Identification of Binary Choice Models with Social Interactions

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This paper provides a set of results on the econometric identifiability of binary choice models with social interactions. Our analysis moves beyond parametric identification results that have been obtained in the literature to consider the identifiability of model parameters when the distribution of random payoff terms is unknown. Further, we consider how identification is affected by the presence of unobservable payoff terms of various types as well as identification in the presence of certain forms of endogenous group membership. Our results suggest that at least partial identification may be achieved under assumptions that in certain contexts may be plausible.

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1. Introduction

This paper analyzes the identifiability of social interactions for binary choice models. In social interactions models, individual decisions are assumed to depend on various characteristics of the groups of which a given agent is a member. Social interactions have become an increasingly influential component of economic reasoning; Brock and Durlauf (2001b) and Manski (2000) survey a range of contexts in which social interactions have been argued to explain individual and aggregate outcomes. From the perspective of economic theorizing, the study of social interactions is important as it integrates substantive sociological ideas with formal economic reasoning. Early applications of social interactions to substantive theoretical problems include analyses of patterns of residential segregation (Schelling, 1971) and racial inequality (Loury, 1977); recent contributions include volatility in financial markets (Brock, 1993), cross-city variation in crime (Glaeser, Sacerdote, and Scheinkman, 1996), and labor market versus welfare dependence (Lindbeck, Nyberg, and Weibull, 1999; Nechyba, 2001). Social interactions models may also be understood as exploring the consequences for individuals of their location in social space (cf. Akerlof, 1997); as such they are a complement to various spatial approaches to modeling; see Anselin (2003) for a discussion of spatial approaches to externalities.

While empirical work has overall lagged behind theoretical analyses of social interactions, a growing number of empirical studies have explored aspects of such dependence.[1] Examples include Crane (1991) and South and Baumer (2000) who

[1] Empirical work on social interaction effects is surveyed in Durlauf (2004).

explore how neighborhood parental characteristics affect teenage fertility, Conley and Topa (2002) and Topa (2001) who analyze social interactions in employment, Aizer and Currie (2004) and Bertrand, Luttmer, and Mullainathan (2000) who link social networks to the use of public assistance, Ioannides and Zabel (2003a,2003b) who identify social determinants of housing demand, Bayer, Pintoff, and Pozen (2004), Glaeser, Sacerdote and Scheinkman (1996) and Sirakaya (2003) who evaluate the role of social interactions in criminal behavior and Young and Burke (2001,2003) who explore conformity in agricultural contracts.

The purpose of this paper is to develop an analysis of the identification problem for social interactions in binary choice models using individual level data. Binary choice environments represent an important leading case for social interactions. One reason that the binary choice context is interesting is that many of the behaviors in which social interactions have been posited to matter, e,g, nonmarital fertility, commission of a crime, use of cigarettes, are binary in nature. In addition, binary choice models of social interactions possess a number of interesting theoretical properties, such as multiple equilibria and phase transition (the potential for qualitative changes in the properties of the model to change with small changes in model parameters) that are not present in linear-in means models.

Relative to previous work, we establish that the identification of social interactions in a binary choice context may be achieved under weaker assumptions than have appeared in the literature. For example, we show that identification does not require prior knowledge of the parametric specification of the distribution function for random utility terms. As such, our analysis represents an extension of Manski (1988), who studies identification of binary choice models without parametric distributional assumptions, to models with social interactions and an extension of Brock and Durlauf (2001a,b), who study identification of social interactions in discrete choice models, to a broader set of environments. Beyond an amalgamation of the analyses in these papers, we also analyze the identification of social interaction in the presence of unobserved group effects.

Section 2 of the paper describes the basic theoretical structure of binary choice social interactions models and discusses econometrically implementable versions of the models. Section 3 provides results on identification when group memberships are randomly assigned. Section 4 analyzes identification in the presence of unobservable group effects. Section 5 analyzes identification when group memberships are nonrandom. Section 6 discusses an approach to identification under heteroskedasticity of random payoff terms. Section 7 provides conclusions.

2. Binary choice with social interactions

A general model of binary choice with social interactions is developed in Brock and Durlauf (2001a,b) and is the template for our identification analysis. We consider a sample of I individuals; individual i is a member of group g; the group memberships are known to the econometrician. There are $g = 1...G$ groups in the population.

Individual choices are coded by $\omega_i \in \{-1,1\}$. These choices are determined by five factors:

1. observable individual-specific characteristics, measured by an r-vector X_i,

2. unobservable individual characteristics summarized by a scalar ε_i,

3. observable group characteristics, measured by an s-vector Y_g; these are known as contextual effects as they relate to how characteristics of a group affect its members,

4. unobservable (to the econometrician) group characteristics, measured by a scalar α_g,[2]

5. subjective expectation by agent i of the average choice in the group $m_{i,g}^e$; this is known as an endogenous effect as it describes how the behaviors of others (mediated by beliefs) affect each individual.

These factors are assumed to produce payoffs for the possible choices, $u_i(1)$ and $u_i(-1)$ such that the difference between these payoffs is additive in the various factors, i.e.

$$u_i(1) - u_i(-1) = k + cX_i + dY_g + Jm_{i,g}^e + \alpha_g - \varepsilon_i . \qquad (1)^3$$

[2] In our analysis, we treat the α_g terms as fixed effects. Treating these as random effects would imply restrictions on the sources of the effects that we believe are likely to be untenable for most data sets.

Choice 1 is made when this difference is positive.

We assume that subjective beliefs are rational, given information on the group level characteristics Y_g and $F_{X|Y_g,\alpha_g}$, the distribution function of X_i conditional on the observable and unobservable (to the econometrician) group characteristics; this last term allows for heteroskedasticity in the distribution of individual characteristics across groups, for example. Hence the subjective expectations $m_{i,g}^e$ coincide with m_g, the mathematical expectation of the average choice in group g given Y_g and α_g. Since

$$E\left(\omega_i \middle| X_i, Y_g, \alpha_g\right) = 2F_{\varepsilon|X_i,Y_g,\alpha_g}\left(k + cX_i + dY_g + Jm_g + \alpha_g\right) - 1 \qquad (2)$$

m_g is defined by the integral

$$m_g = 2\int F_{\varepsilon|X,Y_g,\alpha_g}\left(k + cX + dY_g + Jm_g + \alpha_g\right) dF_{X|Y_g,\alpha_g} - 1. \qquad (3)$$

It is possible for there to exist multiple values of m_g that fulfill (3); intuitively the dependence of the payoff (1) on m_g produces an expectations-based complementarity[4]

[3]In eq. (1) and elsewhere, it is understood that variables indexed by g refer to the group of which i is a member whenever the equation describes an individual-level relationship.

[4]See Cooper (1999) for a general discussion of the relationship between complementarities and multiple equilibria.

between individual decisions that can generate multiple self-consistent behaviors in the population as a whole. As shown in Brock and Durlauf (2001a,b), multiplicity versus uniqueness in the equilibrium expected average choice levels will depend on the interplay of the strength of social interactions, measured by J and features of the distribution of $F_{\varepsilon|X_i,Y_g,\alpha_g}$, specifically the degree of dispersion of the random payoff terms; Theorem 3 of that paper specifically shows that for a given set of parameters k, c, d, J and distribution functions $F_{\varepsilon|X_i,Y_g,\alpha_g}$ and $F_{X|Y_g,\alpha_g}$, there will always exist a threshold $\bar{J}$ (which depends on the parameters and distributions), such that if $J > \bar{J}$, there must exist multiple values of m_g that solve (3) whereas if $0 \le J < \bar{J}$, then m_g is unique. This type of property does not depend on the fact we are analyzing binary choices; Brock and Durlauf (2002,2004) show how multiple equilibria of this type can arise in general multinomial choice contexts and provide multinomial generalizations of eqs. (1)-(3).

Note that when multiple solutions exist, it is assumed that agents know which solution describes expected average choices in g; put differently, agents know which equilibrium expected average choice level is selected. As such, our analysis avoids considering issues of learning and coordination on a given equilibria. An interesting extension of our results would be the consideration of how identification is affected by allowing for different types of learning.

