t - e_{t+1}) + b_t (p_t / p_{t+1}) - q_{t+1} b_{t+1},\]

and use the definition of the firm's budget constraint to get from (3.2):

\[(3.4) \quad v_t = E_t \{\Gamma_{t,t+1} (v_{t+1} - D_{e_t} \tau_{h,t+1} e_t - D_{b_t} \tau_{h,t+1} b_t \]
\[\quad \quad \quad + a_{t+1} g(x_t, \ell_{t+1}, v_t) - w_{t+1} \ell_{t+1} - (x_{t+1} - x_t + \xi_t x_t) - \tau_{f,t+1}\}.\]
Equation (3.4) is a "no arbitrage profits" condition. It says that no money can be made by stockholders and bondholders by purchasing the firm at date \( t \) and selling it at date \( t+1 \) after paying taxes. That is, the task of management is broader than maximizing the after corporate tax discounted cash flow: it is to package receipts so as to maximize the discounted cash flows net of all taxes.

In the case of no taxes, (3.4) can be used to show that the value of a firm is independent of its dividend policy and of its debt structure as measured by \( \{b_t, e_t\} \).

As an aside, it is important to note that in the presence of taxes that maximization of firm valuation will not lead to maximization of equilibrium consumer welfare. The proper way to formulate that particular problem would be to calculate equilibrium expected utility for each firm decision rule and then maximize equilibrium expected utility over a well specified set of decision rules. This leads to a class of optimal tax problems of the Ramsey type which we wish to avoid here. The use of stock market value as a measurable index of performance that has some support from survival arguments. I.e., if management does something blatantly different from maximizing stock market value, its tenure may be threatened.

In fact, the "obvious" way of proceeding, that is, by maximizing (3.4) subject to the firm's budget constraint, is not the best way to proceed since the value of a firm is a convex combination of equity value and bond value. Hence, firm value appears on the R.H.S. of equation (3.4) as well as the L.H.S. This creates a "consistency" problem.
We will attack the problem of choosing financial policy to maximize
(3.4) in the following way. First, we define

\[ \lambda_t \equiv q_t b_t / p_t e_t \]

\[ v_t \equiv q_t b_t + p_t e_t \]

and express the R.H.S. (3.4) in terms of the debt/equity ratio \( \lambda_t \), \( v_t \) and
physical quantities. We will do this by using the necessary conditions of
consumer equilibrium for equities and bonds.

Second, we will observe that because ordinary income tax rates are
larger than capital gains rates, the firm will attempt to minimize dividend
payout and repurchase shares, other things equal. But there are legal
restraints (Section 302 of the Tax Code) on share repurchase in lieu of
dividend payout. Hence the firm will attempt to convert ordinary income
into capital gains through equity repurchase, subject to this constraint.

Third, given the representation of the constraint, we will use it to
substitute for the \( d_t e_t \) terms in the R.H.S. of (3.4) obtained from the
first and second steps above. Then we will recurse (3.4) forward to
develop a present value expression for \( v_{it} \).

Fourth we will choose \( \{ \lambda_t \}_{t=1}^{\infty}, \{ x_t \}_{t=2}^{\infty} \) to maximize the present value
expression. At each date \( t \), specialization in bond finance or equity
finance will take place. This is related to Miller [13] and is explained
by the absence here of heterogeneous households (which would admit
clientele effects) and of significant bankruptcy costs. Since taxes are
an important part of our model, the specialization result is important in
revealing the conditions under which firms lean toward debt finance or
equity financing because of tax considerations. Most importantly, while
the bang-bang formulae for optimal regulation are affected by the form of
financing, the bang-bang result itself holds irrespective of the debt/equity
decision.

Since the derivation outlined in the preceding four steps is messy,
we introduce the following notation. Let

\[(3.7) \quad \Gamma_{t, t+1} \equiv \Gamma_{t+1}.\]

The consumer's necessary conditions of equilibrium for equities and bonds
may be written,

\[(3.8) \quad p_t = E_t \{ \Gamma_{t+1} [p_{t+1} (1 - 2 \alpha_{t+1}) + d_{it} (1 - \alpha_{t+1}) + 3 \alpha_{t+1} p_{t+1}] \} \]
\[(3.9) \quad q_t = E_t \{ \Gamma_{t+1} [1 - m_{t, t+1} (1 - q_t)] / (1 + i_{t+1}) \} \]
\[(3.10) \quad \lambda_{t+1} = m_{t, t+1} \phi_{t+1}, \quad 2 \alpha_{t+1} = m_{t, t+1} \gamma_{t+1}, \quad 3 \alpha_{t+1} = 2 \alpha_{t+1} / (1 + i_{t+1}).\]

Solving (3.9) for \(q_t\) yields

\[(3.11) \quad q_t = (1 - E_t \{ \Gamma_{t+1} m_{t, t+1} / (1 + i_{t+1}) \})^{-1} E_t \{ \Gamma_{t+1} [1 - m_{t, t+1}] / (1 + i_{t+1}) \}.\]

Corporate taxes, by definition, equal

\[(3.12) \quad \tau_{f, t+1} = \tau \{ [b_t (q_t - 1) / (1 + i_{t+1})] + a_{t+1} g(x_t, z_{t+1}, \eta_t) - w_{t+1} \ell_{t+1} - \frac{\ell_t x_t}{(1 + i_{t+1})} - \gamma_t (x_t, x_{t+1}) \}.
\]

Here \(\tau_f\) is a constant, the corporate tax rate. The first term is a credit
for bond interest which the firm pays, expressed in real terms. Since it
appears on the right hand side of the tax equation, it is negative. The
last term in the equation is the investment tax credit. Equation (3.4) may be rewritten thus

\[
(3.13) \quad \nu_t = E_t \{ f_{t+1} [\nu_{t+1} - i_{t+1} q_t e_t + \alpha_{t+1} p_t e_t + B_t q_t b_t
\]
\[ - 2 \alpha_{t+1} p_{t+1} e_t + \phi_{t+1} ] \}
\]

where

\[
(3.14) \quad D_{b_t} = \frac{T_h, t+1}{b_t}
\]

minus corporate tax credit for bond interest = total taxes induced by bonds

\[
(3.15) \quad B_t \equiv TB_{t+1} / (q_t)
\]

\[
(3.16) \quad \phi_{t+1} \equiv (1 - \bar{f}) [a_{t+1} g(x_t, \ell_{t+1}, \nu_t) - w_{t+1} \ell_{t+1}
\]
\[ + \frac{e_t x_t \bar{f}}{1 + i_{t+1}} + \gamma_t (x_t, x_{t+1}) - [x_{t+1} - x_t + \xi_t x_t].
\]

