Empirical and Policy Performance of a Forward-Looking Monetary Model

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Abstract

In this paper we consider the implications of a fully specified dynamic general equilibrium model, developed by Smets and Wouters (2003). This is a relatively large-scale forward looking model, which was shown to provide a good fit to the data. We show that systematically accounting for prior uncertainty may lead to substantially different parameter estimates. However many of the qualitative features of the model remain similar under the alternative estimates that we find. We then formulate and analyze optimal policy rules in the model, focusing on a simple loss function which is commonly used and is independent of the estimates. We determine the optimal equilibrium dynamics for our estimates as well as those of Smets and Wouters, and find that they imply largely similar behavior. We then analyze simple policy rules, finding that these rules perform relatively well and are robust to our different sets of parameter estimates. Overall, our results suggest that the model may be relatively robust in its ability to capture certain aspects of the data. However some caution should be exercised in basing inference on the structural estimates, as these seem to be only weakly identified.

1 Introduction

In recent years, there has been a renewed interest in the study of monetary policy under uncertainty. On the one hand, there has been a large literature studying the effects of uncertainty on policy performance in relatively small macroeconomic models. This includes

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a number of different papers focusing on backward-looking models without explicit micro-foundations, forward-looking models with explicit micro-foundations, or some combination of the two.[1] At the same time, there have been a number of developments in the study of empirical dynamic general equilibrium models.[2] By incorporating significant frictions in the form of nominal rigidities and adjustment costs, these models have been better able to account for the dynamics observed in the data. However to date there has been relatively little work analyzing policy performance in the context of fully specified empirical dynamic general equilibrium models.[3] The goal of this paper is to assess the usefulness of such a model for monetary policy, by looking more closely at its empirical performance and its implications for optimal policy.

In particular, we focus on a leading new empirical model developed by Smets and Wouters (2003), henceforth SW. Building on work by Christiano, Eichenbaum, and Evans (2005), SW develop a fully specified equilibrium model for the Euro area that includes a number of frictions and additions which can induce intrinsic persistence in the propagation of shocks. They then estimate the model using Bayesian methods and show that it fits the data as well or better than a conventional atheoretical vector autoregression. But while these results are striking, some questions remain: first, about the robustness of the empirical results, and then about their implications for policy design.

To confidently utilize the model for policy, it is important to know how strongly the empirical results depend on the prior assumptions. We first reconsider the empirical performance of the model, by providing a relatively comprehensive analysis of prior uncertainty and re-estimating the model under a less informative prior than SW employed. We show that this matters substantially for the estimates of the structural parameters. We also discuss some of the complications associated with estimating such large scale models, as we found numerous local minima which confounded many optimization methods.[4] Interestingly, although our parameter estimates differ greatly, the implied time series of the output gap that we find nearly matches that in SW and the qualitative features of many of the impulse responses are similar. These results suggest that the model may be over-parameterized, and that it may be possible to find a smaller model that would fit nearly as well. To further explore this, we analyze the sensitivity of different qualitative features of the model to changes in the parameters. We find that in order to account for the variation in the output gap and the response of key variables to a monetary policy shock, the specification of the policy rule and the price stickiness are by far the most important aspects of the model.

We then turn to the implications of the model for monetary policy. Although the model is based on utility maximization and thus has a natural welfare criterion, we choose instead to use a more traditional quadratic loss function weighting fluctuations in inflation, the output

[1]This literature is enormous, including the key contributions summarized in Woodford (2003) and the papers in Taylor (1999). Aside from these, some key papers we build on include Fuhrer (2000), Rudebusch (2001), Onatski and Williams (2003), and Levin and Williams (2003).

[2]Again this literature is large, but the key papers we build on include Rotemberg and Woodford (1997), Christiano, Eichenbaum, and Evans (2005), and Smets and Wouters (2003).

[3]Since the first draft of this paper, this literature has grown, and includes Levin, Onatski, Williams, and Williams (2006), Schmitt-Grohe and Uribe (2007), and Schmitt-Grohe and Uribe (2006), among others.

[4]As discussed below, some of the problems we encountered were mitigated by considering a smoother prior.

gap, and the instrument interest rate. In Levin, Onatski, Williams, and Williams (2006) we analyze policy using a utility-based loss in this model, but here we choose a standard (but ad hoc) loss for several reasons. Having a simple loss allows us to simply state and compare the tradeoffs in policy, and fits with the longstanding literature and policy discussion which is phrased in terms of a "dual mandate" looking at output and inflation variability. Moreover, one of our goals is to analyze policy with different structural estimates, and we find it useful to have a common benchmark for evaluating these estimates. With a welfare-based loss, the policy objective would change with the estimates. Thus we find it more transparent here to use a loss function whose weights are independent of the parameters governing the dynamics of the economy, and thus to avoid conflating changes in policymaker preferences with changes in the economic dynamics.

Within the context of our estimated model, we then develop and compare policy rules which are optimal for the chosen loss function. We first focus on deriving the optimal equilibrium dynamics, which in many cases differ rather substantially from the estimated dynamics. We also compare the optimal equilibrium dynamics from our benchmark parameter estimates with those of Smets and Wouters (2003). We find that even though the estimates differ rather substantially, the optimal equilibrium dynamics are roughly similar. This accords with our findings on the qualitative behavior of the economy, and depends crucially on our specification of the simple loss function which is invariant to changes in the model parameters. We then consider the performance of some simple policy rules in which the interest rate responds to some current variables only. Following Taylor (1993) simple rules have been analyzed in a number of contexts and often have been found to both perform relatively well and to be relatively robust (see the contributions in Taylor (1999) for examples). In our case, we find that there is relatively little degradation in performance by switching from optimal to simple rules. Moreover, the optimal simple rule turns out to be same under each set of estimates, a rather remarkable robustness property.

In summary, we find that the model is relatively robust in its ability to capture certain aspects of the data. Many of the qualitative dynamics remain the same under different structural estimates. However some caution should be exercised in using and interpreting the structural estimates, as many of the parameters appear to be only weakly identified. Robustness may be gained by considering simple policy rules, but this in turn may depend on the loss function which is used. If we considered a welfare-based loss function whose weights changed with the parameter estimates, then this robustness result would likely be destroyed. Nonetheless, the model was relatively robust in accounting for the dynamics of a number of variables, and in providing suggestions for stabilizing inflation and output.

2 The Smets-Wouters Model

In an important recent paper, Smets and Wouters (2003) developed and estimated a dynamic stochastic general equilibrium model for the Euro area. Their paper has garnered much attention because they show that the model fits the data as well as a conventional atheoretical VAR.[5] This suggests that explicitly founded models may have finally reached a crucial stage in empirical work in which they can compete with purely empirical specifications. However,

[5]However the model does not fit quite as well as some Bayesian VAR specifications.

as mentioned in the introduction, the policy implications of the model remain largely unexplored, and is our focus. In this section we briefly describe the key equations of the model, which are derived and discussed in more detail in SW.

Building largely on work by Christiano, Eichenbaum, and Evans (2005) (who in turn draw on Erceg, Henderson, and Levin (2000) and Rotemberg and Woodford (1997)), the SW model includes a number of frictions and additions which can induce intrinsic persistence in the propagation of shocks. The frictions include sticky prices and sticky wages, both with partial indexation and adjustment costs in investment. The model also allows for habit persistence and variable capacity utilization with utilization costs. Further, in order to empirically confront seven data series in estimation, the model is supplemented with ten structural shocks, six of which are allowed to be temporally dependent. We will focus on the linearized version of the model which we now lay out. Smets and Wouters (2003) formulate and describe the full nonlinear model as well as deriving the linearization we present here. (See also Levin, Onatski, Williams, and Williams (2006).)

The model consists of a continuum of households who value consumption and leisure. The preferences incorporate external habit persistence in consumption and are subject to two types of temporally dependent preference shocks. The households have some degree of market power in the labor market, as they supply differentiated labor in an imperfectly competitive market. Households trade in a complete market to allocate their consumption over time, with the consumption Euler equation summarizing their optimal behavior:

$$C_t = \frac{h}{1+h}C_{t-1} + \frac{1}{1+h}E_tC_{t+1} - \frac{1-h}{(1+h)\sigma_C}(i_t - E_t\pi_{t+1}) + \frac{1-h}{(1+h)\sigma_C}(\epsilon_t^b - E_t\epsilon_{t+1}^b). \quad (1)$$

All variables are expressed in terms of logarithmic deviations from the steady state. Here C_t is consumption, h is the habit persistence in the additive habit stock, σ_C is the curvature parameter for consumption utility, i_t is the nominal interest rate, π_t is inflation, and ϵ_t^b is a preference shock which scales utility multiplicatively. This preference shock follows an $AR(1)$ process with correlation ρ_b. As an extension of a typical Euler equation, (1) states that the household balances the marginal utility of consumption in successive periods, where now the marginal utility is affected by lagged consumption via the habit stock and is perturbed via the preference shock.