Our objective in the following discussion is to determine conditions under which information about k, c, d, and J may be obtained from data. We will focus on both point identification, i.e. conditions under which the values of these parameters are identified, as well as partial identification,[5] i.e. conditions under which nontrivial restrictions on the possible values of k, c, d, and J may be obtained. Our discussion will be particularly concerned with the social interactions parameters d and J. The J parameter is of particular interest as it is the hypothesis that $J > 0$ that typically underlies theoretical models of social interactions. In fact, while it is relatively uncontroversial to assume that students in a class are affected by teacher quality or educational resources available in a classroom (both of which are contextual effects) it is more controversial to assume that the effort or learning of one student depends on the effort or learning of others. And as indicated above, from a theoretical perspective, the finding that $J > 0$ is of particular importance because of its implications for the presence of multiple equilibria. From the perspective of econometrics, an important class of identification issues is driven by the question of when contextual social interactions effects generated by Y_g may be distinguished from endogenous social interaction effects generated by m_g. The difficulties associated with distinguishing these two types of social interactions occur because, as indicated by (3), m_g is functionally dependent on Y_g; in a seminal paper Manski (1993) named this the reflection problem.

Identification arguments are naturally based on technical conditions concerning the distributions of errors and regressors. As such, it is often difficult to interpret requirements for identification in terms of substantive restrictions on individuals, and in our case, groups. For this reason, we believe it is useful to have an example in mind. We focus on the analysis of classroom peer group effects: a question addressed by Angrist and Lang (2002), Boozer and Cacciola (2001), Hanushek, Kain, Markman and

[5]Manski (2003) provides a definitive analysis of partial identification.

Rivkin (2003) and Hoxby (2000a,b) among others. Studies of this type consider the effects of classroom composition on individual educational outcomes. As such, they naturally involve contextual effects such as teacher or school quality as well as endogenous effects produced by peer influences. The classroom example will be useful in assessing the plausibility of various assumptions we make.

There are a number of previous studies of identification. Manski (1993) studies identification in linear and nonparametric contexts and emphasizes the absence of identification when little prior information is available. Brock and Durlauf (2001b,2002,2004) analyze identification for binary and multinomial choice models. They show that for relatively weak conditions on the joint distribution of X_i and Y_g, the parameters of (1) are identified when $F_{\varepsilon|X_i,Y_g,\alpha_g} = F_\varepsilon$ and F_ε is known a priori. The subsequent discussion shows how to extend this to the case where F_ε is unknown and also explores how various types of unobservable variables affect identification. Other contributions to the study of the identification of social interactions include Bayer and Timmins (2002) who analyze identification in multinomial logit models with various types of unobservables.

In evaluating the conditions under which identification does or does not hold for this model, we will follow the standard practice of assuming that population distributions are known for all observables. Hence we ignore issues of estimation.[6]

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[6]Bisin, Moro, and Topa (2002) discuss estimation strategies for this model.

3. Identification with random assignment and no group level unobservables

In this section, we consider identification for the binary choice model of social interactions when agents are randomly assigned to groups and when there are no group level unobservables present in the individual payoffs defined by (1). Each of these assumptions represents a substantive restriction on the class of environments under study. While random assignment may be appropriate for examples such as classrooms in which schools explicitly follow such policies in forming classes, it clearly does not apply in contexts such as residential neighborhoods; in fact analyses such as Bénabou (1993,1996), Durlauf (1996a,b) and Hoff and Sen (2002) make clear that social interactions are an important factor in understanding neighborhood composition. Similarly, the absence of group level unobservables represents a strong assumption in terms of model specification. In a context such as neighborhoods, a range of social factors such as willingness to contribute to public goods are typically unobservable.[7]

We work with the following set of assumptions in order to provide a baseline for identification analysis. Our strategy is to first understand why identification holds under

[7]Recent efforts to measure these types of group characteristics are currently under way. A very important example is the Project on Human Development in Chicago Neighborhoods, in which neighborhood characteristics such as collective efficacy, which refers to factors such as how a neighborhood provides support for its members, for example, through assistance to neighbors in childrearing are being measured via extensive survey efforts. Important studies based on this project include Sampson, Raudenbush and Earls (1997) and Sampson, Morenoff, and Earls (1999).

this particular set of assumptions and then consider how relaxation of random assignment and the absence of group level unobservables affects what may be learned about social interactions. In the assumptions and elsewhere $\text{supp}(Z)$ denotes the support of the random vector Z.

A.1. Conditional on (X_i, Y_g, α_g), the random payoff terms ε_i are independently and identically distributed according to $F_\varepsilon; F_\varepsilon(0) = .5$.

A.2. F_ε is absolutely continuous with associated density $dF_\varepsilon; dF_\varepsilon$ is positive almost everywhere on the support (L, U) which may be $(-\infty, \infty)$.

A.3. X does not include a constant; there exists a group g_0 such that for at least one element of X, x_j, (with associated nonzero coefficient c_j), x_{ij} varies continuously over R and $\text{supp}(X_{-j})$[8] is not contained in a hyperplane of R^{r-1}. Further, for group g_0, there exists an x_l, (with associated nonzero coefficient c_l) such that for all open intervals $(a_1, a_2) \in R$, $\text{Prob}\left(x_{l,g_0} \in (a_1, a_2) \middle| X_{-l,g_0}, Y_{g_0}\right) > 0$ for almost all values of X_{-l,g_0}, Y_{g_0}.

A.4. Y varies continuously over R^s and $\text{supp}(Y_{-j})$ is not contained in a hyperplane of R^{s-1}.

Variants of these assumptions will be maintained throughout and are based on Cameron and Heckman (1998). They are somewhat more restrictive than those that appear in Manski (1988) but are easier to interpret, and so we employ them. Assumption $A.1$ imposes an i.i.d. assumption on the random payoff terms ε_i within and across groups. The additional condition $F_\varepsilon(0)=.5$ is a normalization given the i.i.d. assumption.[9] $A.2$ imposes a certain degree of smoothness on F_ε. $A.3$ and $A.4$ impose linear independence among the observable individual-specific and group-specific characteristics as well as a "large support" assumption on one element of X_i and all elements of Y_g. Together, these assumptions require a certain amount of variation in the regressors X_i and Y_g. The assumptions do not place any requirement on the number of groups or the size of any group per se.

These assumptions will be augmented with different assumptions on the mechanism by which individuals are assigned to groups. Random assignment is defined by

$RA.$ $F_{X|Y_g,\alpha_g}=F_X$.

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[8]Throughout, for a vector such as Z, Z_{-j} denotes the vector when z_j is omitted.

[9]If heteroskedasticity were allowed in the errors, the condition would be nontrivial and would correspond to median independence; see Manski (1988) for discussion.

RA operationalizes the idea of random assignment by equating it with the independence of the distribution of individual characteristics within a group from the observable and unobservable (to the econometrician) characteristics of the group.

Finally, we define the case of no unobservables through the assumption

NU. $\alpha_g = 0 \ \forall \mathrm{g}$

This is the first case we analyze.

The set of parameters $k,\ c,\ d,\ J$ and the distribution function F_ε are observationally equivalent to the alternative set of parameters $\bar{k},\ \bar{c},\ \bar{d},\ \bar{J}$ and alternative distribution function $\bar{F}_\varepsilon$ if

$$F_\varepsilon\left(k + cX_i + dY_g + Jm_g\right) = \bar{F}_\varepsilon\left(\bar{k} + \bar{c}X_i + \bar{d}Y_g + \bar{J}m_g\right) \ \forall i \tag{4}$$

and

$$m_g = 2\int F_\varepsilon\left(k + cX + dY_g + Jm_g\right)dF_X - 1 = 2\int \bar{F}_\varepsilon\left(\bar{k} + \bar{c}X + \bar{d}Y_g + \bar{J}m_g\right)dF_X - 1 \tag{5}$$

for all elements of $\mathrm{supp}(X)$ and $\mathrm{supp}(Y)$. Identification holds if observational equivalence between the model parameters and distribution function and an alternative implies they are identical, i.e.

$$k=\overline{k},\ \mathrm{c}=\overline{c},\ d=\overline{d},\ J=\overline{J} \text{ and } F_{\varepsilon}=\overline{F}_{\varepsilon}. \tag{6}$$

Notice that in the conditions for observational equivalence, eqs. (4) and (5), we assume that m_g is observable since it is a moment associated with observables.