Equation (3.16) is a representation of the after-corporate-tax real cash flow of the firm. It simply equals after-tax income plus the tax value of depreciation plus the investment tax credit less net investment. Equation (3.13) is gotten from (3.4) by using

\[
(3.17) \quad D_{e_t} = \frac{T_h, t+1}{m_h, t+1} \phi_{t+1} d_t + m_h, t+1 \theta_{t+1} [p_{t+1} - p_t / (1 + i_{t+1})]
\]
\[ = \alpha_{t+1} d_t - \alpha_{t+1} p_t + \alpha_{t+1} p_{t+1}.
\]

It is useful to write the corporate budget constraint in the following notation:
\[(3.18) \quad e_t d_t = p_{t+1}(e_{t+1} - e_t) + q_{t+1} b_{t+1} - \frac{b_t}{(1+i_{t+1})} - \frac{\tau(b_t(q_t - 1)/(1+i_{t+1}))}{(1+i_{t+1})} + \phi_{t+1} \cdot \]

Equation (3.18) indicates that the value of dividends to the firm equals income from the sale of equities plus income from the sale of bonds less the payout made to maturing bonds plus the reduction in corporate taxes due to the deductibility of interest plus after-tax real cash flow. The effect of bonds on marginal taxes is:

\[(3.19) \quad D_b \tau_{h,t+1} = ((1-q_t)/(1+i_{t+1}))m_{h,t+1}. \]

One can write the R.H.S. of (3.13) in terms of \(\lambda_t, v_t, v_{t+1} \) by attempting to solve for \(q_t\) and \(p_{t+1}\) from (3.8) and (3.9) respectively.

In order to get \(p_{t+1}\) out of the R.H.S. of (3.13) we need the crucial Assumptions 3.1 (i). The quantity \(2\alpha_{t+1} = 0\) or (ii) \(e_t\) is required to be equal to \(e_{t+1}\) or (iii) One can solve (3.8) for \(E_t \Gamma_{t+1} P_{t+1}.\)

Assumption 3.1 is needed to solve (3.8) for \(E_t \{\Gamma_{t+1} P_{t+1}\}\) in terms of \(p_t\) so that the R.H.S. of (3.13) may be written in terms of \(v_t, v_{t+1}, d_t, \lambda_t.\) After this is done the four-step procedure outlined earlier in this section may be applied. The only way we have found to solve the problem of maximizing equilibrium firm value in general without Assumption 3.1 involves the messy procedure of solving T period problems and taking T to infinity to get the infinite horizon result.

One case in which the R.H.S. of (3.13) can be conveniently reduced is that of a zero capital gains tax \(2\alpha_{t+1} = 0.\)
This case is not as ridiculous as it sounds because capital gains are not taxed until they are realized. Hence the effective rate of tax on capital gains is much smaller than the realized rate. In this case (3.13) becomes

\[ (3.13') \quad v_t = E_t \{ \frac{\Gamma_{t+1}}{\lambda_t} [v_{t+1} - \lambda_t \alpha_{t+1} d_t e_t + B_t v_t (\frac{\lambda_t}{1 + \lambda_t} + \phi_{t+1})] \}. \]

Hence

\[ (3.20) \quad v_t [1 - (E_t \Gamma_{t+1} B_t) (\lambda_t/(1+\lambda_t))] = E_t \{ \frac{\Gamma_{t+1}}{\lambda_t} [v_{t+1} - \lambda_t \alpha_{t+1} d_t e_t + \phi_{t+1}] \} = v_t C_t (\lambda_t). \]

Formula (3.20) reveals two principles. First, if the ordinary income tax rate is greater than the effective tax rate in capital gains, the firm should minimize dividend payout (taking into account institutional restraints). This is an old result. Second, at each date and for each firm, the debt-equity ratio \( \lambda_t \) should be chosen to solve

\[ (3.21) \quad \text{Minimize } C_t (\lambda_t) \]

for the case in which institutional restraints permit \( d_t = 0 \). Usually, however, \( d_t e_t \) is constrained to be positive by law when there is positive net cash flow available for distribution for the stockholders, (i.e., \( \phi_{t+1} > 0 \)). When this constraint is substituted into (3.13), we obtain a development of the form

\[ (3.22) \quad v_t C_t(\lambda_t) = E_t \{ \frac{\Gamma_{t+1}}{\lambda_t} [v_{t+1} A'_{t+1} + A''_{t+1} \phi_{t+1}] \}, \]
provided that $d_t e_t$ is constrained to be some exogenously given fraction of after tax corporate net cash flow. The multipliers $A'_{t+1}, A''_{t+1}$ are constants. Here, too $\lambda^*_t$ is chosen to minimize $C'_t(\lambda_t)$. Then the investment sequence $\{x_t\}_{t=1}^\infty$ is chosen to maximize the resulting present value expression.

See Brock-Turnovsky [8] for the same kind of derivation carried out in the deterministic case.