Because of their market power, households are wage setters in the labor market. However they cannot reset their wages every period, but instead face nominal wage rigidity of the Calvo (1983) type in which they can only reset wages with probability $1 - \xi_w$. However there is partial indexation, so households that cannot re-optimize have their wages grow at a rate equal to the rate of inflation raised to the power $\gamma_w \in (0, 1)$. Firms combine the differentiated labor supplied by individuals into aggregate labor L_t via a Dixit-Stiglitz aggregator with power $1 + \lambda_{w,t}$. The power in the aggregator is allowed to vary over time to reflect changes in market power, but is assumed to be i.i.d. around a constant mean: $\lambda_{w,t} = \lambda_w + \eta_t^w$. Households set wages subject to their individual labor demand curves, which arise from the firms' input demands. Optimal wage setting by the household leads to the following evolution of the real wage w_t:

$$\begin{aligned} w_t \quad = \quad & \frac{\beta}{1+\beta} E_t w_{t+1} + \frac{1}{1+\beta} w_{t-1} + \frac{\beta}{1+\beta} E_t \pi_{t+1} - \frac{1+\beta\gamma_w}{1+\beta}\pi_t + \frac{\gamma_w}{1+\beta}\pi_{t-1} \\ & - \frac{\lambda_w(1-\beta\xi_w)(1-\xi_w)}{(1+\beta)(\lambda_w+(1+\lambda_w)\sigma_L)\xi_w} \left(w_t - \sigma_L L_t - \frac{\sigma_c}{1-h}(C_t - hC_{t-1}) - \epsilon_t^L - \eta_t^w \right). \end{aligned} \tag{2}$$

The final term captures variation in the current period marginal utility, where $1+\sigma_L$ is the utility parameter for the disutility of labor, β is the subjective discount factor, and ϵ_t^L is a preference shock in labor supply, which is $AR(1)$ with correlation ρ_L. As in usual Calvo pricing models, (2) incorporates forward-looking expectations of future nominal wages, but now includes lagged inflation via the partial indexation.

Households own the capital stock K_t, which they rent to firms at rental rate r_t^k. Households face adjustment costs in investment I_t, where the costs are assumed to be a function of the changes in investment, but subject to a shock ϵ_t^I which is $AR(1)$ with correlation ρ^I. Instead of incurring costs, households may also change the rate of utilization of existing capital which itself entails utilization costs. Around the steady state, the adjustment costs are assumed to be zero and only of second order. Thus the costs do not affect the linearized capital evolution, which is given by:

$$K_t = (1-\tau)K_{t-1} + \tau I_{t-1} \tag{3}$$

where τ is the depreciation rate. The optimal investment decision leads to a linearized Euler equation:

$$I_t = \frac{1}{1+\beta} I_{t-1} + \frac{\beta}{1+\beta} E_t I_{t+1} + \frac{\varphi}{1+\beta} Q_t + \frac{\beta E_t \epsilon_{t+1}^I - \epsilon_t^I}{1+\beta}, \tag{4}$$

where Q_t is the real value of capital and φ is the inverse of the adjustment costs. This equation balances the costs and benefits of investment, with lagged investment and the shocks showing up through the effects of the costs of adjustment. The equilibrium real value of capital itself is determined via a typical asset pricing Euler equation which gives:

$$Q_t = -(i_t - E_t\pi_{t+1}) + \frac{1-\tau}{1-\tau+\bar{r}^k} E_t Q_{t+1} + \frac{\bar{r}^k}{1-\tau+\bar{r}^k} E_t r_{t+1}^k + \eta_t^Q, \tag{5}$$

where here η_t^Q is an i.i.d. equity premium shock (which is not based on the primitives of the model) and $\bar{r}^k$ is the mean real rate of return on capital which is assumed to satisfy $\beta = 1/(1-\tau+\bar{r}^k)$.[6]

On the production side, there are a continuum of intermediate goods producers who are monopolistic competitors. Their products are aggregated into a single final good which is used for consumption and investment via a Dixit-Stiglitz aggregator. As in the labor market, the power in the aggregator is assumed to be stochastic and is i.i.d. around a constant mean: $\lambda_{p,t} = \lambda_p + \eta_t^p$. The intermediate goods producers face fixed costs in production, and are all subject to a common technology shock, ϵ_t^a which is $AR(1)$ with correlation ρ_a. The firms have common constant returns Cobb-Douglas production functions with parameter α, which

[6] SW interpret the equity premium shock as reflecting variation in the external finance premium, but acknowledge that its appearance here is ad hoc. Our equation (5) corrects a minor typo in SW.

leads to the implication that the capital-labor ratio is identical across firms. The linearized version of a firm's cost minimization condition expresses the firms' demand for labor:

$$L_t = w_t + (1+\psi)r_t^k + K_{t-1}, \tag{6}$$

where ψ is the inverse of the elasticity of the capital utilization cost function. Because there is no data for the Euro area on hours worked, Smets and Wouters (2003) model employment as the labor variable. As an admitted shortcut, they assume that firms face Calvo-type rigidity in adjusting employment (e_t) with unobservable hours (L_t) adjusting as a residual. This leads to the evolution of employment:

$$e_t = \beta e_{t+1} + \frac{(1-\beta\xi_e)(1-\xi_e)}{\xi_e}(L_t - e_t), \tag{7}$$

where ξ_e is the fraction of firms who are able to adjust employment in any given period.

As households do in the labor market, firms also face Calvo-type nominal rigidities in price setting, and the model allows for partial indexation. Each firm may change its price in a given period with probability $1-\xi_p$, but those firms that do not re-optimize see their prices increase by a rate equal to the rate of inflation raised to the power γ_p. The firm's optimal price setting condition leads to a generalized New Keynesian Phillips curve as:

$$\pi_t = \frac{\beta}{1+\beta\gamma_p}E_t\pi_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p}\pi_{t-1} + \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta\gamma_p)\xi_p}(\alpha r_t^k + (1-\alpha)w_t - \epsilon_t^a + \eta_t^p). \tag{8}$$

Here lagged inflation arises due to the indexation, and the final term in the equation represents the contribution of marginal costs.

Equilibrium in the goods market obtains when production equals the demand for goods by households and the government. Fiscal policy is assumed to be Ricardian, and variations in government spending are modeled as an $AR(1)$ shock ϵ_t^G with correlation ρ_G. The linearized goods market equilibrium condition is:

$$Y_t = c_y C_t + g_y \epsilon_t^G + \tau k_y I_t + \bar{r}^k k_y \psi r_t^k = \phi\epsilon_t^a + \phi\alpha K_{t-1} + \phi\alpha\psi r_t^k + \phi(1-\alpha)L_t. \tag{9}$$

The left side of the equation expresses the demand for output, where Y_t is output, c_y, g_y, and k_y are the steady state ratios of consumption, government spending, and capital to output. As we show in Appendix A, our equation (9) corrects a slight error in Smets and Wouters (2003) due to the capital utilization costs which enters as the final term on the left side. The right side expresses the supply of output via production, where ϕ is 1 plus the share of fixed costs in production.

Finally, Smets and Wouters (2003) close the model by specifying an empirical monetary policy reaction function. They specify policy in terms of a generalized Taylor-type rule, where the policy authority sets nominal rates in response to inflation and the output gap. To do this, they define a model-consistent output gap as the difference between actual and potential output, where potential output is defined as what would prevail under flexible prices and wages and in the absence of the three "cost-push" shocks $(\eta_t^w, \eta_t^Q, \eta_t^p)$ coming from variations in wage and price markups and the equity premium. Thus the model is supplemented with flexible-price versions of (1)-(9) which determine the potential output Y_t^*. Then the policy rule is assumed to take the following form:

$$\begin{aligned} i_t &= \rho i_{t-1} + (1-\rho)\left(\bar{\pi}_t + r_\pi(\pi_{t-1} - \bar{\pi}_t) + r_y(Y_{t-1} - Y_{t-1}^*)\right) \\ &\quad + r_{\Delta\pi}(\pi_t - \pi_{t-1}) + r_{\Delta y}(Y_t - Y_t^* - (Y_{t-1} - Y_{t-1}^*)) + \eta_t^R. \end{aligned} \tag{10}$$

Here $\bar{\pi}_t$ is an $AR(1)$ shock to the inflation objective with correlation ρ_π, while η_t^R is an i.i.d. policy shock. Note that the rule allows for the Taylor (1993) effects, along with interest rate smoothing, and responses to changes in inflation and the output gap.