Finally, we note that from the perspective of identification arguments, we treat m_g as a data moment. This is an example of the analogy principle. Empirical analogs can be constructed nonparametrically, so there is nothing lost in this approach.

Our various assumptions provide our first identification result:

Proposition 1: Identification of the binary choice model with social interactions and random assignment to groups

Under Assumptions $A.1-A.4$, *RA*, and *NU*, the parameters of the binary choice model with social interactions (1)-(3) are identified up to scale.

Proof. Relative to Manski (1988), the key to establishing identification in the presence of social interactions concerns the treatment of m_g. Put differently, if it were known that $J=0$, so that m_g did not affect individual choices, then identification of the remaining parameters up to scale has been shown in Manski (1988) Proposition 2, Corollary 5. The key features of Manski's proof are 1) the quantile independence of $F_{\varepsilon|Y_g}$, i.e. $F_{\varepsilon|Y_g}(z)=F_{\varepsilon}(z)$ for some z and all g, which is implied by $A.1$ and $A.2$ and 2) the support assumption on X, which corresponds to $A.3$. The regressor m_g, in turn, is a function of F_ε, F_X, Y_g and (if there is more than one solution to (3)) the selection rule that determines the solution for m_g.[10] The extension of Manski's results to the social interactions case requires that the endogeneity of m_g under (3) is accounted for.

First, we consider identification of c and F_ε. Without loss of generality, assume that x_1 is the element of X with nonzero coefficient and continuous variation across R; we normalize $c_1 = 1$. For individuals in group g_0, let $\kappa_{g_0} = k + dY_{g_0} + Jm_{g_0}$ and $\bar{\kappa}_{g_0} = \bar{k} + \bar{d}Y_{g_0} + \bar{J}m_{g_0}$. Hence for members of the group, the probability that $\omega_i = 1$ is $F_\varepsilon\left(\kappa_{g_0} + cX_i\right)$ under the true set of parameters and random payoff distribution function; under the alternative this probability is $\bar{F}_\varepsilon\left(\bar{\kappa}_{g_0} + \bar{c}X_i\right)$.

Identification of c and F_ε requires that for all elements of $\mathrm{supp}\left(X\right)$ among members of g_0,

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[10]From the perspective of identification, it is not necessary to model how an equilibrium is selected for a given group, since m_g is observable as it is a sample moment (as discussed in the text). Intuitively, different choices of parameters imply different sets of self-consistent solutions for m_g and the distance of each from the sample mean can be used to identify which solution has been selected. Hence, for identification, the econometrician does not need prior information on selection. However, the issue of selection does matter for estimation, since the estimation implicitly requires construction of an estimate of m_g that is consistent with the estimated parameters. Some suggestions in this respect are found in Bisin, Moro, and Topa (2002).

$$F_{\varepsilon}\left(\kappa_{g_0}+cX_i\right)=\overline{F}_{\varepsilon}\left(\overline{\kappa}_{g_0}+\overline{c}X_i\right)\ \forall X_i\in\operatorname{supp}\left(X\right)\Rightarrow \\ c=\overline{c},\ F_{\varepsilon}=\overline{F}_{\varepsilon} \tag{7}$$

Identification of c and F_{ε} via Assumption *A*.3 is established in Manski (1988) Proposition 2, Corollary 5. Therefore, the analysis of identification of the remaining model parameters may be conducted under the assumption that c and F_{ε} are known.

We consider the identification of the remaining parameters. Using the random assignment Assumption *RA*, define a mapping $\psi\left(\cdot\right)$ that is invariant across groups[11]

$$\psi\left(\varsigma\right)=2\int F_{\varepsilon}\left(cX+\varsigma\right)dF_X-1 \tag{8}$$

It is obvious that $\psi\left(\cdot\right)$ is monotone increasing and nonlinear in ς. Taking $\varsigma=k+dY_g+Jm_g$, then

$$\psi\left(k+dY_g+Jm_g\right)=2\int F_{\varepsilon}\left(k+cX+dY_g+Jm_g\right)dF_X-1=m_g \tag{9}$$

where the second equality in (9) restates (3).

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[11]Invariance of $\psi\left(\cdot\right)$ follows from the assumptions *A*.1 and *RA* which state that neither F_{ε} nor F_X varies across groups.

This mapping facilitates identification analysis. Monotonicity of $\psi(\cdot)$ implies that the second observational equivalence requirement (eq. (5)) holds if and only if

$$k + dY_g + Jm_g = \bar{k} + \bar{d}Y_g + \bar{J}m_g \;\; \forall Y_g \in \mathrm{supp}(Y) \tag{10}$$

Rewriting (10) as

$$\left(J - \bar{J}\right)m_g = \bar{k} - k + \left(\bar{d} - d\right)Y_g \;\; \forall Y_g \in \mathrm{supp}(Y) \tag{11}$$

it is apparent that observational equivalence between two distinct sets of parameters requires that m_g is a linear function of Y_g on the support of Y_g. Since by $A.4$ Y_g varies continuously across R^s and, by construction, m_g is bounded between -1 and 1, (11) can only hold if $J = \bar{J}$. However, if $d - \bar{d} = 0$ would imply that m_g is constant, which violates the assumption that m_g is not constant across the data set. Therefore J is identified. Put another way, $A.4$ ensures that Y_g varies enough to reveal the nonlinearity of the dependence between it and m_g.

Imposing $J = \bar{J}$, observational equivalence of k and d with $\bar{k}$ and $\bar{d}$ requires that

$$\bar{k} - k + \left(\bar{d} - d\right)Y_g = 0 \;\forall Y_g \in \mathrm{supp}(Y) \tag{12}$$

Under the Assumption $A.4$, this can only hold if $k=\bar{k}$ and $d=\bar{d}$ which completes the proof that the model parameters are k, c, d and J. □

What are the key features of the proof? Assumptions $A.1$- $A.3$ allow one to identify c and F using intra-group data from one group as shown in Manski (1988). Relative to the social interactions model, the conditions allow one to construct m_g across neighborhoods and therefore treat it as an observable, given a set of parameters k, c, d and J. Assumptions *RA* and *NU* mean that the self-consistency condition (5) for observational equivalence of expected average choice levels may be inverted to produce a condition for observational equivalence on the linear payoff differentials between the choices, i.e. (4). Assumption $A.4$ ensures that there is enough variability in Y_g to identify k, d, and J given the transformation of the integral condition (5) into the linear condition (10).

The identification argument exploits the fact that because m_g is bounded between -1 and 1, it cannot be linearly dependent on Y_g under the large support Assumption $A.4$. To be clear, what drives our result is not that Y_g has unbounded support, but rather that Y_g possesses a sufficiently large support to reveal the nonlinear relationship between Y_g and m_g that holds for any given set of parameters and a given random payoff distribution, cf. eq. (3) and its form under random assignment, eq. (5). This is the reason why the so-called reflection problem (Manski, 1993) does not arise in the binary choice case; see Brock and Durlauf (2001a,b) for more discussion. The large support assumption is a simple way of ensuring that the data reveal the nonlinear relationship between m_g

and Y_g. To understand the importance of this nonlinearity, it is useful to recall Manski's (1993) nonidentification result for linear models. Manski considers the model

$$\omega_i = k + cX_i + dY_g + Jm_g^e + \varepsilon_i \tag{13}$$

under the assumption that $Y_g = E\left(X_i \middle| Y_g\right)$, i.e. the contextual variables are the group level averages of the individual characteristics. Rationality of subjective beliefs, using the same information structure as used to derive (3) implies that

$$m_g = \frac{k}{1-J} + \frac{c+d}{1-J} Y_g \tag{14}$$

so that m_g is linearly dependent on the other regressors in (13). This linear dependence is ruled out in binary choice models.[12] [13]

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[12]The boundedness of choices is important in establishing identification in other contexts. For example, it plays a key role in the development of bounds on treatment effects, cf. Manski (1990).

