Mancur Olson and the Group Size Argument

Mancur Olson is often said to have "proved" that larger groups are less able to provide collective goods. He seems to be claiming this, but his equations are actually independent of group size. The following quotation from pp. 39-41 of Gerald Marwell and Pamela Oliver's The Critical Mass in Collective Action: A Micro-Social Theory (Cambridge University Press 1993) is a proof originally published by Pamela Oliver in 1980 in the American Journal of Sociology. Unfortunately, the AJS version is unintelligible because the AJS printer scrambled the lines after the galley proofs were checked.

Let's begin with what Olson said. For Olson, groups come in three theoretically different sizes:

(1) "Small" or "privileged", in which some individual may have enough interest in the collective good to provide some level of it himself;

(2) "Moderate", in which no individual can provide a significant portion of the good himself, but some individuals can make a "noticeable" difference in the level of provision of the collective good, i.e., affect it enough that it seems to have increased a small amount;

(3) "Large", in which no individual can make even a noticeable difference (p.44).

These definitions would seem to define a large group as one in which no contribution is noticeable, and thus make the "group size" thesis vacuous. Olson himself notes that this would be tautological (pp. 48-9n), and recasts his position as "the (surely reasonable) empirical hypothesis that the total costs of the collective goods wanted by large groups are large enough to exceed the value of the small fraction of the total benefit that an individual in a large group would get" (p. 49n). Hence, Olson argues, no rational individual in a large group would ever contribute towards the provision of a public good.

It is actually not necessary to invoke any additional evidence to show the flaw in Olson's "size" argument. His claim fails to stand on its own terms. Olson's verbal arguments have a persuasive ring, and his mathematical equations seems to imply that the size argument has been "proved," but careful inspection of his equations reveals that they are actually independent of group size. Olson assumes a linear relation between the value of the good to an actor, $\mathbf{V_i}$, and the level $\mathbf{T}$ at which it is provided with the equation $\mathbf{V_i = F_i S_g T}$ (p. 23), where $\mathbf{S_g}$ is the "size" of the group (in value units) and $\mathbf{F_i}$ is the fraction an individual's value is of the total group value, i.e. $\mathbf{F_i = V_i/V_g}$. For any particular group, $\mathbf{F_i}$ and $\mathbf{S_g}$ are constants, so the relation between $\mathbf{T}$ and $\mathbf{V_i}$ is linear. This linear relation makes the decision to contribute independent of group size, despite appearances to the contrary.

Olson supports his "size" argument by taking derivatives and solving for the point at which the marginal cost equals the marginal value. He gives two versions of his result, $\mathbf{dC/dT = F_i S_g}$ (page 23) and $\mathbf{dC/dT = F_i(dV_g/dT)}$ (page 24); the latter he interprets by saying: ". . . the rate of gain to the group ($\mathbf{dV_g/dT}$) must exceed the rate of increase in cost ($\mathbf{dC/dT}$) by the same multiple that the group gain exceeds the gain to the individual concerned ($\mathbf{1/F_i = V_g/V_i}$)" (p. 24). His implication in this passage, and in the subsequent references he makes to his results, clearly is that the likelihood of the marginal gain to the group exceeding the marginal cost by the appropriate multiple declines as the group size increases, since $\mathbf{F_i}$ gets small as the group size gets large. But this is not true. Notice that we have two equations involving $\mathbf{F_i}$, $\mathbf{V_i = F_iS_gT}$, which implies $\mathbf{F_i = (V_i/S_gT)}$ and $\mathbf{F_i = V_i/V_g}$. Equating these two, we find $\mathbf{(V_i/S_gT) = V_i/V_g}$, which implies $\mathbf{S_gT = V_g}$, i.e. that the group value is simply the group size times the level of provision. But $\mathbf{F_i}$, which has $\mathbf{V_g}$ in the denominator, is always paired with $\mathbf{S_g}$ or $\mathbf{V_g}$ in the numerator; the "size" terms seem to cancel out.

We may nail this down formally with a simple algebraic proof. Compare a group of size $\mathbf{S_g}$ with a larger group of size $\mathbf{S_g' = S_g + d}$. The linearity assumption $\mathbf{V_i = F_iS_gT}$ implies $\mathbf{F_i = (V_i/S_gT)}$, so the individual's fraction of the larger group is $\mathbf{F_i' = [V_i/(S_g+d)T]}$. The level of $\mathbf{T}$ which should rationally be purchased in the augmented group occurs when $\mathbf{(dC/dT)' = F_i'S_g' = (S_g+d)[V_i/(S_g+d)T]}$. Note that the augmented group size occurs in both the numerator and denominator of this expression and thus cancels out, leaving $\mathbf{(dC/dT)' = F_i'S_g' = V_i/T = F_iS_g = dC/dT}$. That is, the solution is the same for both the original group and the larger group. It is <u>independent of group size</u>. This result also holds for the equation $\mathbf{dV_g/dT = 1/F_i(dC/dT)}$ (page 25) which is simply an algebraic rearrangement of the previous result.

To show that Olson's mathematics are independent of group size is not to prove that group size never has an effect. Of course it does. But group size effects cannot fall of out simple all-purpose equations. Instead, one must begin with assumptions (or facts) about the group size effect, and then derive group size results that are direct consequences of the group size assumptions. This does not mean that there is nothing interesting to be learned about group size, but rather that what we need to know is more complex and interesting than simple aphorisms. Scholars have added assumptions about the elasticity of the good or the nature of interdependence among the actors and shown that these alter the effect of group size on collective action (Chamberlin 1974; Frohlich and Oppenheimer 1970; McGuire 1974). We believe that the interaction of group size with the degree of jointness of supply is the most important avenue to pursue. (See Hardin 1982, pp.38-49 for a very thorough treatment of this issue.)