This completes the specification of the model. It is relatively large scale, consisting of the equations (1)-(10) and their flexible price counterparts which determine the behavior of ten endogenous variables (and their flexible price versions) and which are subject to ten different stochastic shocks. We now turn to estimation of the model, which presents some difficulties.

3 Estimation

This section describes the results of our estimation of the Smets and Wouters (2003) model described above. As in Smets and Wouters (2003) we estimate the model using Bayesian methods, which in our case is equivalent to maximum likelihood estimation over bounded ranges. We first describe the formulation of our prior, then turn to estimation of the model. Finally, we look at the implications of the estimates for certain qualitative features of the model.

3.1 Prior Specification

In order to judge the usefulness of the model for policy purposes, it is crucial to know first how sensitive the empirical performance of the model is to different prior assumptions. One option would be to consider a classical estimation method, but the relative size of the model and the number of parameters to be fit made Bayesian estimation almost a necessity. In our case the prior simply delimits the range of possible parameter values over which we maximize the likelihood function.[7] Moreover, while there are few directly comparable studies to Smets and Wouters (2003), there are a number of studies in the literature which provide insight on some of the key parameters of the model. Thus we formulate our own prior, which is significantly less informative along many dimensions than that used by Smets and Wouters (2003), but does incorporate knowledge from the literature about the range of reasonable parameter values.

First, we follow Smets and Wouters (2003) in fixing several parameters throughout. We use the same values they do, which are all relatively standard. This includes setting the wage markup to $\lambda_w = 0.5$, the Cobb-Douglas production parameter to $\alpha = 0.3$, the subjective discount factor to $\beta = 0.99$, and the (quarterly) depreciation rate to $\tau = 0.025$. As Smets

[7] As our estimates hit a number of boundaries from our prior, we have also estimated the model with a smoother prior which yielded estimates in the interior of the parameter space. Our results were largely similar.

Parameter	Meaning	Range	Low Value	Source	High Value	Source
φ	Inv. Adj. Cost	0.12-0.28	0.13	ACEL	0.28	CEE
σ_C	Cons. Utility	1-4	Common	-	Common	-
h	Cons. Habit	0.4-0.9	0.57	SW	0.9	BCF
σ_L	Labor Utility	1-3	Common	-	Common	-
ϕ	Fixed cost	1-1.8	Lower bd.	-	1.8	SWUS
ψ	Cap. Util. Cost	2.8-10	2.9	SWUS	10	KR
ξ_w	Calvo wages	0.65-0.85	0.66	RW	0.85	SWUS
ξ_p	Calvo prices	0.4-0.93	0.42	ACEL	0.93	SWUS
ξ_e	Calvo employment	0.4-0.8	0.6	SW	0.6	SW
γ_w	Wage indexation	0-1	Lower bd.	-	Upper bd.	-
γ_p	Price indexation	0-1	Lower bd.	-	Upper bd.	-
r_π	Policy, inflation	1-4	Lower bd.	-	Upper bd.	-
$r_{\Delta\pi}$	Policy, inf. gr.	0-0.2	Lower bd.	-	0.18	SWUS
ρ	Policy, lag interest	0.6-0.99	0.63	S	0.96	SW
r_y	Policy, output gap	0-1	0.04	SWUS	0.98	JR
$r_{\Delta y}$	Policy, out. gap gr.	0-1	0.03	JR	Upper bd.	-

Sources: ACEL=Altig, Christiano, Eichenbaum, and Linde (2002), BCF =Boldrin, Christiano, and Fisher (2001), CEE=Christiano, Eichenbaum, and Evans (2005), JR= Judd and Rudebusch (1998), KR= King and Rebelo (1999), RW=Rotemberg and Woodford (1999), S=Sack (1998), SW=Smets and Wouters (2003), SWUS=Smets and Wouters (2007).

Table 1: Prior specification for estimation of structural parameters.

and Wouters (2003) note, most of these parameters govern ratios of different variables in the steady state, but the Euro-area data set which we use is demeaned which precludes calculation of these ratios.

As is common, we set independent priors for each of the parameters which are combined to form the prior for the model. As already mentioned, in each case we take the prior to be uniform over a bounded range, which we viewed as a natural, relatively uninformative prior. Of course, at the edges of the range the prior is dogmatic, but we found it more natural to specify possible ranges for the estimates than say to calibrate parameters of a distribution of some different assumed form.[8] We had little prior information about the parameters in the shock processes, so we set quite loose priors. For each of the shocks, we set the prior on the standard deviations to range from zero to 10 times the posterior mode estimated by Smets and Wouters (2003). For the persistent shocks, we set the range of the prior on the autocorrelation parameters to the entire unit interval. As we discuss below, this matters for the identification of some of the structural shock processes, as in some cases separate persistent and i.i.d. shocks enter additively.

For the structural parameters, we did a brief survey of the related literature, which is summarized in Table 1. We make no claim that this survey is exhaustive, but it serves to delimit the range of reasonable parameter values. As we noted above, the paper by

[8]See Berger (1985) for a discussion of alternative priors and means of prior elicitation. Also recall our discussion above, where we note that largely similar results obtained with a smoother prior specification.

Smets and Wouters (2003) is the only one that estimates a model of this form on this data set. However recent papers by Christiano, Eichenbaum, and Evans (2005) and Altig, Christiano, Eichenbaum, and Linde (2002) work with similar models on US data, using different estimation/calibration methods than Smets and Wouters. The follow-up paper Smets and Wouters (2007) considers a slightly modified version of the Euro-area model and fits it to US data. For some of the other parameters, we looked at studies focusing on real models (King and Rebelo (1999) and Boldrin, Christiano, and Fisher (2001)) as well as papers focusing on monetary policy in smaller models (Rotemberg and Woodford (1997), Judd and Rudebusch (1998), Sack (1998)). We also consulted a number of other papers which we do not list whose estimates fell within the ranges we outlined above. Finally, for some parameters we set ranges with less direct guidance from the literature. Our prior for the utility curvature parameters reflect relatively standard ranges (although they exclude the very high curvature found in Rotemberg and Woodford (1997)). For the price and wage indexation parameters, we included the entire range from no indexation to full indexation. For the parameter governing the Calvo-style adjustment of employment, which does not really have any precedent in the literature, we consider an evenly spaced interval around the point estimate of Smets and Wouters (2003). Finally, for the policy reaction to inflation we included a large range from the critical value of 1 (hence we respect the "Taylor-principle") to a very high value of 4. Most of the literature estimates coefficients from 1-1.8, which is well within our range.

3.2 Estimating the Model

With the prior specified, we now turn to the estimation of the model. As in Smets and Wouters (2003), we look for a parameter vector which maximizes the posterior mode, given our prior and the likelihood based on the data. Once again, this is equivalent in our case to simply maximizing the likelihood over the support of our prior. We use the same Euro-area data set, described in Fagan, Henry, and Mestre (2001), which provides quarterly data from 1970-1999, and we use the same detrending procedure (taking separate linear trends out of each variable). Relative to SW, however we use the whole data set, whereas they use the data from the 1970s to "initialize their estimates" and use only the data from 1980 onward for estimation.[9]

While estimation of the model is conceptually straightforward, there are a number of difficulties which we met in practice. To estimate the model, we must maximize the posterior over all the parameters, which is a challenging numerical task. In addition to the 16 structural parameters detailed in Table 1, we also have to find the 10 standard deviation parameters for the stochastic shocks, and the 6 autocorrelations for the persistent shocks, for a total of 32 parameters. Although many numerical optimization methods can in principle handle such a large parameter vector, all of the methods that we tried settled in to local maxima for a range of starting values. We tried the simplex algorithm in Matlab, Chris Sims's algorithms designed to avoid common problems with likelihood functions (available on his web page), and a global search genetic algorithm. The method that produced the greatest value of the

[9]We also re-estimated the model using the SW prior including the data from the 1970s. This had very little consequence for the estimates.