[13]This same nonlinearity argument also implies that parametric multinomial choice models are identified when group memberships are randomly assigned and when there are no group level unobservables; see Brock and Durlauf (2006) for a formal statement and proof.

This analysis indicates that the unobservability of F_ε is not an impediment to identification; however, it is important to note that accurate estimates of the magnitude of social interactions may depend on the accuracy of estimates of this density, at least to the extent that one needs to construct proxies for m_g using (3). In a context such as classrooms, it may be problematic to estimate F_ε using data from a single group. In such cases, it will be important to employ estimation methods that allow information on F_ε to be constructed from data from all groups.

4. Group level unobservables

As we have suggested, one important limitation to Proposition 1 is that it assumes that no group level unobservables are present in the individual payoff functions. Such unobservables are likely in many contexts even when there is random assignment. In the case of classrooms, differences in teacher quality are one such unobservable. In this section, we relax Assumption *NU* and consider some ways to achieve identification when group-level unobservables are present. The analysis will show that prior information on the distribution of group level unobservables is necessary to achieve even partial identification of social interactions; we provide examples of prior information that is useful in this respect.[14] Our analysis will employ the same assumptions as before, except NU will no longer be employed.

When group level unobservables are present, our identification definition needs to account for their presence. Modifying eqs. (4) and (5), identification in the presence of unobservable group effects requires that for a given set of parameter values k, c, d, J, set of unobservables α_g, and random payoff distribution F_ε, that if there exists an alternative set of parameters $\bar{k}, \bar{c}, \bar{d}, \bar{J}$, set of unobservables $\bar{\alpha}_g$, and distribution function $\bar{F}_\varepsilon$ such that

$$F_\varepsilon\left(k + cX_i + dY_g + Jm_g + \alpha_g\right) = \bar{F}_\varepsilon\left(\bar{k} + \bar{c}X_i + \bar{d}Y_g + \bar{J}m_g + \bar{\alpha}_g\right) \ \forall i \tag{15}$$

and

$$\begin{aligned} m_g &= 2\int F_\varepsilon\left(k + cX + dY_g + Jm_g + \alpha_g\right)dF_X - 1 \\ &= 2\int \bar{F}_\varepsilon\left(\bar{k} + \bar{c}X + \bar{d}Y_g + \bar{J}m_g + \bar{\alpha}_g\right)dF_X - 1 \end{aligned} \tag{16}$$

for all elements of $\operatorname{supp}(X)$ and $\operatorname{supp}(Y)$, it must be the case that eq. (6) holds. Relative to the earlier definition of identification, notice that the values of the

[14]When individual agents face more than two choices, additional routes to identification may exist beyond those we study, as shown in Bayer and Timmins (2002); the basic idea in this work is that comparisons among subsets of the choices may be used to identify unobservable choice-specific payoff terms.

unobservables, i.e. α_g and $\bar{\alpha}_g$ must be allowed to differ when comparing alternative sets of parameters.

Without any additional assumptions, identification fails for the social interactions model with unobserved group effects. This is easy to see. Clearly, identification of c and dF_ε is unaffected by the presence of α_g; the argument in Section 3 is unaffected since in the current case all that changes is that $\kappa_{g_0} = k + dY_{g_0} + \alpha_{g_0} + Jm_{g_0}$. Hence, we proceed taking these as known.

We again work with the mapping $\psi(\varsigma)$ defined by equation (8). For $\varsigma = k + dY_g + Jm_g + \alpha_g$,

$$\psi\left(k + dY_g + Jm_g + \alpha_g\right) = 2\int F_\varepsilon\left(k + cX + dY_g + Jm_g + \alpha_g\right)dF_X - 1 = m_g \qquad (17)$$

Comparing (17) and (9), observational equivalence requires, recalling that $\psi(\varsigma)$ is increasing, monotonic and invariant across groups,

$$k + dY_g + Jm_g + \alpha_g = \bar{k} + \bar{d}Y_g + \bar{J}m_g + \bar{\alpha}_g \;\; \forall Y_g \in \mathrm{supp}(Y) \qquad (18)$$

If one chooses $\bar{\alpha}_g = \alpha_g + Jm_g$ and $\bar{J} = 0$, then $k + dY_g + Jm_g + \alpha_g = k + \bar{d}Y_g + \bar{\alpha}_g$ $\forall Y_g \in \mathrm{supp}(Y)$, which means that J and d are not identified. We can therefore state

Proposition 2. Nonidentification with unobserved contextual effects

Under the Assumptions $A.1 - A.4$ and RA, the parameters of the binary choice model with social interactions (1)-(3) are not identified.

What sorts of additional assumptions are needed to allow for identification of social interactions when unobservable group effects are present? A number of possibilities exist that may, depending on the particular environment, be plausible. We consider two alternative classes of identification strategies.

i. identification based on restrictions of the joint distribution of observables and unobservables

One way in which identification may be achieved in the presence of unobservables is via the use of prior restrictions on the distribution of these unobservables either in isolation or in terms of their relationship to various observables. The utility of this approach will of course depend on the plausibility of these restrictions.

a. restrictions on the support of the unobservables

In the proof of Proposition 1 a large support assumption, combined with the intrinsic nonlinearity of the binary choice model, produced identification. We first explore how the logic of the large support assumption may be adapted to the model with group-level unobservables. An analogous large support assumption in the unobservable case is

U.1. $\operatorname{supp}\left(\left(d-\bar{d}\right)Y_g+\alpha_g-\bar{\alpha}_g\right)$ is unbounded.

This assumption produces identification of J but does not permit identification of d, as stated in Proposition 3.

Proposition 3. Partial identification of the binary choice model with social interactions and unobservables with unbounded support

Under Assumptions $A.1-A.4$, *RA*, and *U.1*, the parameter J in the binary choice model with social interactions (1)-(3) is identified up to scale but k and d are not identified.

Proof. Rewrite (18) as

$$\left(\bar{J}-J\right)m_g=k-\bar{k}+\left(d-\bar{d}\right)Y_g+\alpha_g-\bar{\alpha}_g \tag{19}$$

Under $U.1$, the right hand side of (19) will possess full support. Since $m_g\in\left[-1,1\right]$, (19) cannot hold unless $J=\bar{J}$, so J is identified. Imposing this on (19),

$$k-\bar{k}+\left(d-\bar{d}\right)Y_g+\alpha_g-\bar{\alpha}_g=0 \tag{20}$$

which imposes no restrictions on k and d since α_g and $\bar{\alpha}_g$ are unrestricted. This verifies the proposition. □

In our judgment, this is a relatively unappealing route to identification since it is difficult to fairly imagine a priori cases in which the unbounded support Assumption $U.1$ can be made credible. Specifically, the proposition requires that one can make claims about the relationship between the variation in the difference in unobservables under two candidates sets of parameters and the variation in Y_g, which seems hard to justify. Perhaps one could identify cases where, for example, government policies of some type have introduced large unmeasured heterogeneity into schools via lumpy investments in facilities that can pin down the nature of the variation in the unobservables. We include this proposition primarily for the purpose of completeness and illustration.

b. restrictions on the relationship between observables and unobservables

A second approach to restricting the unobservable contextual effects so as to achieve some sort of identification is through restrictions on the relationship between the unobservables and observable contextual effects. We consider two sorts of restrictions of this type.

stochastic monotonicity

One way to restrict the relationship between observables and unobservables is to impose stochastic monotonicity between the types of contextual effects. This type of assumption is analogous to those studied in Manski (1997) and Manski and Pepper (2000). Without loss of generality, assume that each element of Y_g is measured so that $d \geq 0$ [15] with strict inequality for at least one element of d. We consider the following assumption:

M.1. First order stochastic monotonicity of group level unobservables. If $Y_g > Y_{g'}$, then the conditional distribution of unobservables in $g', F_{\alpha_{g'}|Y_{g'}}$, is first order stochastically dominated by $F_{\alpha_g|Y_g}$.

