

**Economics 719**  
Problem Set 3  
Due Nov 6, 2007

Consider a sample  $\{X_1, \dots, X_n\}$  of independent and identically distributed random variables. The density of  $X$  is

$$f(x) = \frac{\theta e^{-x}}{(1 + e^{-x})^{\theta+1}} \quad \theta \in (0, \infty),$$

Note  $\theta$  is a shape parameter.

**Question:**

1. Show that  $X$  is a member of the exponential family.
2. Write the density in canonical form.
3. Find a sufficient statistic,  $T(X_1, \dots, X_n)$ , for the sample  $\{X_1, \dots, X_n\}$ .
4. Use properties of the exponential families to find  $E[T(X_1, \dots, X_n)]$  and  $Var [T(X_1, \dots, X_n)]$ .