Parameter	Shock	Prior Range	Our Estimate	SW Estimate	SW SE
σ_a	Productivity	0-6	0.343	0.598	0.113
σ_π	Inflation objective	0-1	1.000	0.017	0.008
σ_b	Preference	0-4	0.240	0.336	0.096
σ_G	Govt. spending	0-4	0.354	0.325	0.026
σ_L	Labor supply	0-36	2.351	3.520	1.027
σ_I	Investment	0-1	0.059	0.085	0.030
σ_R	Interest rate	0-1	0.000	0.081	0.023
σ_Q	Equity premium	0-7	7.000	0.604	0.063
σ_p	Price markup	0-2	0.172	0.160	0.016
σ_w	Wage markup	0-3	0.246	0.289	0.027
ρ_a	Productivity	0-1	0.957	0.823	0.065
ρ_π	Inflation objective	0-1	0.582	0.924	0.088
ρ_b	Preference	0-1	0.876	0.855	0.035
ρ_G	Govt. spending	0-1	0.972	0.949	0.029
ρ_L	Labor supply	0-1	0.974	0.889	0.052
ρ_I	Investment	0-1	0.943	0.927	0.022

Table 2: Point estimates for the shock processes, with the σ parameters being standard deviations and the ρ parameters autocorrelations, compared to the estimates of Smets and Wouters (2003) and their standard errors.

posterior was the genetic algorithm, which in turn was initialized after extensive previous search and then locally refined with Sims's algorithm. In particular, we first ran a very long random search algorithm, drawing points from our prior distribution and calculating the posterior. From this, we kept the 40 best points, which we then used as the initial population for a genetic algorithm. Once this algorithm appeared to converge, we applied Sims's algorithm to the resulting value, which resulted in further localized improvement in the likelihood. We examined many other initial conditions and never obtained a result which improved upon this value, but of course we cannot guarantee that we have found a true global maximum.[10]

In Table 2 we report the estimates of the parameters in the exogenous shock processes, with the estimates of Smets and Wouters (2003) given for comparison.[11] While a number of our estimates appear roughly consistent with theirs, there are also a number of marked differences. The inflation objective shock that we estimate has much more volatile innovations (by a factor of nearly 6), but is much less persistent. Our i.i.d. equity premium shock is also much more volatile, by an order of magnitude. This may reflect a difference in the scaling of how the shock enters the structural equation. In addition, the i.i.d. policy shock is effectively zeroed out in our estimation. This can be understood by noting that in (10) the "inflation objective" shock $\bar{\pi}_t$ and the "policy" shock η_t^R enter additively. Thus we effectively estimate a combination of the two, while SW were only able to separately identify them by

[10]Once again, our analysis with a smoother prior gave largely similar results, but the gradient-based optimization worked much better.

[11]The calculation of standard errors for our estimates is an ongoing task, which is complicated by the fact that our estimates are on a number of boundaries in the parameter space.

Parameter	Meaning	Prior Range	Our Estimate	SW Estimate	SW SE
φ	Inv. Adj. Cost	0.12-0.28	0.152	0.148	0.022
σ_C	Cons. Utility	1-4	2.178	1.353	0.282
h	Cons. Habit	0.4-0.9	0.400	0.573	0.076
σ_L	Labor Utility	1-3	3.000	2.400	0.589
ϕ	Fixed cost	1-1.8	1.800	1.408	0.166
ψ	Cap. Util. Cost	2.8-10	2.800	5.917	2.626
ξ_w	Calvo wages	0.65-0.85	0.704	0.737	0.049
ξ_p	Calvo prices	0.4-0.93	0.930	0.908	0.011
ξ_e	Calvo employment	0.4-0.8	0.400	0.599	0.050
γ_w	Wage indexation	0-1	0.000	0.763	0.188
γ_p	Price indexation	0-1	0.323	0.469	0.103
r_π	Policy, inflation	1-4	4.000	1.684	0.109
$r_{\Delta\pi}$	Policy, inf. gr.	0-0.2	0.181	0.140	0.053
ρ	Policy, lag interest	0.6-0.99	0.962	0.961	0.014
r_y	Policy, output gap	0-1	0.062	0.099	0.041
$r_{\Delta y}$	Policy, out. gap gr.	0-1	0.319	0.159	0.027

Table 3: Point estimates for the structural parameters compared to the estimates of Smets and Wouters (2003) and their standard errors.

prior restrictions. Even though it is persistent, it decays rather quickly and is thus more appropriately thought of as a (negative) shock to monetary policy than as a change in the inflation objectives of the central bank.

Less striking but still sizeable are the differences in the labor supply, productivity, and preference shocks, with our estimates less volatile but more persistent in each case. Thus we find that overall the innovations from the shocks with less direct micro-foundations, the equity premium shock and the inflation objective shocks, are much larger under our estimates, while many of the structural shocks have smaller innovations but are more persistent. Of course the proportion of volatility explained by any of the shocks depends on the structural parameters, which we turn to next.

Table 3 gives the estimates of the structural parameters, with the estimates and standard errors of Smets and Wouters (2003) again given for comparison. A number of our estimates are on the boundaries of our prior range. To the extent that we set our prior to reflect reasonable ranges of estimates, this is not terribly troubling in itself. But it does suggest that the data may favor some parameter values which may be implausible from an economic viewpoint, and hence are not in the support of our prior.[12] Turning now to the differences with SW, again we find that a number of the parameter estimates are similar but there are also significant differences. For example, the parameters describing agents' preferences are fairly different. Our estimates of the curvature of preferences is greater for both consumption and labor, but the habit stock parameter is much smaller. We find relatively large fixed costs, amounting to 80% of the share of production. If we assume that profits are zero in the steady

[12]Our smoother prior also allowed for wider parameter ranges, and our estimates there were interior. Again, this had little effect on our results.

state as in Christiano, Eichenbaum, and Evans (2005), then this amounts to an average price markup factor of $\lambda_p = 0.8$ as well, which is substantial. We also find a relatively small value capital utilization adjustment parameter, which implies relatively large costs as ψ is the inverse elasticity. Although large, both of these estimates are close to the values found by Smets and Wouters (2007) in their analysis of a similar model for US data. For the wage and price dynamics, our estimates of the Calvo adjustment parameters are similar, although our employment adjustment parameter is much smaller. However we find much less of a role for indexation, including no indexation of wages. Finally, the policy reaction function that we estimate is substantially different. The inflation response coefficient is very large in absolute terms and more than double what SW estimate, representing a strong response to inflation. Moreover, our estimated output gap response is somewhat smaller, but the response to output gap growth is nearly double that in SW.

Overall, we note that in many cases our estimates are far outside reasonable confidence intervals around the SW estimates, suggesting that the different prior specifications have important effects. However it is a bit difficult to directly gauge the implications of the different parameter estimates on the behavior of the model. We next turn to some implications of the model under the different sets of estimates which makes this more clear.

3.3 Impulse Responses and the Output Gap

Although the estimates of some of the structural parameters are of interest in themselves, the impulse responses of the model show how the different parts of the model interact. We now plot the impulse responses of consumption, inflation, investment, and wages for each of the nine structural shocks. Recall that the policy shock is zeroed out in our estimates. In each case, we show the response under our estimates in a solid line and the estimates of SW in a dashed line. Overall, the qualitative features of the impulse responses are similar under the different sets of estimates. In each case, the variables respond in the same direction in response to a shock and the dynamics are rather similar. However there are some differences in the magnitude and persistence of the response to the different variables.

The most important case which exhibits substantial differences is shown in Figure 1 which plots the impulse responses of the variables to an inflation objective shock. But recall that here we are effectively identifying a different structural shock process than SW, one consistent with a policy shock. As such, the response of the variables under our estimates is roughly consistent with the literature on policy shocks (see Christiano, Eichenbaum, and Evans (1999)). In response to the (expansionary) policy shock, inflation and consumption rise, with the largest effect occurring after roughly four quarters. Investment and wages have more delayed, hump shaped effects. While the estimates of Smets and Wouters (2003) are orders of magnitude smaller, our results are qualitatively similar to their policy shock estimates.