This assumption is sufficient to facilitate partial identification of social interactions.

Proposition 4. Pattern reversals and partial identification of endogenous social interactions

Under Assumptions $A.1-A.4$, *RA* and *M*.1, if there exists a pair of groups such that

[15] Vector inequalities are measured in the usual way, i.e. $V_1 > V_2$ means each element of V_1 is greater than the corresponding element of V_2.

$$Y_g > Y_{g'} \text{ and } m_g < m_{g'} \tag{21}$$

then it must be the case that $J > 0$ and J is large enough to produce multiple equilibria, for the binary choice model with social interactions (1)-(3).

Proof. From (3), m_g is defined by

$$m_g = 2\int F_\varepsilon\left(k + cX + dY_g + Jm_g + \alpha_g\right) dF_X \, dF_{\alpha_g|Y_g} - 1 \tag{22}$$

Notice that the only cross-group variation in m_g is due to variation in Y_g. If the observables for group g are ordered according to $Y_g > Y_{g'}$ so that by $M.1$ $F_{\alpha_g|Y_g}$ first-order stochastically dominates $F_{\alpha_{g'}|Y_{g'}}$, and if $J = 0$, then $m_g > m_{g'}$ since $F_\varepsilon\left(k + cX + dY_g + Jm_g + \alpha_g\right)$ is increasing in Y_g (recall that Y_g is measured so that $d \geq 0$) and α_g. This monotonicity can only be broken if $J > 0$ and there are multiple equilibria, so that the equilibrium expected average choice levels of groups g and g' are such that g has coordinated at an equilibrium other than the one with the highest expected average choice level whereas g' has coordinated at an equilibrium other than the lowest expected average choice; otherwise the pattern reversal in (21) could not have occurred. □

One may consider alternatives to $M.1$; one natural example is stochastic dominance based on an index, i.e.

$M.2$. First order stochastic monotonicity of an index of unobservables. There exists a scalar index function $I(\cdot)$ such that if $I(Y_g) > I(Y_{g'})$, then the conditional distribution of unobservables in g', $F_{\alpha_{g'}|I(Y_{g'})}$, is first order stochastically dominated by $F_{\alpha_g|I(Y_g)}$.

This allows one to state a Corollary to Proposition 4.

Corollary 1. Pattern reversals with an index and partial identification of social interactions

Under Assumptions $A.1-A.4$, *RA* and *M*.2, if there exists a pair of groups such that

$$Y_g > Y_{g'},\ I(Y_g) > I(Y_{g'}) \text{ and } m_g < m_{g'} \tag{23}$$

then it must be the case that $J > 0$ and J is large enough to produce multiple equilibria for the binary choice model with social interactions (1)-(3).

This corollary includes cases such as $I(Y_g) = dY_g$. Its utility relative to Proposition 4 will depend on context. One can imagine cases where the effects of changes in individual elements of Y_g on the density $F_{\alpha_g|Y_g}$ is unclear whereas one can plausibly make such assumptions for $F_{\alpha_g|I(Y_g)}$ given an index $I(Y_g)$. For example, it may not be clear how, by itself, years of experience shifts the distribution of teacher quality, but one might conclude that the fitted estimate of teacher salary, in which years of experience appears, does have an unambiguous effect.

Are monotonicity assumptions plausible? This is of course a matter of judgment; nevertheless we believe there are cases where this is so. For example, consider the case of teenage smoking. According to one recent national survey in 2003, 10.0% of black teenagers had smoked a cigarette in the last 30 days whereas 29.4% of white teenagers had. As ethnicity is a natural social group, this sort of behavioral difference seems hard to understand without endogenous social interactions, since one would generally expect that the contextual factors that are omitted by these raw percentages, e.g family income, etc. would have the property of making the black teenage smoking rate higher than the white rate.[16] Similarly, we could conjecture that for unobservable teacher quality, stochastic monotonicity might hold with respect to observable variables such as average family income or per pupil expenditure.[17]

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[16]Various social interaction effects have been directly estimated in studies such as Krauth (2002).

[17]The power of the stochastic monotonicity assumption also implies the importance of testing it when possible. Barrett and Donald (2003) have recently proposed methods for doing this that may be relevant in this context.

linearity

Alternatively, one can assume a linear relationship between the observable and unobservable contextual effects.

L.1. $\alpha_g = f_0 + f_1 Y_g + \mu_g$, $E\left(\mu_g \middle| Y_g\right) = 0$.

This is a much stronger assumption than stochastic monotonicity, but will in turn provide an important increase in the restrictions that $J = 0$ places on data. To see this, we construct the variable $Z_i = F_\varepsilon^{-1}\left(\text{Prob}\left(\omega_i = 1 \middle| X_i, Y_g, \alpha_g\right)\right)$ which by (1) equals

$$Z_i = k + cX_i + dY_g + Jm_g + \alpha_g \tag{24}$$

One can compare this to the regression equation

$$Z_i = \pi_0 + \pi_1 X_i + \pi_2 Y_g + \phi_i \tag{25}$$

Under $J = 0$ and Assumption $L.1$, $E\left(\phi_i \middle| X_i, Y_g\right) = 0$ whereas if $J \neq 0$ there is a nonlinear dependence among ϕ_i and X_i and Y_g, given the nonlinearity inherent in the relationship between these vectors and m_g as shown in (3). Assumption $L.1$ thus places a restriction on Z_i.

Proposition 5. Nonlinearity of transformed outcome probabilities in the presence of endogenous social interactions

Under Assumptions $A.1 - A.4$, RA and $L.1$, if $J = 0$, then Z_i is linear in X_i and Y_g whereas if $J \neq 0$, then Z_i is not linear in X_i and Y_g, for the binary choice model with social interactions (1)-(3).

This implication is easily testable if one can construct an empirical analog to Z_i and so estimate (25) as a regression. Granger and Terasvirta (1996) provide methods for testing whether the linear relationship in (25) holds. One can also use the BDS test (Brock, Dechert, LeBaron, and Scheinkman, 1996) on estimated residuals from (25) to see whether the residuals are i.i.d.; acceptance of this null rules out the nonlinearity described in the proposition.

Can one construct such an empirical analog? One possibility is suggested by the classroom example. Suppose that students answer a number of true/false questions and that the proportion for student i is $\hat{s}_i$. Assume that the probability of a correct answer follows (1), i.e. the probability of a given answer being correct is $F_\varepsilon\left(k + cX_i + dY_g + Jm_g + \alpha_g\right)$. If $\hat{F}_\varepsilon$ is an estimate of F_ε, which is identified under our assumptions, then $\hat{F}_\varepsilon^{-1}\left(\hat{s}_i\right)$ is a sample estimate of Z_i.

restrictions on the density of unobservables

A third route to identification in the presence of unobserved group effects lies in restricting the density of the unobserved group effects. One such restriction is

$P.1.$ dF_{α_g} is unimodal .

This assumption leads to Proposition 6.

Proposition 6. Partial identification of endogenous social interactions when unobservables are unimodally distributed.

Under Assumptions $A.1-A.4$, RA and $P.1$, if $J=0$, then there must exist a vector π such that $dF_{\pi Y_g|m_g}$ is unimodal.