The family of formulae (3.13), (3.20), (3.22) are related to the recent outpouring of NBER work on taxation (e.g., Auerbach [1], Auerbach-King [2]). Our work should be viewed as an attempt to integrate the existing taxation literature with the macro-finance literature on rational expectations (cf. Brock [7], Prescott-Mehra [16], Danthine-Donaldson [9]) and the A.J. literature. We hope to gain a better understanding of the A.J. firm by embedding it in such models.
4. The Bang-Bang Result

In this section, we use the model developed in the previous sections to describe the effects of regulation of an individual utility which is assumed to be small relative to the economy as a whole. Over time, the size of the regulated firm and the quantity of services which it provides will be determined by (1) stock traders forming expectations on future cash flows and trading stock; (2) consumers and utilities gaming the regulators and vice-versa; (3) stock market traders forming expectations on the effect of the gaming on future cash flows and hence future stock values; and (4) stock market clearing. While we cannot solve all of these problems, we attack most of them explicitly. In particular, we attack a dynamic version of the Averch-Johnson [3] problem. First, we assume that regulatory agencies independently set a rate of return on the utility's capital; the rate set cannot be affected by the behavior of the utility's management or by consumers. Second, given this return, we assume that the utility managers invest capital to maximize its market value: that is, they act in their shareholder's interest. If the utility can profit by building up its rate base, we assume it will do so. Third we assume that demand is infinitely inelastic. Based upon these assumptions, we obtain two results.

The first is a bang-bang result: unless the regulators set the allowed rate of return properly, the optimal investment policy of the utility will lead it to invest very little (undercapitalize) or invest too much (overcapitalize). Mathematically, the utility will implode or explode. The result holds assuming perfectly inelastic demand for the
utility's output. Obviously the result does not hold for the utility sector as a whole. This is so because all utilities could not expand or contract without affecting the rate of return to capital. The rate of return to capital is parametric to an individual utility. We are concentrating on the problem facing an individual utility that is assumed to be atomistic relative to the whole economy. In this sense we are still doing partial equilibrium analysis.

Second, the rate of return which regulators should apply to a particular regulated firm to avoid the bang-bang extremes (the "bang-bang" rate of return, BR) is affected by most, though not all, important general equilibrium variables and policies: intertemporal consumer preferences (the yield curve), discount rates, and most tax rates. BR is also dependent upon firm-specific variables such as the depreciation rate of the firm's capital. If regulators do not adjust the allowed rate of return in response to changes in BR, regulation can have unfortunate consequences.

**Case 1: The Simplest Case**

We will work on the simplest possible case first: no taxes, no money, no government, etc. At the beginning of period $t+1$, the rate base for a regulated firm is defined by

$$ rb_{t+1} = (1 - \xi_t) x_t, $$

i.e., the undepreciated capital stock. In this section we continue to omit the firm subscripts. Regulators are assumed to allow a firm-specific rate of return $\rho_{t+1}$ at date $t+1$ on the base $rb_{t+1}$. The sequence $\{\rho_t\}$ is
assumed to be exogenously given so that it is outside of the control of the utilities and of consumers. We are interested in solving for the sequence of regulated returns $\rho_t$ which would generate behavior by the utility which is equivalent to "perfectly competitive behavior."

Assume that the price of the regulated firm's output in period $t+1$, $a_{t+1}$, satisfies the Averch-Johnson [3] constraint:

$$rb_{t+1} \rho_{t+1} \equiv (1 - \xi_t)x_t \rho_{t+1} \geq a_{t+1} g(x_t, \ell_{t+1}, v_t) - w_{t+1} \ell_{t+1} - \xi_t x_t.$$  

Recall that $g$ is the production function which is a function of capital $x_t$, variable factors $\ell_{t+1}$ which are hired after capital is in place and after the random shock $v_t$ is revealed. The capital $x_t$ is installed before $v_t$ is revealed. Thus, the regulated firm's net revenue cannot exceed the allowed return on the rate base.

In the unregulated case $a_{t+1} = 1$, since this is a one-good model.

In a two-good model with the first good as a competitively produced numeraire, $a_{t+1}$ in terms of the numeraire would be the marginal rate of substitution (the marginal utility of the second good divided by the marginal utility of the numeraire good).

It is not necessary to develop a complete general equilibrium treatment of multiproduct demand in deriving the bang-bang result. But it is important that we show how the regulated firm copes with the bang-bang phenomenon under alternative assumptions about product demand. Unless specified otherwise, capital in the model is putty-putty: it can be unbckcled and transferred to alternative uses costlessly.
Perfectly Inelastic Demand

If demand for the output of regulated firms is totally unresponsive to price, we write

\[ g(x_t, \ell_{t+1}, v_t) = Q_{t+1} \]

where \( Q_{t+1} \) is the level of the regulated firm's output demanded by consumers in period \( t+1 \). The regulated firm will maximize

\[ v_1 = \Gamma_2 (a_2 g_2 - w_2 \ell_2 - (x_2 - x_1 + \xi_1 x_1) \]

\[ + \Gamma_2 \Gamma_3 (a_3 g_3 - w_3 \ell_3 - (x_3 - x_2 + \xi_2 x_2) + \ldots \]

subject to (4.2) and (4.3). Recall that \( \Gamma_t \) is a random discount which is the marginal rate of substitution between date \( t-1 \) and date \( t \) goods. The solution requires that for any \( x_t \), \( Q_{t+1} \), there is an \( \ell_{t+1} \) such that (4.3) holds; this rules out Leontief fixed-coefficient production. Since \( x_1 \) is exogenous (historically given) and \( Q_2, Q_3, \ldots \) are determined independently by consumers, the sequence \( \ell_2, \ell_3, \ldots \) is fixed by

\[ Q_2 = g(x_1, \ell_2, v_1) \]
\[ Q_3 = g(x_2, \ell_3, v_2) \]
\[ \vdots \]
\[ Q_t = g(x_t, \ell_{t+1}, v_t). \]

Obviously there are more degrees of freedom the larger the number of variable factors. Finally, note that maximization of stock market value by the utility in (4.4) subject to meeting market demand in (4.3) implies that the regulated firm hires variable factors \( \ell_t \), so that the Averch-Johnson constraint (4.2) holds with equality.
When (4.2) and (4.3) are inserted into (4.4), we find that the regulated firm will maximize

\[
(4.6) \quad v_1 = \Gamma_2 \{(1 - \xi_1) x_1 \rho_2 - (x_2 - x_1) + \Gamma_2 \Gamma_3 \{(1 - \xi_2) x_2 \rho_3 - (x_3 - x_2) + \ldots\}.
\]

This completes the proof of the bang-bang result. Notice that the value of the firm is a linear function of the rate base, \(\{x_t\}\), so that except for hairline cases, the regulated firm will set \(x_t\) equal to either zero or plus infinity. If regulators set the rate of return properly to avoid the extremes of explosion or implosion (i.e., the bang-bang rate of return, \(BR\)), this rate equals

\[
(4.7) \quad 1 = E_t\{[BR_{t+1} (1 - \xi_t) + 1]\}, \quad t = 1, 2, \ldots.
\]

The reason for the "bang-bang" result is linearity of the rate base and, hence, linearity of allowed income in \(\{x_t\}\). If the firm increases capital \(x_t\) by one unit today, it earns \(\rho\) on this capital and still has \(1 - \xi\) left over. In equilibrium, the discounted value of these must equal 1, the opportunity cost of the machine. Otherwise, arbitrage opportunities are possible.

