

Midterm Exam – Economics 713

1. (15 points)

Suppose that two players repeatedly play the following normal form game:

		2		
		T	M	B
1	T	3, 3	2, 1	-2, 4
	M	1, 2	1, 1	-1, 3
	B	4, -2	3, -1	0, 0

Suppose that strategy σ_i for player i is described as follows:

Begin play in stage I.

- (I) If T has always been played, play T . Otherwise, begin stage II.
- (II) If M has always been played since stage II began, play M ; otherwise, begin stage III.
- (III) Play B .

For what values of δ is strategy profile $\sigma = (\sigma_1, \sigma_2)$ a subgame perfect equilibrium?

2. (15 points)

Let $G = \{N, \{A_i\}_{i \in N}, \{u_i\}_{i \in N}\}$ be a normal form game. Let $\Gamma = \{N, \{T_i\}_{i \in N}, p, \{A_i\}_{i \in N}, \{u_i\}_{i \in N}\}$ be a Bayesian game with common prior distribution $p \in \Delta T$, and with the same action sets A_i and utility functions $u_i : A \rightarrow \mathbf{R}$ as G . Note that u_i does not condition on players' types.

- (i) Describe S_i , the set of of player i 's pure strategies in Γ .
- (ii) Under what conditions is pure strategy profile $s^* \in S$ is a Nash equilibrium of Γ ?
- (iii) Let $\rho^* \in \Delta A$ be the correlated strategy induced by Nash equilibrium s^* :

$$\rho^*(a) = \sum_{t: s^*(t)=a} p(t).$$

Show that ρ^* is a correlated equilibrium of G .

3. (20 points)

Let Γ be a signaling game whose type set T , message set M , and response set R are finite. Prove that every perfect Bayesian equilibrium of Γ is also a sequential equilibrium of Γ .

4. (50 points)

Ace-King-Queen Poker is a two-player card game that is played using a deck consisting of three cards: an Ace (the high card), a King (the middle card), and a Queen (the low card). Play proceeds as follows:

- Each player puts \$1 in a pot in the center of the table.
 - The deck is shuffled, and each player is dealt one card. Each player only sees the card he is dealt.
 - Player 1 chooses to Raise (R) or Fold (F). A choice of R means that player 1 puts an additional \$1 in the pot. Choosing F means that player 1 ends the game, allowing player 2 to have the money already in the pot.
 - If player 1 raises, then player 2 chooses to Call (c) or Fold (f). A choice of f means that player 2 ends the game, allowing player 1 to have the money already in the pot. A choice of c means that player 2 also puts an additional \$1 in the pot; in this case, the players reveal their cards and the player with the higher card wins the money in the pot.
- (i) Draw the extensive form of this game.
 - (ii) Find all sequential equilibria of this game.
 - (iii) If you could choose whether to be player 1 or player 2 in this game, which player would you choose to be?
 - (iv) Suppose we modify the game as follows: Instead of choosing between Raise and Fold, player 1 chooses between Raise and Laydown (L). A choice of L means that the game ends, the players show their cards, and the player with the higher card wins the pot. Answer parts (ii) and (iii) for this modified game.

In answering this question, refer to player 1's card as A , K , or Q , and refer to player 2's card as a , k , or q .