Proof. We prove the Proposition with $\pi = d$; we do not condition on d in the statement of the proposition because it is not identified. Recall from (17) that, if $J=0$, $m_g = \psi\left(k + dY_g + \alpha_g\right)$; ψ is invertible, so $\psi^{-1}\left(m_g\right) = k + dY_g + \alpha_g$ which means that $\alpha_g = \psi^{-1}\left(m_g\right) - k - dY_g$. Therefore,

$$\begin{aligned} F_{dY_g|m_g}(\xi) &= \Pr\left(dY_g \le \xi \middle| m_g\right) = \Pr\left(-k-\alpha_g+\psi^{-1}\left(m_g\right) \le \xi\right) \\ &= \Pr\left(\alpha_g \ge -k-\xi+\psi^{-1}\left(m_g\right)\right) = 1-\Pr\left(\alpha_g \le -k-\xi+\psi^{-1}\left(m_g\right)\right) \end{aligned} \tag{26}$$

Differentiation of both sides of (26) gives

$$dF_{dY_g|m_g}(\xi) = dF_{\alpha_g}\left(-k - \xi + \psi^{-1}(m_g)\right) \tag{27}$$

Since dF_{α_g} is unimodal, it has a unique maximum with respect to ξ; which by (27) implies that $dF_{dY_g|m_g}(\xi)$ has a unique maximum as well, and so is unimodal, as stated in the Proposition. □

This Proposition differs from 4 and 5 in that it focuses on the density of the contextual effects given the expected average choice in the group, rather than the expected average choice given the contextual effects. The reason for this is that the relationship between unimodality and multiple equilibria is in fact quite subtle. If one thinks of the conditional density of $dF_{m_g|Y_g}(\xi)$ multiple equilibria may not induce multimodality; intuitively, the density is, for multiple equilibria, a mixture (where the components of the mixture correspond to the densities conditional on the equilibria) and there is no reason why a mixture must be multimodal. On the other hand, it is possible for $dF_{m_g|Y_g}(\xi)$ to exhibit multimodality without social interactions since it is a nonlinear transformation of α_g, so that unimodality may not be preserved under the transformation. Our conditioning of $dF_{\pi Y_g|m_g}(\xi)$ avoids this latter possibility.

The plausibility of unimodality will of course depend on context. We conjecture that for a variable such as unobserved teacher quality, the assumption of unimodality is reasonable.

Taken together, Propositions 4-6 and Corollary 1 demonstrate how multiple equilibria can play a key role in identification of endogenous social interactions when unobserved group effects are present. In some sense, since the one property that models with endogenous social interactions can exhibit, whereas models without them cannot, is multiple equilibria. In this sense, our identification results exhibit an analog to phase transition as applicability of the results changes according to whether or not the parameter J is large enough to produce multiple equilibria.

ii. identification in panels

Our discussion thus far has focused on identification in cross-section contexts. An alternative possibility for addressing group level unobservables lies in the use of panel data. In the linear context, panels have been shown to affect identification in Brock and Durlauf (2001b) and Graham and Hahn (2005). In the linear context, panels facilitate identification for two reasons. First, differencing of the data permits elimination of group-level fixed effects. Second, the timing of contextual and endogenous social interactions (eg. whether $\omega_{i,t}$ depends on $Y_{g,t}$ and $m_{g,t}$ or $Y_{g,t-1}$ and $m_{g,t-1}$) can break the linear dependence between the contextual and endogenous effects that is the basis of Manski's (1993) finding of nonidentification in the linear case. We will focus on the analog of differencing to see how the availability of panel data can affect identification; as such, our argument is an example of the analysis pioneered in Chamberlain (1984).

Implications of time varying group membership characteristics

Identification may be facilitated if group characteristics evolve across time. This is the key idea used to identify peer effects in Hoxby (2000b); here we provide a formalization and generalization of her argument. To do this, we replace (1) with

$$u_{i,t}(1) - u_{i,t}(-1) = k + cX_{i,t} + dY_g + eY_{g,t} + Jm_{g,t} + \alpha_g - \varepsilon_{i,t} \tag{28}$$

where, relative to (1), Y_g and $Y_{g,t}$ are vectors of group-specific characteristics that do not and do vary across time respectively. In the context of classroom interactions, we are interpreting Y_g as incorporating variables such as observed teacher quality, for example, whereas we are interpreting $Y_{g,t}$ as incorporating variables such as the average parental income. Relative to the cross-section model, we are not imposing any additional assumptions on individuals as expectations are only formed about the contemporary behavior of group members. Also, notice that group composition may change across time.

Following previous arguments, one can construct a map $\psi_t(\varsigma)$

$$\psi_t(\varsigma) = 2\int F_\varepsilon(cX + \varsigma)\, dF_{X_t} - 1 \tag{29}$$

In (29), F_{X_t} is the density of X at time t. We are therefore assuming that the distribution of $\varepsilon_{i,t}$ is time invariant but are allowing the distribution $X_{i,t}$ to change temporally. As before, $\psi_t(\varsigma)$ is monotonic increasing and invariant across *g*; the only difference relative to (17) is that the map is allowed to vary across t. In parallel to our earlier discussion, $\varsigma = k + dY_g + eY_{g,t} + Jm_{g,t} + \alpha_g$, producing

$$\begin{gathered}\psi_t\left(k + dY_g + eY_{g,t} + Jm_{g,t} + \alpha_g\right) = \\ 2\int F_\varepsilon\left(k + cX + dY_g + eY_{g,t} + Jm_{g,t} + \alpha_g\right)dF_{X_t} - 1 = m_{g,t}\end{gathered} \tag{30}$$

The invariance of $\psi_t\left(\varsigma\right)$ across groups means that observational equivalence requires that for each time t

$$k + dY_g + eY_{g,t} + Jm_{g,t} + \alpha_g = \overline{k} + \overline{d}Y_g + \overline{e}Y_{g,t} + \overline{J}m_{g,t} + \overline{\alpha}_g \tag{31}$$

which provides a linear panel structure when we consider (31) at different t's.

The panel structure of (31) allows one to achieve identification. Formally, we have Proposition 7.

Proposition 7. Partial identification in binary choice models with panel data

Under Assumptions $A.1 - A.4$ and RA, the parameters, c, e, and J are identified up to scale whereas k and d are not identified, for the panel binary choice model of social interactions, (28) and (30).

Proof. c is identified as shown in Proposition 1. To analyze the remaining parameters, difference (31) to produce

$$\left(e-\bar{e}\right)\left(Y_{g,t}-Y_{g,t-1}\right)+\left(J-\bar{J}\right)\left(m_{g,t}-m_{g,t-1}\right)=0 \tag{32}$$

Eq. (32) implies that the $m_{g,t}-m_{g,t-1}$ must be linear for all elements of the support of $Y_{g,t}-Y_{g,t-1}$. If the domain of this support is large enough, then our nonlinearity argument will apply, so that $e=\bar{e}$ and $J=\bar{J}$ and so these parameters are identified. The nonidentifiability of k and d is immediate from the absence of any restrictions on the cross-second moments of the vectors 1 and Y_g with α_g. □

The logic of this proposition may be extended to more alternative frameworks. Hoxby's (2000b) analysis provides a variant of this model in which black and white students generate distinct social interactions within classroom. Formally, it is assumed that within group g there are two individual types $T\in\{B,W\}$ such that within-group social interactions that are time varying only occur among members of a common type. Thus, she works with a model of the form

$$u_i\left(1\right)-u_i\left(-1\right)=k+cX_i+dY_g+eY_{T,g,t}+Jm_{T,g,t}+\alpha_g-\varepsilon_{i,t} \tag{33}$$

The specific idea in her work is that $Y_{T,g,t}$ measures the percentage of T in the classroom; fluctuations in the percentage of blacks in a classroom are then used to identify social interactions. Her approach suggests a strategy for identifying social interactions that exploits the two dimensional nature of groups at a point in time.