The other case where our results differ substantially from those in SW concerns the equity premium shock, which is shown in Figure 2. Recall that we estimate the standard deviation of this shock to be more than a factor of ten larger than what SW estimate, which accounts for the larger differences in the magnitude of the responses of the variables to the shock. While it is a bit difficult to see from the figure, the qualitative dynamics are actually similar. An increase in the equity premium increases the demand for investment

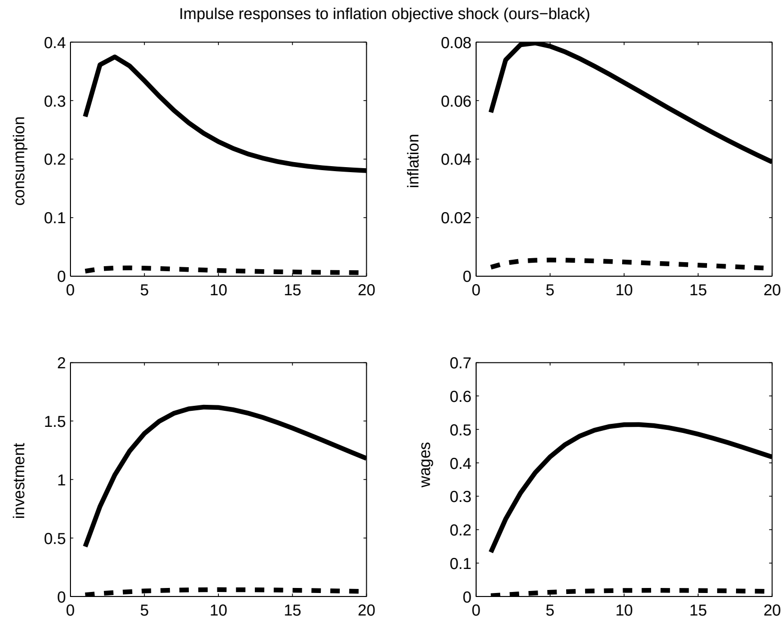

Figure 1: Impulse responses of selected variables to the inflation objective shock under Smets and Wouters's (2003) estimates (dashed lines) and our estimates (solid lines).

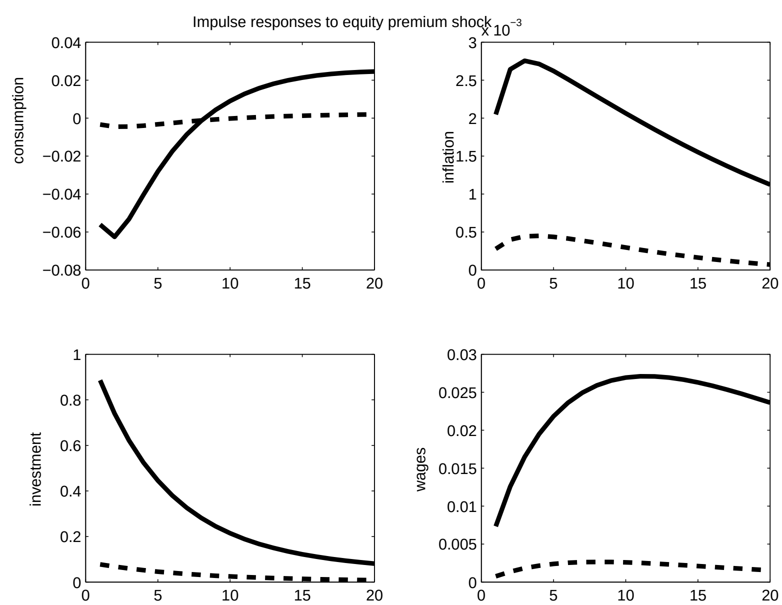

Figure 2: Impulse responses of selected variables to the equity premium shock under Smets and Wouters's (2003) estimates (dashed lines) and our estimates (solid lines).

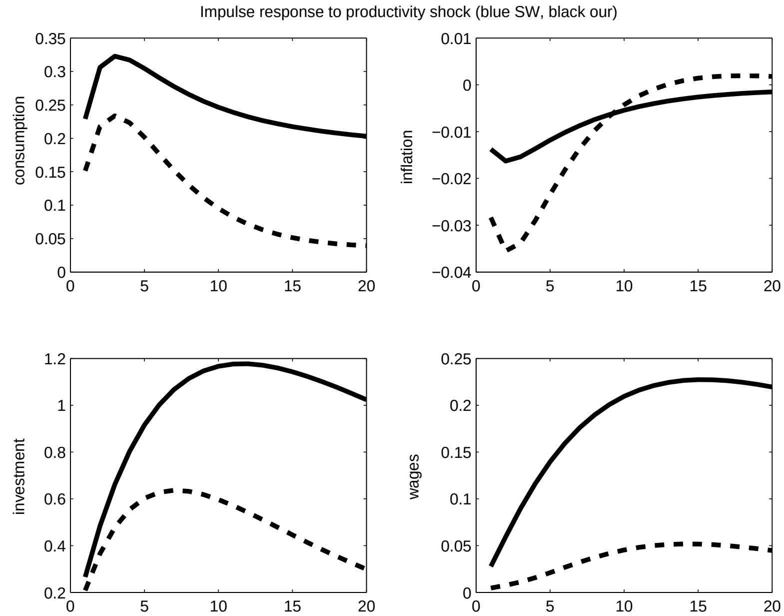

Figure 3: Impulse responses of selected variables to the productivity shock under Smets and Wouters's (2003) estimates (dashed lines) and our estimates (solid lines).

and decreases consumption, and results in a rise in inflation and real wages.

For all of the other shocks, the impulse responses seem economically reasonable, and the differences under the different estimates are not as substantial. In some cases, our estimates give larger and more prolonged responses than SW, but in some cases the reverse happens. For example, Figure 3 plots the impulse responses for a productivity shock, again showing a hump shaped increase in consumption peaking after roughly four quarters, and much more prolonged hump shaped positive responses of investment and real wages. Increased productivity negatively impacts inflation, with the peak again coming after roughly a year. Although the effects are qualitatively similar for our estimates and those of SW, for all variables but inflation our responses are larger and more persistent. Figure 4 shows that in response to an investment shock, investment, inflation, and wages rise, while consumption falls, and in each case the response is hump shaped and relatively persistent. In this case, our estimates are smaller and less persistent than SW.

The impulse responses to fluctuations in the wage markup and price markup are shown in Figures 5 and 6. In each case, the increased markup leads to a fall in consumption and investment and a rise in inflation, as the increased market power feeds through the economy. However real wages increase in response to a wage markup (unsurprisingly), while they fall in response to a price markup (as inflation increases and nominal wages are slow to adjust).

The impulse responses for the preference shock and the labor supply shock are shown in Figures 7 and 8. Here we find that for other than consumption, the effects are nearly mirror images of each other. In response to a positive preference shock or negative labor supply shock, investment and wages fall and inflation rises. However consumption rises in response to a preference shock as it increases the utility benefit of current consumption, while a negative labor supply shock makes the consumption/labor tradeoff less attractive,

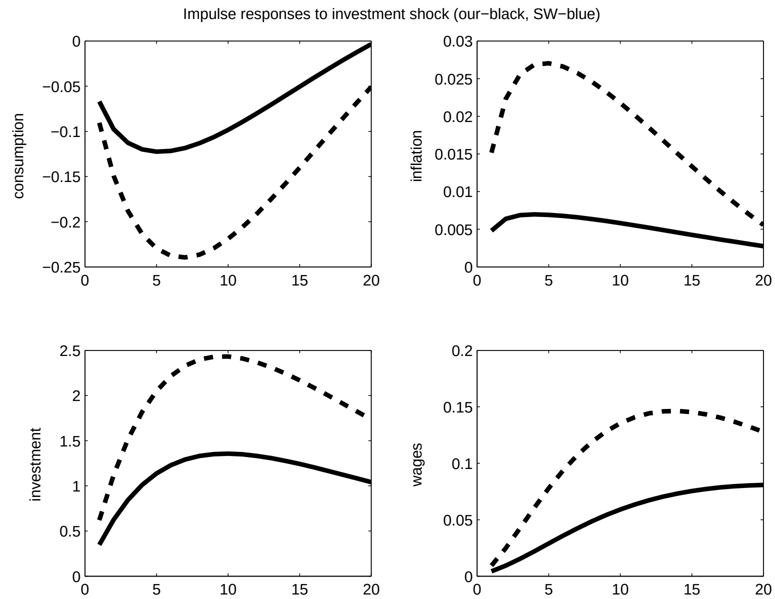

Figure 4: Impulse responses of selected variables to the investment shock under Smets and Wouters's (2003) estimates (dashed lines) and our estimates (solid lines).