If regularity return is set below the bang-bang rate (\(\rho < BR\)), the firm will remove the capital in a putty-putty world. In the more realistic putty-clay case, the utility will let is rate base contract at the rate of physical depreciation each period.

Notice that if \(rb_{t+1} = x_t\) then depreciation factor \(\xi_t\) does not enter (4.7) because depreciation is "fully covered" by the rate-making
machinery in this case. Formulae of type \((4.7)\) may be worked out for different definitions of \(rb_{t+1}\) following the procedure laid out in this paper.

A related procedure as above holds when demand is responsive to price. Assume that we are in a putty-putty world.

If \(\rho_{t+1}\) is set so that

\[
(4.7a) \quad 1 > E_t\{\Gamma_{t+1}(\rho_{t+1}(1 - \xi_t) + 1)\}
\]

then the maximum income that the firm can obtain from an extra unit of capital installed at date \(t\) will not cover its cost. This is because the A-J constraint \((4.2)\) is binding in a putty-putty world when \((4.7a)\) holds. Hence a rational firm in the face of zero adjustment cost (or covered adjustment cost) will contract its rate base at date \(t\). Hence the "contraction part" of the bang-bang result holds regardless of whether demand is perfectly elastic or not.

If the firm is required to meet demand it will do so by hiring variable factors.

It is when

\[
(4.7b) \quad 1 \geq E_t\{\Gamma_{t+1}(\rho_{t+1}(1 - \xi_t) + 1)\}
\]

that demand considerations must be taken into account.

We divide the analysis into two cases. Case (i) the gross revenue function:

\[
R(Q) = D(Q)Q
\]

is unbounded. Case (ii) the gross revenue function is bounded.
In case (i) revenues can be made as large as you please by suitably adjusting quantity. Hence the A-J constraint (4.2) will always be binding when (4.7b) holds. Hence the firm will expand when (4.7b) holds. It is important to notice that in the case where demand is inelastic that expansion will be accomplished by expanding capital, shrinking variable factors, shrinking quantity and raising price. It is unlikely that any commission will allow such a scenario to be carried out in actual practice. The bang-bang formula is only intended to uncover central tendencies. I.e., we are calculating "r" in Baumol-Klevorick [5] language. If (4.7a) holds, i.e., "s < r" in Baumol-Klevorick [5] language the utility goes out of business.

In case (ii) a careful study of the A.J. problem along the lines of El-Hodiri-Takayama [10] must be undertaken in the presence of uncertainty and taxation. This is beyond the scope of the present paper.

Nevertheless just as "s ≥ r" (Baumol-Klevorick [5], Takayama [20]) must hold for the utility to voluntarily remain in business so also must (4.7b) hold. Our contribution in the above section and the sections below is to show how the viability inequality "s ≥ r" must be modified in the presence of taxes and uncertainty as well as showing how it is influenced by macro variables such as the price of systematic risk.

**Case 2: Corporate Taxes**

We now modify case 1 to include corporate taxes. We continue to assume that the firm is all equity financed and that all after-tax corporate income is paid out as dividends. However, no personal taxes are levied on these dividends. We assume that regulators adjust the price of
regulated output so that the rate of return on undepreciated capital covers variable costs, depreciation and corporate taxes:

\[ r_b(t+1) = (1 - \xi_t)x_t \rho_{t+1} \]

\[ \geq a_{t+1}g() - w_{t+1}d_{t+1} - \xi_t x_t \]

\[ - \tau_{t+1}^{-F}(a_{t+1}g() - w_{t+1}d_{t+1} - \xi_t x_t) \]

\[ = (1 - \tau_{t+1}^{-F})(a_{t+1}g() - w_{t+1}d_{t+1} - \xi_t x_t). \]

Asset markets will evaluate the utility at the beginning of period t as the expected value, properly discounted of the firm's value at the beginning of period t+1 plus dividends earned during period t.

\[ v_t = E_t^t [\Gamma_{t,t+1}(v_{t+1} + \phi_{t+1})] \]

Equation (4.10) simply reflects our assumption that all after-tax corporate earnings are paid out as dividends and that no shares are repurchased by the firm. Recall from equation (3.16) that \( \phi_{t+1} \) is defined as follows:

\[ \phi_{t+1} = (1 - \tau_{t+1}^{-F})(a_{t+1}g() - w_{t+1}d_{t+1} + \xi_t x_t) - (x_{t+1} - x_t) \]

\[ = (1 - \tau_{t+1}^{-F})(a_{t+1}g() - w_{t+1}d_{t+1} - \xi_t x_t) - (x_{t+1} - x_t). \]

Notice, however, that the first term in equation (4.11) simply equals the allowed rate of return set by the regulators in equation (4.9). Thus, we can write after corporate tax net cash flow as follows:
\( (4.12) \quad \phi_{t+1} = (1 - \xi_t) x_t \rho_{t+1} - (x_{t+1} - x_t). \)

Finally, the valuation of the firm can be written as follows:

\( (4.13) \quad v_t = E_t \{ \Gamma_{t+1} [v_{t+1} + (1 - \tau^{-F})(a_{t+1} g(\cdot) + w_{t+1} \ell_{t+1} - \xi_t x_t) - (x_{t-1} - x_t)] \}

= E_t \{ \Gamma_{t+1} [v_{t+1} + (1 - \xi_t) x_t \rho_{t+1} - (x_{t+1} - x_t)] \} \}.