Suppose that one observes two types within a classroom. This will allow the construction of the difference between two means of the two groups, $m_{W,g,t} - m_{B,g,t}$ which will prove useful in controlling for the common group effects Y_g and α_g. To see why, note first that for (33), the required self-consistency condition for type-specific expected average choices is

$$m_{T,g,t} = 2\int F_\varepsilon\left(k + cX + dY_g + eY_{T,g,t} + Jm_{T,g,t} + \alpha_g\right)dF_{T,X,t} - 1,\ T \in \{B,W\} \qquad (34)$$

where $dF_{T,X,t}$ is the density of individual-specific characteristics of type T in group *g* at time *t*. By analogy to (30) and (31), (34) implies that

$$k + dY_g + eY_{T,g,t} + Jm_{T,g,t} + \alpha_g = \bar{k} + \bar{d}Y_g + \bar{e}Y_{T,g,t} + \bar{J}m_{T,g,t} + \bar{\alpha}_g \qquad (35)$$

so that if one subtracts (35) with $T = W$ from (35) with $T = B$

$$\left(e - \bar{e}\right)\left(Y_{W,g,t} - Y_{B,g,t}\right) + \left(J - \bar{J}\right)\left(m_{W,g,t} - m_{B,g,t}\right) = 0 \qquad (36)$$

eliminating Y_g and α_g. If there is sufficient variation in $Y_{W,g,t} - Y_{B,g,t}$ across groups, then the nonlinear relationship between $Y_{W,g,t} - Y_{B,g,t}$ and $m_{W,g,t} - m_{B,g,t}$ will require $e - \bar{e} = J - \bar{J} = 0$. This suggests how memberships in multiple groups (e.g. classrooms and race) can facilitate identification.

5. Identification with nonrandom assignment

i. Assignment based on observables

We first consider the case where agents are nonrandomly assigned, but where the assignment is a function of observable variables. What this means is that Assumption *RA* no longer holds, i.e. different groups will be associated with different distributions of observables $F_{X|Y_g}$. We will preserve the independence of this distribution from the group identity and so replace *RA* with

NRA.1. $F_{X|Y_g,\alpha_g} = F_{X|Y_g}$ and is independent of g.

We will also maintain Assumption *NU* that there are no unobservable group level variables; this may be replaced with a version of $M.1$; we omit doing this for expositional purposes. For this case, the Assumption $A.1$ that the conditional distribution of the individual-specific random payoff terms is independent of the group characteristics, our analysis would correspond to the case of strong ignorability in the treatment effects literature, cf. Rosenbaum and Rubin (1983).[18]

[18]In Rosenbaum and Rubin (1983), even though different individuals have different treatment probabilities, the conditional distribution of the effects of treatment is independent of whether treatment occurs, conditional on those variables that determine

Relative to Proposition 1, selection on observables (as we have defined it), requires additional analysis because the argument that the $\psi(\cdot)$ mapping defined by (8) is invariant across groups is no longer valid. The original argument can be amended, however. Consider the conditional mapping

$$\psi\left(\varsigma \middle| Y_g\right) = 2\int F_\varepsilon\left(cX + \varsigma\right) dF_{X|Y_g} - 1. \tag{37}$$

For a fixed value of Y_g, $\psi\left(\cdot \middle| Y_g\right)$ is monotonic, increasing, and invariant across groups just as $\psi(\cdot)$ was in the analysis of Proposition 1. This means that if we set $\varsigma = k + dY_g + m_g$, observational equivalence between parameters k, c, d, J and random payoff distribution F_ε and alternative parameters $\bar{k}, \bar{c}, \bar{d}, \bar{J}$, and distribution function $\bar{F}_\varepsilon$ will require that (4) and

$$m_g = 2\int F_\varepsilon\left(k + cX + dY_g + Jm_g\right)dF_{X|Y_g} - 1 = 2\int \bar{F}_\varepsilon\left(\bar{k} + \bar{c}X + \bar{d}Y_g + \bar{J}m_g\right)dF_{X|Y_g} - 1 \tag{38}$$

imply (6). The argument that verifies that identification holds in Proposition 1 also implies that (38) implies (6). Therefore, we can conclude with this proposition.

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the treatment probability. In modeling the dependence of group memberships on observables, we are allowing memberships to depend on the observable Y_g, but the median of the random term in the payoff function is independent of these variables.

Proposition 8. Identification with selection on observables

Under Assumptions *A*.1-*A*.4, *NRA*.1, and *NU*, the parameters of the binary choice model with social interactions (1)-(3) are identified up to scale.

ii. assignment based on unobservables

We finally consider identification when group memberships are related to unobservable individual characteristics. To do this, we treat the membership question as the outcome of a matching problem and place some restrictions on the equilibria that emerge from the matching.[19] We assume that matching occurs with respect to indices A_i and T_g respectively. We assume that these terms may be modeled as

$$A_i = cX_i - \varepsilon_i \tag{39}$$

and

[19]This strategy is particularly useful when instrumental variables approaches to accounting for self-selection are not available. See Evans, Oates, and Schwab (1992) for an example. See also Ioannides and Zabel (2003b) for an interesting analysis of how self-selection can provide additional information in identifying social interactions in linear models.

$$T_g = dY_g \,. \tag{40}$$

In the context of peer effects in classrooms, A_i may be thought of as student ability and T_g as teacher quality.

We assume that matching occurs on the basis of individual and group quality. To do this, we assume that the individual characteristics X_i are measured so that $c \geq 0$. Individuals and groups are matched in the sense that higher group quality is associated with higher individual quality:

NRA.2. For any two groups g and g', if $T_g > T_{g'}$, then $F_{A|T_g}$ first order stochastically dominates $F_{A|T_{g'}}$.

This assumption is weaker than one which imposes strict assortative matching between better groups and higher ability individuals; the latter is predicted by models such as Becker (1973). The assumption is qualitatively consistent with a range of payoff functions that relate groups and individuals, see Sattinger (1993) for a useful survey of equilibrium matching problems. Notice that the assumption replaces *A*.1 since it places an implicit restriction on $F_{\varepsilon|Y_g}$

This assumption leads to the following proposition.

Proposition 9. Partial identification of endogenous social interactions under assortative matching

Under Assumptions $A.2$-$A.4$, $NRA.2$ and NU, suppose that $Y_g > Y_{g'}$ and $F_{X|g}$ first order stochastically dominates $F_{X|g'}$. Then, if $m_g < m_{g'}$, it must be the case that $J > 0$ and J is large enough to produce multiple equilibria, for the binary choice model with social interactions and assortative matching.

Proof. The structure of the proof mirrors that of Proposition 4 and so is omitted. Note that $m_g = E\left(\bar{\omega}_g \left| Y_g, F_{X|g}\right.\right)$.

One can develop more subtle implications of assortative matching for the presence of endogenous social interactions. Specifically, one can find testable restrictions concerning J that condition on the value of T_g. While T_g is not directly observable, the following proposition implies an implicit restriction on the relationship between d and J. Alternatively, if there is any prior basis for ranking the relative values of T_g across neighborhoods, then the proposition can be employed in a straightforward fashion. One possible source of such information might be the membership price for a group, e.g. the tuition charged by a school.

Proposition 10. Partial identification of endogenous social interactions under assortative matching

Let $m_{g|T_g} = E\left(\bar{\omega}_g \middle| T_g\right)$. Under Assumptions $A.2$-$A.4$, $NRA.2$ and NU, suppose that $T_g > T_{g'}$. Then, if $m_g < m_{g'}$, it must be the case that $J > 0$ and J is large enough to produce multiple equilibria, for the binary choice model with social interactions and assortative matching.

Proof. Consider the binary choice model (1) under the assumption $J = 0$. It is sufficient to show that $\text{Prob}\left(k + A_i + T_g > 0 \middle| T_g\right)$ is increasing in T_g, since $m_{g|T_g} = E\left(\omega_i \middle| T_g\right)$ and $E\left(\omega_i \middle| T_g\right) = 2\text{Prob}\left(k + A_i + T_g > 0 \middle| T_g\right) - 1$. Rewriting this probability as

$$\text{Prob}\left(k + A_i + T_g > 0 \middle| T_g\right) = \text{Prob}\left(A_i > -T_g - k \middle| T_g\right) = 1 - F_{A|T_g}\left(-T_g - k\right) \qquad (41)$$

It is immediate from $NRA.2$ that $F_{A|T_g}\left(-T_g - k\right)$ is decreasing in T_g hence $\text{Prob}\left(k + A_i + T_g > 0 \middle| T_g\right)$ is increasing in T_g. Therefore, if $m_g < m_{g'}$, following the proof of Proposition 4, it must be the case that group g has coordinated on an equilibrium expected average choice level other than the largest of the possible equilibria associated with it while group g' has coordinated on an equilibrium other than the lowest possible expected average choice level among those it could have attained, implying $J > 0$. □

This use of assortative matching to facilitate identification may be extended to panel data. To do this, modify (39) and (40) so that $A_{i,t} = k + cX_{i,t} + \varepsilon_{i,t}$ and $T_{g,t} = dY_g + eY_{g,t}$ and Assumption *NRA*.2 is modified to

$NRA.2'$. For any two groups g and g', if $T_{g,t} > T_{g',t}$, then $F_{A_t|T_{g,t}}$ first order stochastically dominates $F_{A_t|T_{g',t}}$.

