Problem Set 2

1. Suppose you have a model based on latent random variables. Your latent random variable is

$$Y_i^* = X_i\beta + \varepsilon$$

while your observed random variable is

$$Y_i = \begin{cases} 1 \text{ if } Y_i^* > 0 \\ Y_i^* \text{ if } Y_i^* \leq 0 \end{cases}$$

Assume ε has some distribution function, call it $F()$.

a) What is the likelihood of this model?

b) Can we identify the variance of ε? Why?

2. Now let's show some important properties of the Weibull. Remember if x is distributed Weibull with shape parameter α then

$$F(x) = e^{-e^{-(x+\alpha)}}.$$

a) Show that the Weibull distribution is closed under maximization (i.e., that the maximum of n Weibulls is a Weibull). That is, show that $\Pr(\max_i \varepsilon_i \leq c)$ where ε_i is Weibull with parameter α_i will be a Weibull.

b) Show that the Weibull indeed generates a logistic model. That is, show that the difference between two Weibulls is a logit so show that $Pr(\varepsilon_2 - \varepsilon_1 \leq 0)$ will generate a logisitic.

3. Gronau's female labor supply model. (See Heckman 1974 for a much more general model).

Suppose you have a utility function over leisure L and some good x: $u(L, x)$. The budget constraint is given by $x = wH + v$ with $H = 1 - L$ and v is asset income. Take the wage rate w as given and we are assuming $P_x = 1$. From basic economic theory we know that

$$\frac{\frac{\partial u}{\partial L}}{\frac{\partial u}{\partial x}} = w \qquad \text{when} \qquad L < 1$$

and that the reservation wage $\left(w^R\right)$ is given by

$$MRS|_{H=0} = w^R.$$

Assume that the wage of a person is given by $w_i = X_i\beta + \varepsilon$ and that the reservation wage is given by $w^R = Z_i\gamma + \eta$.

a) What is the observed wage?

b) What is the likelihood generated by this model?

c) Can you identify all the parameters? Which ones can you identify?