This valuation formula results in the hairline case of perfect regulation defined (4.14):

\( (4.14) \quad E_{t+1} \{ \Gamma_{t+2} [(1 - \xi_{t+1}) \rho_{t+2} + 1] \} = 1. \)

The important result in equation (4.14) is that the corporate income tax rate does not affect the rate of return which should be set on public utilities. Thus, if the corporate tax rate were doubled or halved, this would still not affect the utilities' decision of whether or not to expand or contract during the period. The reason is that the rate allowed public utilities covers all of its corporate taxes so that a change in the corporate tax level cannot affect the investment decision.

**Case 3: Corporate and Personal Taxes**

We assume the presence of both personal income taxes \( (1 \alpha > 0) \) and capital gains taxes \( (2 \alpha > 0) \) as well as the corporate taxes considered in the previous section. We continue to assume that the firm is all equity financed so that all after-tax income is paid out as dividends (there is no share repurchasing). Regulators continue to behave as postulated in
equation (4.9), namely, they set the price of utility services such as to yield a return on the rate base which covers labor cost, depreciation and corporate taxes. The valuation formulae of the firm are now more complex. Consider first capital gains. The capital gains tax rate, \( 2^\alpha_{t+1} \), is applied to the nominal value of the change in the equity price. Since \( p_t \) is a real price, we must adjust by the change in the aggregate price level (which is denoted \( Q_t \)). Capital gains taxes at date \( t+1 \) on the change in the value of equities from \( t \) to \( t+1 \) in real terms can be written as follows:

\[
(4.15) \quad \text{cgt}_{t+1} = (2^\alpha_{t+1}(p_{t+1} Q_{t+1} - p_t Q_t)/Q_{t+1})e_t \\
= 2^\alpha_{t+1}[p_{t+1} - p_t/(1+i)]e_t \\
= 2^\alpha_{t+1} v_{t+1} - 3^\alpha_{t+1} v_t
\]

where, it will be recalled, \( 3^\alpha \) are real capital gains taxes. We can now rewrite equation (4.10): after-tax corporate cash flow is distributed as dividends and taxed at ordinary tax rates \( 1^\alpha_{t+1} \) and capital gains taxes can be written according to (4.15). Thus the value of the firm is

\[
(4.16) \quad v_t = E_t [\Gamma_{t+1}(v_{t+1} + (1-1^\alpha_{t+1})\phi_{t+1} + 3^\alpha_{t+1} v_t - 2^\alpha_{t+1} v_{t+1})].
\]

For convenience, we define

\[
(4.17) \quad T_t \equiv (1 - E_t \Gamma_{t+1} 3^\alpha_{t+1}).
\]

Thus,

\[
(4.18) \quad v_t = E_t [\Gamma_{t+1}(v_{t+1} + (1-1^\alpha_{t+1})\phi_{t+1} - 2^\alpha_{t+1} v_{t+1})]/T_t.
\]
As before, we can replace \( \phi_{t+1} \) with (4.12) so that

\[
(4.19) \quad v_t = E_t \left[ \Gamma_{t+1} (v_{t+1} + (1 - \alpha_{t+1}) \{(1 - \xi_t) x_t \rho_{t+1} - (x_{t+1} - x_t)\}) \right. \\
- \left. 2^{\alpha_{t+1}} (v_{t+k}/T_t) \right]/T_t.
\]

With both personal and corporate taxes, the hair-line formula is

\[
(4.20) \quad 1 - \alpha_{t+1} = (1 - 2^{\alpha_{t+1}}) E_{t+1} \left[ \left( \Gamma_{t+2}/T_{t+1} \right) (1 - \alpha_{t+2}) \{(1 - \xi_{t+1}) \rho_{t+2} + 1\} \right].
\]

Several conclusions emerge from this formula.

1. If the personal income tax rate on ordinary income is constant through time, the \((1 - \alpha)\) terms are equal and cancel in (4.20). Thus, they will not affect the rate of return which should be allowed on public utilities. (In the conclusions which follow, we assume ordinary income tax rates are constant.)

2. Not surprisingly, the higher the economic depreciation rate on capital \((\xi)\) and the higher the real discount rate, the higher should be the allowed return on capital. Recall that if the marginal utility of consumption is constant through time, \(\Gamma_{t+2}\) is just the discount factor \(\beta\).

3. As before, the optimal return is independent of corporate tax rates.

What does formula (4.20) predict for the real return on capital? Consider the following numbers as purely illustrative. At a marginal personal income tax rate of 20%, the capital gain rate would be .4 (20%) or 8%; thus, \(2^{\alpha} = .08\). Assuming a real discount rate of 4 percent implies \(\Gamma = \beta = 1/1.04\). Equipment lines of 40 years have a depreciation rate of \(\xi = .025\) per year; lines of 10 years have \(\xi = .10\) per year. A
long-run inflation rate of 8% implies that $i = .08$. The tax and
depreciation adjustments in (4.20) suggest that the real rate of return on
capital should be 5.1 percent ($\rho = .051$) for capital lasting 40 years and
5.5 percent ($\rho = .055$) for capital lasting 10 years. Note that we have
not yet incorporated in the model the common practice of setting the rate
base equal to the historical value of capital. The results just reported
from (4.20) should be applied to the current value of (depreciated)
capital, not the historical values.

**Implications of Formula (4.20)**

Formula (4.20) generates other implications that are worth
extracting.