This modification allows one to identify implications for $m_{g,t}$ as time varies. For example, this Corollary to Proposition 10 is immediate.

Corollary 2. Equality of average outcomes with equal observable contextual effects.

Under Assumptions $A.2$-$A.4$, $NRA.2'$ and *NU*, if $J = 0$ or $J > 0$ but sufficiently small that $m_{g,t}$ is unique, then $Y_{g,t} = Y_{g,t'}$ implies $m_{g,t|T_g} = m_{g',t|T_{g'}}$, for the binary choice model with social interactions (1)-(3).

This corollary is useful because it indicates how evidence of endogenous social interactions may be adduced from temporal changes in $m_{g,t}$ when $Y_{g,t}$ does not vary. As such this result complements the identification strategy of Hoxby which relies on fluctuations in $Y_{g,t}$.

6. Heteroskedasticity

Our analysis has been designed to highlight the fundamental roles of nonlinearity and multiple equilibria in permitting identification of social interactions. In doing this, we have employed, via *A*.1, the strong assumption that the random payoff terms ε_i are independent and identically distributed across *i*. As originally analyzed in Manski (1988) and developed in Horowitz (1998), one can achieve nonparametric identification results for the binary choice model using the much weaker assumption of median independence, i.e.

$$MI.\ median\left(\varepsilon_i \middle| X_i = \bar{x}, Y_g = \bar{y}, m_g = \bar{m}\right) = 0\ \forall\ \bar{x}, \bar{y}, \bar{m} \text{ in the support of } X, Y, m$$

Horowitz (1998) emphasizes that the assumption that the random payoffs ε_i are homoskedastic is unappealing in many applications. We therefore discuss what can be identified under the weaker assumption of median independence.

To do this, we follow Horowitz (1998) Theorem 3.1, which is a slight extension of Manski's original results. Following Horowitz' proof, define $Z = (1, X, Y, m)$, $\beta = (k, c, d, J)$, and $b = (\bar{k}, \bar{c}, \bar{d}, \bar{J})$. Assume that the c_1 is normalized to equal 1 and let $Z_{-1} = Z$ with the x_1 component deleted; the corresponding coefficient vectors are β_{-1} and b_{-1}. Construct the sets

$$S1(b) = \{Z \,|\, -b_{-1}Z_{-1} \leq x_1 < -\beta_{-1}Z_{-1}\} \tag{42}$$

and

$$S2(b) = \{Z \,|\, -\beta_{-1}Z_{-1} \le x_1 < -b_{-1}Z_{-1}\}\,. \tag{43}$$

As shown in Horowitz (1998, pg. 60), if the density of x_1 conditional on $Z_{-1} = z_{-1}$ is positive for all z_{-1} in the support of Z_{-1}, then $S1(b)$ has positive probability whenever $-b_{-1}z_{-1} < -\beta_{-1}z_{-1}$ and $S2(b)$ has positive probability whenever $-\beta_{-1}z_{-1} < -b_{-1}z_{-1}$. This implies that β_{-1} is identified if

$$\text{Prob}\left(\beta_{-1}Z_{-1} = b_{-1}Z_{-1}\right) < 1\,. \tag{44}$$

Identification holds because $\text{Prob}\left(S1(b) \cup S2(b)\right) > 0$ if $\beta_{-1} \ne b_{-1}$, i.e there is a positive probability that $b_{-1}Z_{-1}$ does not equal $\beta_{-1}Z_{-1}$. Thus identification requires showing that

$$\text{Prob}\left(\beta_{-1}Z_{-1} = b_{-1}Z_{-1}\right) = 1 \Rightarrow \beta_{-1} = b_{-1} \tag{45}$$

which is equivalent to the condition

$$Z_{-1} \text{ does not lie in a proper linear subspace of } R^{r+s+1} \tag{46}$$

What sort of variation in X_i and Y_g are required for (46) to hold? The basic reasoning of our previous proofs will still apply. Intuitively, sufficient variation of X_i within a group will ensure that (46) holds for these elements. The inherent nonlinearity in the relationship between Y_g and m_g via the mapping defined in (3), even under general heteroskedastic error distributions, should prevent Z_{-1} from lying in a proper linear subspace of R^{r+s+1} under appropriate regularity conditions. For example, if one places a large support assumption on Y_g will force $d = \bar{d}$ since m_g is bounded between -1 and 1. But even without a large support assumption, a genericity argument over the space of possible error distributions in (3) plus the nonlinearity inherent in (3) should be sufficient to ensure that Y_g cannot linearly depend on m_g, which will ensure that $d = \bar{d}$. Then, variation in Y_g across groups will imply that $J = \bar{J}$.

It is beyond the scope of this paper to fully develop these types of arguments; we leave this to future research. However, we believe that this heuristic argument demonstrates potential new routes to identification under weaker error assumptions and indicates how the nonlinearity that underlies our identification results is the key to identification of social interactions for the class of models we have studied.

7. Conclusions

This paper provides a set of conditions under which identification holds for the binary choice model with social interactions. Relative to previous work, the analysis establishes identification conditions without assuming that the distribution for random payoff terms is logistic, as is done in Brock and Durlauf (2001a,b). In addition, some partial identification results are developed for models with unobserved group level variables.

Our approach to identification does not rely on the use of market information to reveal the presence of social interactions. Clearly, for contexts such as residential neighborhoods, price data may provide an additional route to identification. Recent work on hedonic models by Ekeland, Heckman, and Nesheim (2002,2004) and Nesheim (2002) is important in this regard.

With respect to further work, we mention two difficulties with the analysis found here. The analysis in the paper is heavily dependent on the fact that the distribution of random payoff terms may be identified based on within-group observations. For contexts such as classrooms, in which the number of group members is small, accurate finite sample approximations to this distribution may prove to be problematic. This suggests the importance of developing ways to combine information across groups to improve the accuracy of the estimation of this distribution. Second, the paper assumes that the groups within which interactions occur are themselves known a priori. As Manski (2000) has emphasized, this is a very strong assumption and may not be plausible in many cases. The absence of a coincidence between measured social groups and true social groups will induce complicated patterns of interdependences in errors across individuals as well as make it difficult to assess counterfactuals such as the effects of changes in the compositions of measured groups; examples of such counterfactuals include changes in the location of public housing or changes in educational policies regarding the tracking of students. Hence, the extension of the measuring of social interaction effects to policy evaluation will require assessing potential model uncertainty of this type. Some of this work is currently under way.

Finally, we would reiterate the importance of developing estimation methods that correspond to the identification results we have developed. In addition to the issues associated with multiple equilibria that are addressed by Bisin, Moro, and Topa (2002), the presence of a phase transition in social interactions models suggests there may be interesting issues of asymptotics that need to be explored, at least around parameter values associated with the phase transition. In the statistics literature, these types of issues have been analyzed in the context of a class of local interaction models by Pickard (1976,1977,1979). For behavioral models of the type studied here, Brock (1993) and Brock and Durlauf (2001a) develop some results on laws of large numbers and central limit theorems and near phase transitions that are "nonstandard", but there is much more to be done here. For example these nonstandard asymptotics may assist in separating social interactions from correlated unobservables, selection effects and contextual effects. We plan to do this in future research.

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