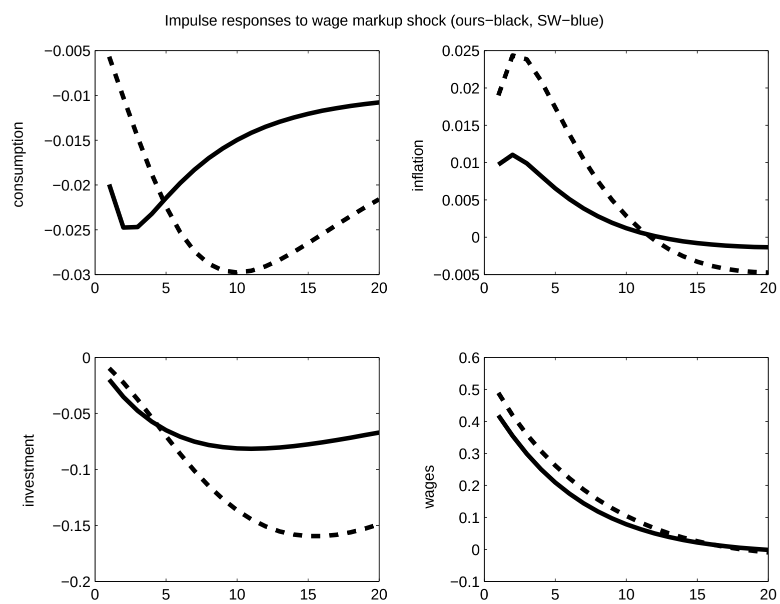

Figure 5: Impulse responses of selected variables to the wage markup shock under Smets and Wouters's (2003) estimates (dashed lines) and our estimates (solid lines).

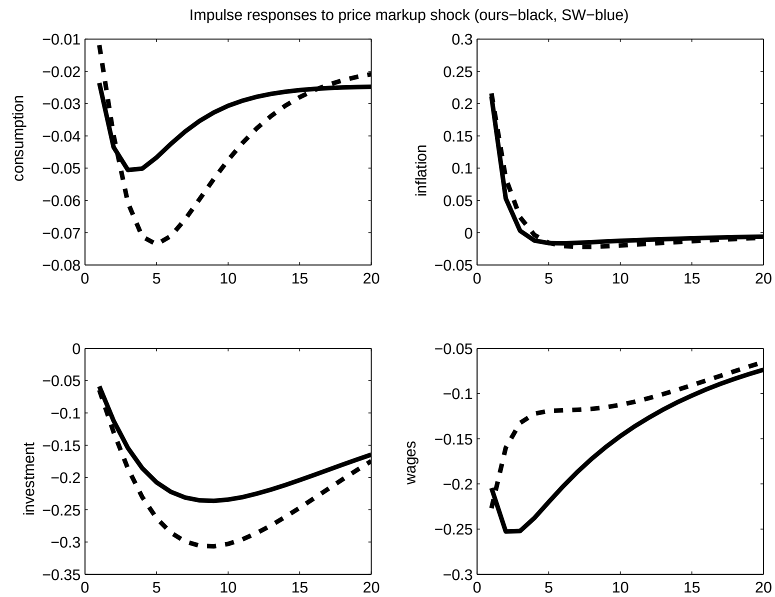

Figure 6: Impulse responses of selected variables to the price markup shock under Smets and Wouters's (2003) estimates (dashed lines) and our estimates (solid lines).

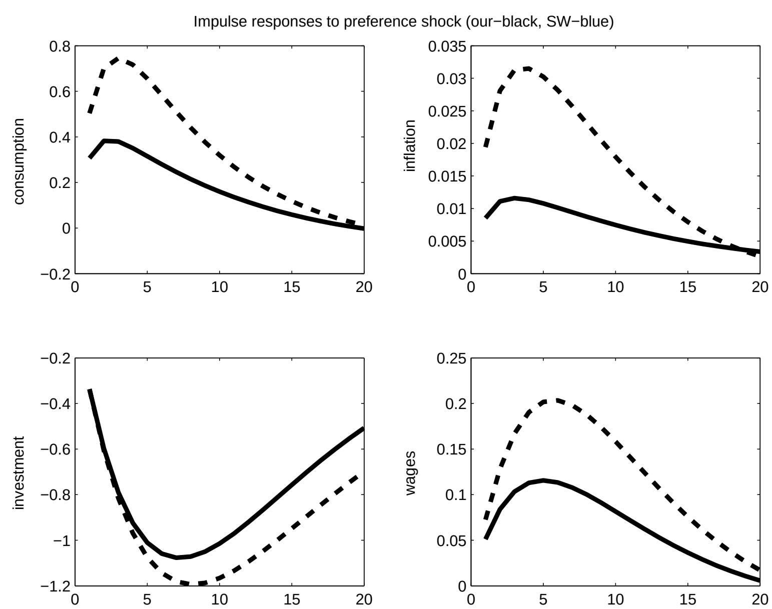

Figure 7: Impulse responses of selected variables to the preference shock under Smets and Wouters's (2003) estimates (dashed lines) and our estimates (solid lines).

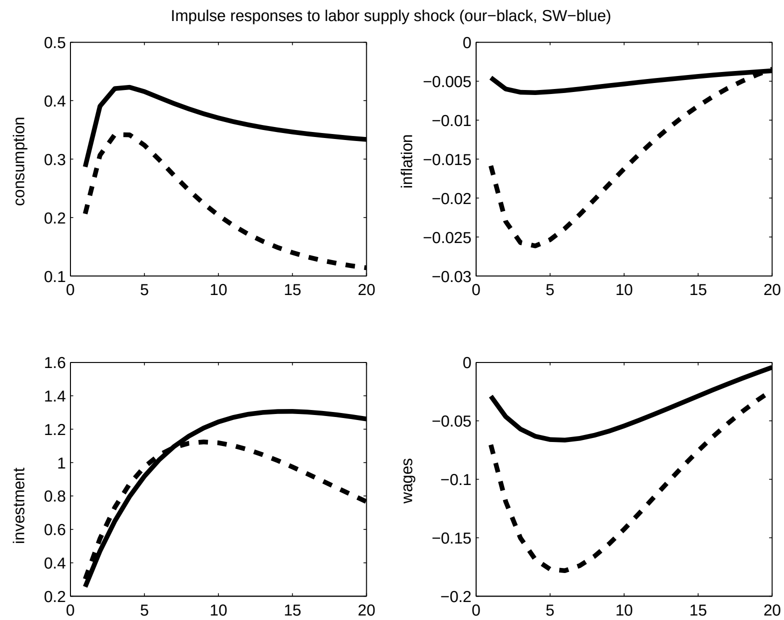

Figure 8: Impulse responses of selected variables to the labor supply shock under Smets and Wouters's (2003) estimates (dashed lines) and our estimates (solid lines).

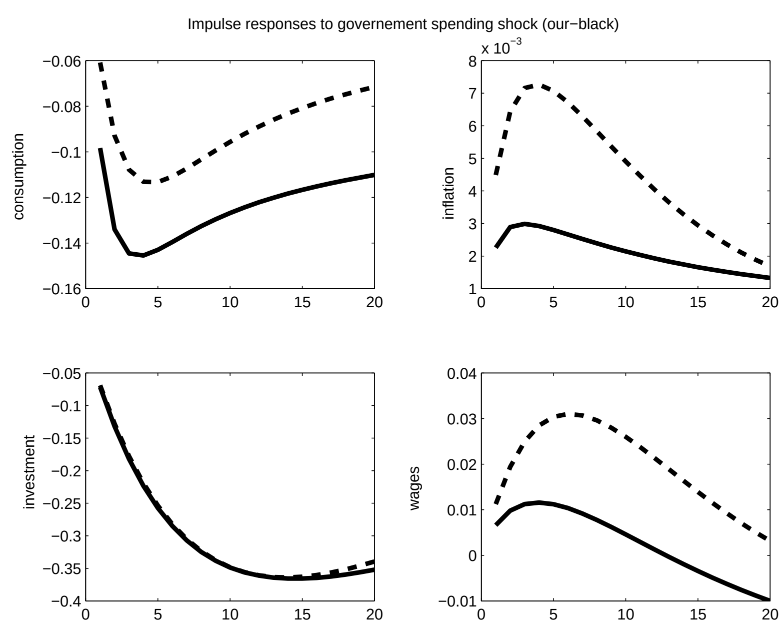

Figure 9: Impulse responses of selected variables to the government spending shock under Smets and Wouters's (2003) estimates (dashed lines) and our estimates (solid lines).

