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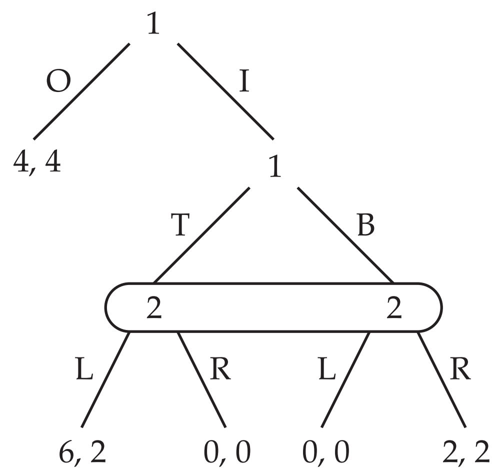
Midterm Exam – Economics 713

1. (35 points)
Consider the extensive form game Γ below:



(i) Compute all subgame perfect equilibria of Γ.
(ii) Compute all sequential equilibria of Γ.
(iii) Compute all proper equilibria of the reduced normal form of Γ.
(iv) The least demanding concept used in the analysis of games is the elimination of strictly dominated strategies: no such strategy should ever be chosen by a Bayesian-rational player. Given this fact, it seems natural to seek solution concepts whose predictions are unaffected when a strategy that is strictly dominated (in the reduced normal form) is removed from a game. Using your answers to parts (i)-(iii), discuss how subgame perfect equilibrium, sequential equilibrium, and proper equilibrium fare under this criterion.

2. (10 points)
An experimenter is evaluating a subject's preferences over lotteries with prizes in the finite set Z. The experimenter finds that the subject strictly prefers each lottery in $P \subset \Delta Z$ to each lottery in $Q \subset \Delta Z$. Suppose that the convex hulls of P and Q are disjoint. Must the experimenter's observations be consistent with the subject's preferences being representable by a von Neumann-Morgenstern utility function? Prove that this is the case, or provide a counterexample.

3. (30 points)

Consider an infinite repetition of the following two-player normal form game, G:

		2: C	D	P
	C	2, 2	0, 3	0, 0
1	D	3, 0	1, 1	1, 0
	P	0, 0	0, 1	0, 0

Suppose that player i's repeated game strategy σ_i is described as follows:

(I) Play C initially, or if C was played last period.
(II) If there is a deviation from (I), play P once and then restart (I).
(III) If there is a deviation from (II), then restart (II).

Now answer the following questions:

(i) For what values of δ is strategy profile $\sigma = (\sigma_1, \sigma_2)$ a subgame perfect equilibrium?
(ii) Suppose that in the stage game G, action profile (P, P) results in both players receiving a payoff of $\frac{1}{2}$ rather than a payoff of 0. In this case, what are the values of δ for which strategy profile $\sigma = (\sigma_1, \sigma_2)$ is a subgame perfect equilibrium?
(iii) Give intuitive explanations for any differences in the results of your analyses of parts (i) and (ii).

4. (25 points)

Fix a normal form game $G = \{N, \{S_i\}_{i \in N}, \{u_i\}_{i \in N}\}$ and a correlated strategy $\rho \in \Delta S$ for this game.

Now consider the following interaction: A central planner will select a pure strategy profile $s \in S$ at random according to the joint distribution $\rho \in \Delta S$. But before this profile is selected, each player i must commit to one of these two behaviors: (a) the player can agree to play his part s_i of whatever pure strategy profile s is chosen by the planner; (b) the player can choose a pure strategy $\hat{s}_i$ of his liking that he will play regardless of which pure strategy profile the planner selects. When making this decision, player i knows the joint distribution ρ to be employed by the planner, but does not know anything about the outcome of the planner's randomization.

If in the above interaction, each player finds it optimal to choose option (a) when the others also do so, we call ρ an *ex ante correlated equilibrium*.

(i) Describe the set of ex ante correlated equilibria, and show that it is convex.
(ii) Is every ex ante correlated equilibrium also a correlated equilibrium? Prove that this is the case, or provide a counterexample.
(iii) Is every correlated equilibrium also an ex ante correlated equilibrium? Prove that this is the case, or provide a counterexample.