First since commissions set the price before the sales are realized
therefore it is in the very nature of regulation that utilities with
procyclical sales generate procyclical earnings. Hence the allowed
earnings $\rho_{t+2}$ may be stylistically modelled as procyclical. Suppose
that for utility $i$

$$i\rho_{t+2} = i\alpha + i\beta \delta_{t+2} + i\varepsilon_{t+2}$$

where $\delta_{t+2}$ represents a measure of systematic risk such as the market
portfolio minus its mean and $i\varepsilon_{t+2}$ represents risk specific to the
particular utility under consideration. Construct $\delta, \varepsilon$ so that each has
zero mean. Then $i\alpha$ is the expected mean return from an extra unit of
i-rate base and $i\beta$ is the standard stock market beta of that extra unit.
Suppose that taxes and depreciation are deterministic. Furthermore
suppose that i-specific risk can be diversified away as in Ross [17] so
that investment markets will impute zero price to i-specific risk. Thus formula (4.20) gives us

\[(4.20a) \quad 1 - \alpha_{t+1} = (1 - 2\alpha_{t+1})E_{t+1}\{(T_{t+2}/T_{t+1})(1 - \alpha_{t+2})
\]
\[- \cdot [(1 - \alpha_{t+1})(\alpha_i + 1\beta t_{t+2}) + 1].\]

The term

\[(4.21) \quad -E_{t}\{T_{t+2}\delta_{t+2}\}\]

is proportional to the slope of the security market line (or Ross's [17] price of risk) in a tax free world. Hence (4.20a) defines a tax adjusted security market line for regulated firms. It shows that firms with high $\beta$ must be allowed higher expected returns on a unit of rate base. But Uncle Same shares part of the risk through capital gains taxes with less offset. Hence $(1 - 2\alpha_{t+1})$ appears as an adjustment as well as $T_{t+1}$. Furthermore a machine purchased today generates $1 - \xi_{t+1}$ units of rate base tomorrow. Therefore $(1 - \xi_{t+1})$ appears as an adjustment factor also.

Notice however that personal tax rates $(1 - 1\alpha_{t+1}),(1 - 1\alpha_{t+2})$ do not affect the "tax adjusted security market line" if they remain constant over time.

The above type of general equilibrium approach to Ross's arbitrage pricing theory (Ross [17]), the security market line, the capital asset pricing model, and the Sharpe-Lintner formula is developed in Brock [7]. Formulae (4.20) and (4.20a) are just tax adjusted versions of formulae in Brock [7] adapted to regulated firms.

A second implication of (4.20) concerns market to book ratios.
The market value of a firm with $x_t$ units of capital installed at date $t$ is

\[(4.22) \quad v_t = v_t(x_t).\]

Assume there is no investment credit for new investment. Differentiate (4.18) w.r.t. to $x_t$, use the envelope theorem and obtain

\[(4.23) \quad v'_t(x_t) = (1-\alpha_{t+1})/(1-\alpha_t) \quad (=\quad \text{if } x_t > 0).\]

Hence in the case of constant returns or perfect regulation (cf. (4.20)) the market to book ratio at date $t$ is

\[(4.24) \quad \frac{M_t}{B_t} = \frac{v_t(x_t)}{x_t} = \frac{v'_t(x_t)x_t}{x_t} = (1-\alpha_t)/(1-\alpha_t),\]

regardless of whether the firm is regulated or not. In particular it follows that in a taxed world a firm should not be regulated using a target market to book ratio of unity. We repeat that an implication of (4.22) is that a market to book ratio less than unity cannot be used as an argument that a utility is being poorly regulated.

In general if the production function is concave increasing then $v_t(x)$ is concave increasing so that

\[(4.25) \quad v_t(x) - v'_t(x)x > 0, \quad x > 0\]

when constant returns does not prevail. Hence there are "pure rents" or "good will" present. In this case the value of the firm is greater than its book value in a non-taxed world and the market to book ratio may be greater than unity in a taxed world even though

\[(1-\alpha_t)/(1-\alpha_t) < 1.\]
Case 4: Corporate Taxes, Personal Taxes, Debt, and Equity

The general case where firms finance with both debt and equity is difficult. In our model firms will specialize in debt or equity financing except for hairline cases. This is due to several unrealistic assumptions that we imposed in order to obtain tractability. These assumptions are: (1) interest rates on bonds are assumed to be independent of the debt equity ratio, (2) there are no bankruptcy costs either to equity-holders or to the firm's claimants as a whole, (3) there is only one class of consumers.

While the specialization result is useful in identifying the tilt that taxes impose towards debt or equity finance it is not useful for deriving formulae of the form (4.20) that have any use for rate case testimony. Fortunately we can use the method of derivation of (4.20) together with imposition of observed empirical regularities concerning debt-equity ratios and quantity of outstanding stock in order to derive a useful formula.

The approach is as follows. It has been observed that debt equity ratios within a particular industry grouping are quite stable over time (cf. Miller [13]). Furthermore the quantity of stock outstanding is quite stable over time for established companies (cf. Auerbach [1]). Therefore let us posit constant debt equity ratios and constant outstanding stock and work out the analogue of (4.20). In this way we find the hairline formula assuming $\lambda_t = \lambda$, $e_t = e$ are constant at their observed empirical values.
This procedure is much like the commonly used procedure of calculating the cost of capital by calculating the cost of equity capital and the cost of debt separately. Then the total cost of capital is the weighted average of the cost of equity capital and the cost of debt capital. Let us get into the derivation. Start with (3.13), use $p_{t+1}e_t = p_{t+1}e_{t+1}$ to obtain,

\begin{equation}
(4.26) \quad v_t = E_t \{ T_{t+1} [v_{t+1} - \alpha_{t+1} e_{t} + 3^\alpha_{t+1} p_{t+1} e_{t} + B_t q_t b_t \\
- 2^\alpha_{t+1} p_{t+1} e_{t} + \phi_{t+1})] \}
\end{equation}

\begin{equation}
= E_t \{ T_{t+1} [v_{t+1} - \alpha_{t+1} e_{t} + 3^\alpha_{t+1} v_t/(1+\lambda_t) + \frac{\lambda_t}{(1-\lambda_t)} v_t \\
- 2^\alpha_{t+1} v_{t+1}/(1+\lambda_{t+1}) + \phi_{t+1}] \}.
\end{equation}