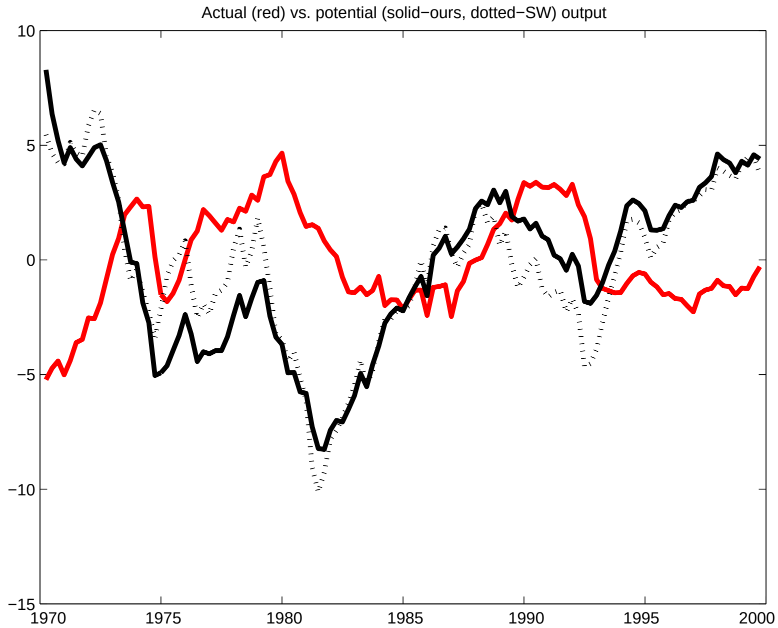

Figure 10: Actual output (red line) and the model-based estimate of potential output under Smets and Wouters's (2003) estimates (dashed line) and our estimates (solid line).

and thus leads to a fall in consumption. Finally, the responses to a government spending shock are shown in Figure 9. Again the qualitative features are similar, with increases in government spending leading to reductions in consumption and investment, and increases in the real wage and inflation.

One other aspect of the model that is interesting to compare is the time series of the potential output, and hence the output gap, which is implied by the model. In Figure 10 we plot the time series of output along with the level of potential output which is implied by our estimates and those of SW. Again we see that the qualitative features are very similar, with out estimated potential series being slightly smoother than that in SW. In particular, our estimates of potential are consistently lower over the first third of the sample and consistently higher over the second half. This suggest some slight changes in business cycle behavior, most notably implying that the boom early on was more pronounced, while that in the 1980s was much less substantial.

Although we have noted the dimensions along which our results differ from those in Smets and Wouters (2003), the overall message of our empirical results must be that the main features are quite similar. Even though our point estimates differ substantially for a number of key parameters, the impulse responses suggest that the model behaves similarly in response to exogenous shocks. In addition, the two sets of estimates imply very similar time series for the model-based measure of potential output. These findings suggest that the model may be over-parameterized: different sets of estimates lead to roughly the same empirical conclusions. However, as we see below, the different estimates lead to different policy conclusions. Thus there is some scope for formulating smaller models which may continue to capture most of the empirical features, while being more stable for policy purposes. This remains a difficult challenge. As a first step in this direction, and of independent interest in

Qualitative Feature	Sum of Squared Level/Difference	Most Influential Parameters	Coordinates of Max Hessian Eigenvector
Potential Output	Levels	ρ, ξ_p, ρ^I	0.96, 0.25, 0.09
	Differences	ρ, ξ_p, r_π	0.99, 0.07, 0.04
Impulse response of	Levels	ρ_π, ξ_p, r_π	0.87, 0.48, 0.08
π to policy shock	Differences	ρ_π, ξ_p, ξ_w	0.72, 0.68, 0.10
Impulse response of	Levels	ρ_π, ρ, r_π	0.96, 0.19, 0.15
Y to policy shock	Differences	ρ_π, ξ_p, ρ	0.68, 0.62, 0.36
Impulse response of	Levels	ρ_π, ξ_p, ρ	0.92, 0.32, 0.18
i to policy shock	Differences	ρ_π, ξ_π, ρ	0.82, 0.53, 0.14

Table 4: The most influential parameters for different qualitative features of the model.

its own right, we now analyze the relative importance of different aspects of the model in leading to the different qualitative features.

3.4 Qualitative Features of the Model

Given the size and complexity of the model, it is difficult to determine directly which parts of the model are important for different aspects of our results. In this section we provide one means of getting at this issue by seeing which parameters of the model are most influential in changing some of the key qualitative features of the model. Our results are summarized in Table 4. The table was constructed in the following way. First, we listed several qualitative features of the model which we felt were important to explain. Because of its interest in characterizing business cycles and hence for policy purposes, we consider first the time series of potential output. Additionally, much of the literature has focused on the response of different variables to a monetary policy shock. In particular, the paper by Christiano, Eichenbaum, and Evans (2005) which forms much of the basis of the Smets and Wouters (2003) model explicitly focuses on the effects of a policy shock. Thus we also analyze the impulse responses of inflation, output, and the nominal interest rate to a policy shock (which recall in our case is captured by the "inflation objective" shock $\bar{\pi}_t$).

For each of these qualitative features, we take the values from our estimates as the baseline values. Then we formulate a criterion which measures how the feature changes when we change the parameters, looking at the sum of squared differences in the levels and the differences, where we look at the entire candidate time series and the first 20 quarters of the impulse responses. We then look at the eigenvector corresponding to the maximum eigenvalue of Hessian matrix of each criterion. This measures direction of the steepest rate of increase in the criterion around our estimates.[13] The table then lists the three most influential parameters in each case, along with the values of their coordinates in the unit eigenvector.

The table shows that for each of these qualitative features, a few key parameters turn out to be very influential. For the estimated potential output time series, the interest rate smoothing coefficient ρ in the policy rule (10) is by far the most influential parameter. This suggests that the estimate of potential output may be very sensitive to the *a priori* specification of the policy rule. The Calvo price stickiness parameter ξ_p also matters somewhat for potential output. For the impulse responses to the policy shock, the persistence of the shock ρ_π is clearly and unsurprisingly very influential. However the price stickiness parameter is also quite important, particulary for the inflation response, and the interest rate smoothing parameter ρ is again of significant importance, particularly for the output and interest rate responses. We also see that response to inflation in the policy rule r_π is of some importance as well. This suggests that although the model has a number of frictions and sources of stickiness, for these features of the model the price stickiness and policy rule specifications are of the most importance.

4 Policy Analysis

We now turn from the empirical aspects of the model to its use in policy evaluation and design. In particular, we first analyze the optimal equilibrium dynamics, comparing our results to the equilibrium dynamics under the estimated policy rule. We then compare the optimal equilibrium dynamics from our estimates with the corresponding dynamics with the estimates of SW. Finally, we discuss the performance and robustness of simple policy rules in this environment.

4.1 The Loss Function

An obvious prerequisite for policy analysis is a specification of the policy objectives. We suppose that policymakers minimize an intertemporal loss function which is the discounted sum of all future and current one period losses. We consider a standard, but ad hoc one period loss given by:

$$\Lambda_t = \pi_t^2 + \Lambda_y (Y_t - Y_t^*)^2 + \Lambda_i i_t^2, \tag{11}$$

where as above Y_t^* is the potential output and Λ_y and Λ_i are positive weights. Note that as in Woodford (2003) the loss function penalizes variation in the level of the interest rate, but that an interest rate smoothing objective is not assumed. We consider a range of different

[13] More explicitly, say $\{I_t^\pi(\theta^*)\}$ gives the impulse response of inflation to a policy shock for $t = 1, \ldots, 20$ for our parameter vector θ^*. Then we analyze the sum of squared differences in levels L and differences D as:

$$L(\Delta) = \sum_{t=1}^{20} \left[I_t^\pi(\theta^* + \Delta) - I_t^\pi(\theta^*)\right]^2, \; D(\Delta) = \sum_{t=1}^{20} \left[(I_t^\pi(\theta^* + \Delta) - I_{t-1}^\pi(\theta^* + \Delta)) - (I_t^\pi(\theta^*) - I_{t-1}^\pi(\theta^*))\right]^2.$$

Then we construct the Hessian matrices $\left.\frac{\partial^2 L(\Delta)}{\partial \Delta \partial \Delta'}\right|_{\Delta=0}$ and $\left.\frac{\partial^2 D(\Delta)}{\partial \Delta \partial \Delta'}\right|_{\Delta=0}$.

possibilities for the relative weights on inflation, output gap, and interest rate variability. We will abuse notation and refer to different loss functions by their ordered pair of weights, thus we denote $\Lambda = (\Lambda_y, \Lambda_i)$.

While loss functions of this type are common in the literature, and have micro-foundations in some simpler models, it is natural in our setting to also consider the loss function consistent with agents' preferences. We do so in Levin, Onatski, Williams, and Williams (2006), where we show that the welfare-based loss function includes substantial weights on a number of different economic variables, with wage inflation perhaps being the most important. However, as discussed above, in this paper we focus solely on the simple loss function. Such a loss function seems to reflect the tradeoffs expressed in actual policy discussions and policy conduct. Moreover, it allows us to easily compare the consequences of different policies and different structural estimates. We've seen that many of the structural estimates differ across prior specifications, with the preference parameters in particular being substantially different. Thus the utility-based loss function would likely change dramatically with the different estimates, and so would policymaker preferences. We focus on the implications of the model for inflation and output stabilization, rather than conflating changes in the dynamics with changes in weights and objectives.

