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Midterm Solutions – Economics 713

1. Suppose to the contrary that τ_1 strictly dominates σ_1. Then $u_1(\tau_1. s_2) > u_1(\sigma_1, s_2)$ for all $s_2 \in S_2$. Therefore, since G is zero sum, $u_2(\tau_1, s_2) < u_2(\sigma_1, s_2)$ for all $s_2 \in S_2$, and so $\min_{\rho_1 \in \Delta S_1}(\max_{s_2 \in S_2} u_2(\rho_1, s_2)) \leq \max_{s_2 \in S_2} u_2(\tau_1, s_2) < \max_{s_2 \in S_2} u_2(\sigma_1, s_2)$. That is, σ_1 does not minmax player 2.

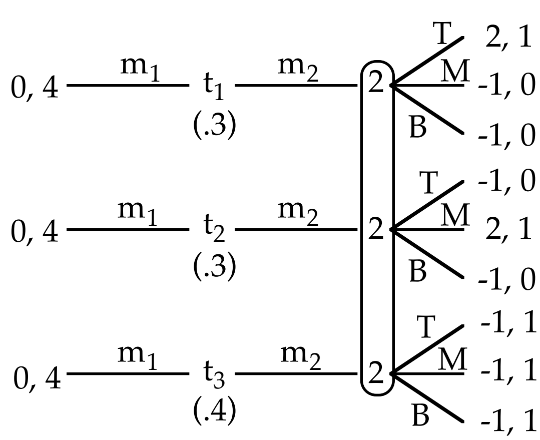
(Alternatively, one could prove this statement by appealing to the Minmax Theorem. The Minmax Theorem tells us that if σ_1 is a minmax strategy for player 1, then (σ_1, σ_2) is a Nash equilibrium, where σ_2 is some minmax strategy for player 2. Hence, σ_1 is a best response to σ_2, and so cannot be a strictly dominated strategy.)

2. This is a bad idea. While a player will never choose a dominated strategy in a Nash equilibrium of a normal form game, it is quite possible for him to play a dominated stage game action in a SPE of a repeated game. The "grim trigger" equilibrium of the repeated prisoner's dilemma provides one example.

3. (i) This condition says that the change in a player's payoffs when he unilaterally deviates is always identical to the change in potential.

(ii) Suppose that pure strategy profile s maximizes P on S. Then the definition of the potential function says that no player has a profitable unilateral deviation, which is precisely the definition of Nash equilibrium. (In fact, the Nash equilibria of G are precisely the local maximizers of the potential function on S.)

4. One possibility for the payoffs is as follows:



With these payoffs, if all types play m_1, player 2 should believe that a deviator is either t_1 or t_2, as m_2 is dominated for type t_3. Hence, player 2's possible best responses are T, M, and randomizations between them. For the "all m_1" outcome to fail the intuitive criterion, we would need to find a type of player 1 who wants to deviate against all of these best responses. Since t_1 does not want to deviate against M and t_2 does not want to deviate against T, no such deviator exists.

On the other hand, for this outcome to fail equilibrium dominance, it is enough to find one deviator for each best response of player 2. Now t_1 strictly prefers to deviate as long as player 2 places probability strictly greater than $\frac{1}{3}$ on T, while t_2 strictly prefers to deviate as long as player 2 places probability strictly greater than $\frac{1}{3}$ on M. The union of these two sets of mixed strategies of player 2 includes all of player 2's best responses to her beliefs; therefore, the "all m_1" equilibria all fail equilibrium dominance.

5. This example is due to Kohlberg and Mertens (1986).

(i) In game Γ_1, strategy C is strictly dominated by strategy B, and so is not played in any Nash equilbrium. Therefore, if player 1 plays B with positive probability, then player 2 must play L, in which case player 1 plays B with probability 1. On the other hand, if player 1 plays A with probability 1, then it is easy to check that player 2's beliefs are unconstrained. As long as she plays R with probability at least $\frac{2}{3}$, it is optimal for player 1 to play A. R is optimal for player 2 as long as $\mu(y) \geq \frac{3}{4}$. To sum up, the sequential equilibria are (B, L) (with $\mu(x) = 1$), $((A, R), \mu(y) \geq \frac{3}{4})$, and $((A, (1 - \alpha) L + \alpha R)$ with $\alpha \geq \frac{2}{3}, \mu(y) \geq \frac{3}{4})$.

Game Γ_2 has a unique subgame perfect equilibrium of $((N, B), L)$, and so this (with $\mu(x) = 1$) is also the unique sequential equilibrium of Γ_2.

(ii) Both games have the same reduced normal form modulo strategy names:

		2: L	R
	A	4, 2	4, 2
1	B	6, 3	3, 0
	C	0, 0	2, 1

(iii) This example shows that two extensive form games with the same reduced normal form need not have the same sets of sequential equilibrium outcomes. Since games like Γ_1 and Γ_2 which have the same RNF are in some sense "strategically equivalent", this example exhibits a weakness of sequential equilibrium as a solution concept.

6. Cross-player consistency implies that $\mu(w) = \mu(y)$ and $\mu(x) = \mu(z)$, and it is easy to check that consistency imposes no other restrictions on beliefs: to get $\mu(w) = c \in (0, 1)$, choose $\sigma_1(B) = c\varepsilon$ and $\sigma_1(C) = (1 - c)\varepsilon$; to get $\mu(w) = 1$, choose $\sigma_1(B) = \varepsilon$ and $\sigma_1(C) = \varepsilon^2$; and to get $\mu(w) = 0$ choose $\sigma_1(B) = \varepsilon^2$ and $\sigma_1(C) = \varepsilon$.

7. The argument fails because while the normalized version of the vector p is always a correlated strategy of players 2 through n, it need not correspond to a mixed strategy profile for those players—it only does so if it is a product measure on $S_{-1} = \prod_{j \neq 1} S_j$. (This issue does not arise in the two player case since the mixed strategies of a single player are identical to his "correlated strategies".) The statement in the question would be true if we replaced the words "some mixed strategy profile of players 2 through n" with the words "some correlated strategy for players 2 through n".