By (3.18) we have, using $e_{t+1} = e_t$,

\begin{equation}
(4.27) \quad e_t d_t = \phi_{t+1} + v_{t+1} \lambda_{t+1}/(1+\lambda_{t+1})
\end{equation}

\begin{equation}
+ (v_t \lambda_t/(1+\lambda_t))((1/(1+i_{t+1})\frac{1}{q_t})(\frac{1}{q_t} - 1)
\end{equation}

\begin{equation}
\equiv v_{t+1} \lambda_{t+1}/(1+\lambda_{t+1}) + (v_t \lambda_t/(1+\lambda_t))A_t + \phi_{t+1}.
\end{equation}

Insert (4.27) into (4.26) to get

\begin{equation}
(4.28) \quad v_t = E_t \{ T_{t+1} [v_{t+1} W_{1,t+1} + v_t W_{2,t} + (1-\alpha_{t+1})\phi_{t+1}] \}
\end{equation}

where

\begin{equation}
(4.29) \quad W_{1,t+1} \equiv 1 - (\alpha_{t+1} \lambda_{t+1} + 2^\alpha_{t+1})/(1+\lambda_{t+1})
\end{equation}

\begin{equation}
(4.30) \quad W_{2,t} \equiv (3^\alpha_{t+1} + B_t \lambda_t - 1^\alpha_{t+1} \lambda_t A_t)/(1+\lambda_t).
\end{equation}
So far we have only used the assumption that $e_t = e_{t+1}$ in order to derive (4.28). Notice the tax effects of leverage. The term $B_t$ corresponds to the usual "gains from leverage" which is proportional to the difference between the corporate tax rate and the marginal personal rate (cf. Miller [13]). But the sale of extra bonds at $t$ causes extra dividend income to appear at the stockholder level. Hence the $1^{\alpha_{t+1}}$ term appears in the "tax distortion" term $W_{2t}$. Extra capital gains income is also generated via debt at the stockholder level, This then is taxed which causes appearance of the $2^{\alpha_1}$ terms in $W_{1,t+1}, W_{2t}$.

In order to derive the hairline formula analogous to (4.20) replace $\phi_{t+1}$ in (4.28) by (4.12) and obtain

$$v_t \quad \tilde{W}_t = v_t [1 - E_t \Gamma_{t+1} W_{2t}]$$

$$= E_t \{\Gamma_{t+1} [v_{t+1} W_{1,t+1} + (1 - 1^{\alpha_{t+1}})/(1 - \xi_t) x_t \rho_{t+1} - (x_{t+1} - x_t)]\}$$

(4.32) \( 1 - 1^{\alpha_{t+1}} = E_{t+1} \{V_{1,t+1}^{\tilde{W}_{t+1}} \Gamma_{t+2} (1 - 1^{\alpha_{t+2}})/(1 - \xi_{t+1}) \rho_{t+2} + 1\} \).

The procedure used is the same as that used to derive (4.20).

Implications regarding market to book ratios may be derived. Follow the same procedure used to obtain (4.24) to obtain from (4.31)

$$v_{t+1} W_{1t+1} \leq 1 - 1^{\alpha_{t+1}}.$$

Hence in the case of perfect regulation or constant returns (where $v_t(x_t)$ is linear in $x_t$) we get

$$\frac{M_t}{B_t} = \frac{v_t(x_t)}{x_t} = (1 - 1^{\alpha_t})/W_{1t}.$$
Notice from (4.29) that an increase in the leverage ratio

\[ \frac{\lambda_t}{(1 + \lambda_t)} \]

increases the market to book ratio. If \( \lambda_t = 0 \) we get

\[ \frac{M_t}{B_t} = \frac{(1 - 1\alpha_t)/(1 - 2\alpha_t)}{1\alpha_t} < 1, \quad \text{if} \quad 1\alpha_t > 2\alpha_t, \]

as before. If \( \lambda_t/(1 + \lambda_t) = 1, \ (\lambda_t = \infty) \) we get

\[ \frac{M_t}{B_t} = \frac{(1 - 1\alpha_t)/(1 - 1\alpha_t)}{1\alpha_t} = 1. \]

Hence when the effective marginal rate of taxation on capital gains is less than the effective marginal rate of taxation on dividends the market to book ratio is always less than one except in the case where the firm is all debt financed.

Other observations concerning the impact on \( \rho_{t+2} \) may be as easily extracted from (4.32) as in the case of (4.20).

We close this section with an important caveat. In a full optimization \( (\lambda_t)_{t=1}^\infty \) would be chosen over time to maximize the initial value of the firm. Since our model is not rich enough to avoid a rather silly specialization result we believe that the best way to use it in the interim is to use (4.32) with \( (\lambda_t)_{t=1}^\infty \) fixed at empirical values. We have worked with more general models enough to believe that a formula with the rough quantitative features of (4.32) will emerge in a more complete general equilibrium setup.
5. **Summary**

Rather than reiterate our findings above let us close with suggestions for future research.

First the paper should be extended to the case of elastic demand and adjustment costs. The formula for \( s \geq r \) will not change but the capital expansion path of the utility will change. The paper by El-Hodiri-Takayama [10] should be useful in carrying out such a study.

Second, the model should be enriched so that dividend payout ratios and debt equity ratios are determined by sharp economic tradeoffs rather than by the type of specialization that appears in our model. This is a hard and controversial area in finance (cf. Auerbach [1], Feldstein and Green [11], Miller [13], Miller and Scholes [14] and their references). We have avoided such difficulties and suggest that until the profession develops a convincing model for dividend policy and debt equity ratios that empirical estimates of these variables be inserted into formulae like (4.28). Then proceed as we did on page 38 to determine the value of the firm. In this way one can obtain an estimate of the cost of capital to the A.J. firm and see how it is influenced by basic macroeconomic magnitudes as well as the tax environment.
FOOTNOTES

1 The authors would like to thank R. Willig and B. Greenwald for perceptive questions that improved the paper. W. A. Brock gratefully acknowledges the financial support of the National Science Foundation and many helpful discussions with José Scheinkman. Of course none of the above is responsible for any errors or shortcomings that are in this paper.

2 Also homogeneous households prevent treatment of clientele effects that also would help to explain simultaneous presence of debt and equity. Nevertheless our preliminary work on clientele effects and real opportunity cost of leverage in multi-agent equilibrium models indicates that the analytical methods of the homogeneous household case will be essential to understanding the heterogeneous household case.

3 Since no personal taxes are paid, therefore equation (3.4) yields (4.10) regardless of whether the firm repurchases stock or issues bonds.

4 In more detail the derivation follows. Recurse (4.19) forward. Differentiate the two consecutive terms involving \( x_{t+1} \) w.r.t. \( x_{t+1} \) and set the result to zero. This gives

\[
1 - 1^{\alpha_{t+1}} = (1 - 2^{\alpha_{t+1}})E_{t+1} \left\{ \frac{T_{t+2}}{T_{t+1}} (1 - 1^{\alpha_{t+2}})[(1 - \xi_{t+1})^{\rho_{t+2}} + 1] \right\}
\]

which is formula (4.20).

5 Formula (4.23) is easy to derive from (4.18). Assume there is no investment tax credit. Group all terms common to \( x_{t+1} \) in (3.16) into
one expression. Call this $\psi_{t+1}$. Note that

$$\phi_{t+1} = \psi_{t+1} - x_{t+1}. $$

Now maximum value, $v_t$, in (4.18) may be written as a function of $x_t$. Similarly $v_{t+1} = v_{t+1}(x_{t+1})$. Gather up terms common to $x_t, x_{t+1}$ in (4.18) and rewrite it thus:

$$v_t(x_t) = E_t \left[ (1 - 2^\alpha_{t+1})v_{t+1} - (1 - 1^\alpha_{t+1})x_{t+1} + (1 - 1^\alpha_{t+1})\psi_{t+1} \right].$$

Since $x_{t+1}$ must be optimal given $x_t$ we must have

(a) $$(1 - 2^\alpha_{t+1})v'_{t+1} \leq (1 - 1^\alpha_{t+1})$$

(= if $x_{t+1} > 0$).

This is (4.23).

6 Extension of our work to elastic demand presents no problem when there are no adjustment costs present in the case of certainty. Work with the formula (4.18) for example. Recurse (4.18) forward to get a development of the form

(a) $$v_t = \sum_{\tau=1}^{\infty} \Theta_t \phi_{\tau}$$

where $\phi_{\tau}$ is defined by (3.16). Now perform the discrete time analogue of "integrating by parts" upon (a) to show that at each date $t$ a static A.J. problem is solved by $x_t$ subject to A.J. constraints (4.2). Follow the static A.J. literature to analyze each A.J. problem at each date $t$.

The result that continuous time dynamic A.J. models without adjustment costs "jump to the A.J. point" is fairly well known in the continuous time deterministic A.J. literature (cf. El-Hodiri-Takayama [10], Spann [19]).
A problem that is open in the literature is to extend the A.J. work to uncertainty in the face of adjustment costs. It seems to us that the problem could be approached by recursing (4.18) forward to get a development like (a). Then apply received literature on stochastic growth problems and their properties to characterize the stochastic development path of the utility. But this is a topic for future work.
REFERENCES


Symbols

\( \Gamma_{t,t+1} \)  
ratio of the present value of the marginal utility of consumption in period \( t \).

\( \beta \)  
discount factor.

\( \nu_{it} \)  
random shock hitting firm \( i \) after \( x_{it} \) is installed but before labor \( \ell_{i,t+1} \) is hired.

\( \xi_{it} \)  
depreciation rate during period \( t \) on firm \( i \)'s capital.

\( \pi_{it} \)  
accounting profits of firm \( i \) during \( t \).

\( \rho_{jt} \)  
allowed "fair rate of return" for firm \( j \) at time \( t \).

\( \tau_{ht} \)  
taxes paid during \( t \) by households.

\( \tau_{it} \)  
taxes paid by firm \( i \) during \( t \).

\( \tau_{rt} \)  
lump-sum transfers to households by the government.

\( Df_x \)  
derivative of \( f \) with respect to \( x \).

\( E_t \)  
mathematical expectation conditional on information at date \( t \).

\( M_t \)  
nominal cash balances held at end of period \( t \).

\( P_t \)  
economy-wide price level at date \( t \).

\( a_{i,t} \)  
price charged by firm \( i \) at date \( t \).

\( b_{i,t} \)  
number of bonds outstanding by firm \( i \) at end of period \( t \).

\( c_t \)  
consumption at date \( t \).

\( d_{it} \)  
dividends paid by firm \( i \) at the end of period \( t \).

\( e_{it} \)  
number of equity shares held in firm \( i \) at date \( t \).

\( g_i \)  
physical level of output of firm \( i \).

\( i \)  
firm number \((i = 1, \ldots, n)\).

\( \ell_{i,t} \)  
labor employed by firm \( i \) at date \( t \).

\( n \)  
(index) number of firms.

\( n_{it} \)  
profits of \( i \) less gross investment at \( t \).
$n_{it}^*$ net cash flow of firm $i$ at date $t$ (net of all marginal taxes).

$o_{it}$ gross investment of firm $i$ at date $t$.

$p_{it}$ exdividend price of equity of firm $i$ at beginning of period $t$.

$q_{it}$ bond price at date $t$.

$rb_{j,t+1}$ rate base of firm $j$ at $t+1$ ($\equiv (1 - \xi_j) x_{jt}$).

$s_{t+1}$ state of the world at time $t+1$.

$u$ utility function.

$v_{it}$ value of equities in firm $i$ at beginning of period $t$.

$w_t$ wage in period $t$.

$x_{i,t}$ capital held by firm $i$ at date $t$.

$m_{h,t+1}$ marginal tax bracket of households payable at date $t+1$.

$\phi_{n,t+1}$ tax preference factor for earnings income of type $i$ at household level payable at date $t+1$.

$\theta_{i,t+1}$ tax preference factor for capital gains income of type $i$ payable at date $t+1$.

$\gamma_{it}(x_{it},x_{i,t+1})$ investment tax credit applicable at date $t+1$.

$\phi_{i,t+1}$ after corporate tax net cash flow of firm $i$ — defined in equation (3.16).

$B_{it} q_{it} b^F_{it}$ total tax drain induced by bonds during period $t$. This is negative if $m_{h,t+1} < \frac{f}{1}$. 