4.2 Optimal Equilibrium Formulation

With preferences specified, we now turn to the determination of the optimal equilibrium. For this analysis, it is convenient to represent the model in the form:

$$\begin{aligned} \begin{bmatrix} x_{1t+1} \\ \Gamma E_t x_{2t+1} \end{bmatrix} &= A \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} i_t + \begin{bmatrix} C\eta_{t+1} \\ 0 \end{bmatrix}, \text{ or :} \\ \hat{\Gamma} E_t x_{t+1} &= A x_t + B i_t + \hat{C}\eta_{t+1}, \end{aligned} \tag{12}$$

where the second line defines $\hat{\Gamma}$, B and $\hat{C}$. Here x_t is the state vector, which is partitioned into (x_{1t}, x_{2t}) so that x_{1t} is a n_1-dimensional vector of predetermined variables in the sense of Klein (2000). That is, the prediction error $x_{1t+1} - E_t x_{1t+1}$ is an exogenously given martingale difference process and the initial value x_{10} is given. x_{2t} is a n_2-dimensional vector of non-predetermined variables, i_t is the nominal interest rate, and η_t are serially uncorrelated shocks having unit covariance matrix.

Since our goal is the analysis of optimal policy, we drop the empirical policy reaction equation (10) from the model and represent the remaining system (1)-(9) in the above form. The optimal policy for models (12) with Γ equal to identity matrix was studied in Backus and Driffill (1986). Svensson and Woodford (2003) generalize the analysis to allow for a singular Γ matrix and unobservable variables.

To compute the optimal equilibrium, we rewrite the loss (11) in a more general form:

$$\Lambda_t = x_t' \Lambda_x x_t + 2 i_t' \Lambda_{ix} x_t + i_t' \Lambda_i i_t.$$

Then we formulate the Lagrangian associated with the loss and (12):

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ x_t' \Lambda_x x_t + 2 i_t' \Lambda_{ix} x_t + i_t' \Lambda_i i_t + 2\lambda_{t+1}' \left(A x_t + B i_t - \hat{\Gamma} E_t x_{t+1} \right) \right\}.$$

Taking the first order conditions and solving the resulting system we find, assuming that the solution exists and is unique, that:

$$\begin{bmatrix} x_{1t+1} \\ \lambda_{2t+1} \end{bmatrix} = F_1 \begin{bmatrix} x_{1t} \\ \lambda_{2t} \end{bmatrix} + S\eta_{t+1}, \quad \lambda_{20} = 0 \tag{13}$$

$$\begin{bmatrix} x_{2t} \\ i_t \\ \lambda_{1t} \end{bmatrix} = F_2 \begin{bmatrix} x_{1t} \\ \lambda_{2t} \end{bmatrix} \tag{14}$$

where F_1 and F_2 are matrices of constant coefficients and λ_{1t} and λ_{2t} are the sub-vectors of the vector of Lagrange multipliers corresponding to the first n_1 and last n_2 equations of the system (12) respectively. An algorithm for computing F_1 and F_2 is given, for example, in Söderlind (1999). These equations characterize the optimal equilibrium dynamics of the economy under a yet-to-be-specified optimal policy rule.

As was stressed by Backus and Driffill (1986) and Currie and Levine (1993), and more recently by Woodford (2003), there may exist many different forms of policy rules resulting in the same equilibrium dynamics of the economy. However, we can compute the optimal loss without actually specifying the form of the optimal policy rule. Indeed, equation (14) shows that in equilibrium the non-predetermined variables, x_{2t}, can be expressed as functions of the predetermined variables and multipliers, which we stack as $X_t = (x'_{1t}, \lambda'_{2t})'$. Hence the optimal equilibrium loss can be reformulated as quadratic function of these variables only. Formally, the one period loss can be expressed as:

$$\Lambda_t = X'_t \Lambda_X X_t,$$

for an appropriate choice of the matrix Λ_X. This loss and equation (13) constitute a standard purely backward-looking system. Therefore the loss can be computed using the standard formula:

$$E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t = X'_0 V X_0 + \Omega, \tag{15}$$

where the matrices V and Ω satisfy:

$$\begin{aligned} V &= \beta F'_1 V F_1 + \Lambda_X \\ \Omega &= \frac{\beta}{1-\beta} \mathrm{tr}\left(\mathrm{S'VS}\right). \end{aligned}$$

Since we focus on unconditional losses, the first term in (15) drops out.

4.3 Optimal Equilibrium Results

For a range of different relative weights, we computed the optimal loss and the corresponding loss implied by the estimated policy reaction function from (10). Using the simple loss from (11), we focused on a 900 point logarithmically-spaced grid over the weights $\Lambda \in [10^{-5}, 1] \times [10^{-5}, 1]$.[14] We started the grid from small positive values instead to insure against

[14] Since for the optimal rule the monetary policy and inflation target shocks are assumed to be zero, for the purpose of compatibility, we set these shocks to zero for the suboptimal rule, too.

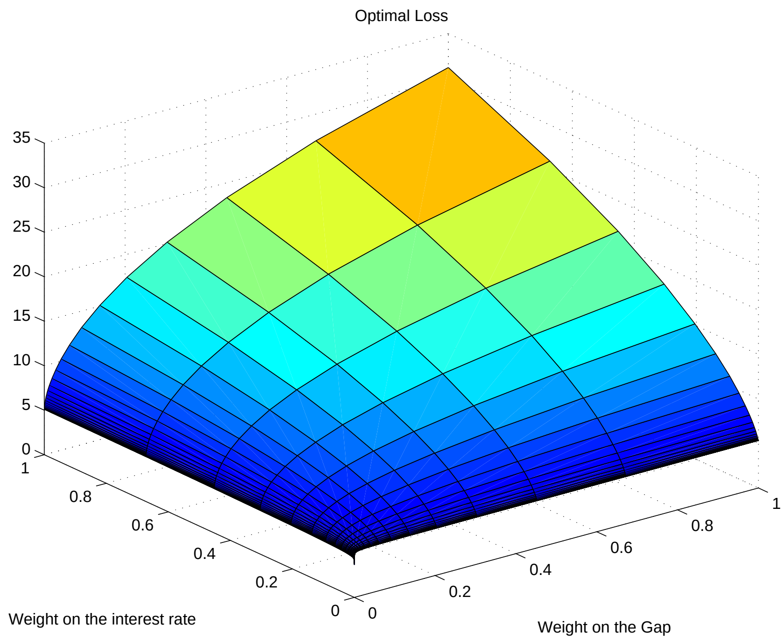

Figure 11: The optimal loss function versus the weights $\Lambda = (\Lambda_y, \Lambda_i)$ on output gap and interest rate variability.

problems that might be caused by the "singularity" of the loss function. The optimal loss is shown in Figure 11. We see that increasing weight on the output stabilization and, at the same time, penalizing the variability of the interest rate instrument increases the value of the loss dramatically. In particular, the loss rises from about 5 when the variability of the output gap and the interest rate are of no concern ($\Lambda \approx (0, 0)$) to about 32 when there are equal weights on the inflation, the output gap, and the interest rate in the loss function ($\Lambda = \bar{\Lambda} \equiv (1, 1)$). Loosely speaking, it is a difficult task for policymakers to stabilize inflation and the output gap with a stable interest rate. On the other hand, it is possible to control both inflation and the gap very effectively if there is no concern about instrument stability. Similarly, it is possible to keep inflation stable without interest rate variability, but at the cost of increased output gap variability.

Figure 12 reports the logarithm of the ratio of the "estimated" loss to the optimal loss. The estimated rule becomes extremely suboptimal for the loss function with equal weights on the inflation and output gap variability and no concern about the variability of the interest rate ($\Lambda \approx (1, 0)$). Interestingly, the estimated rule is only slightly less efficient than the optimal rule when policy makers care almost exclusively about inflation. The minimum ratio on our grid of 1.14 is achieved for weights we denote $\Lambda = \Lambda^* \equiv (0.0017, 0.0039)$. We can think of at least two reasons why the estimated rule is nearly optimal for Λ^*. First, it may be that European policy makers indeed have cared mostly about inflation and are not concerned much about the output gap or interest rate variability. In this case, our results suggest that the policy makers are using a policy that is nearly the best for controlling inflation. Alternatively, it may be that the interest rate simply does not have much effect on inflation in the SW model. In this case, any rule policymakers would use would result in nearly the same inflation variability. In particular, the inflation component of the loss would

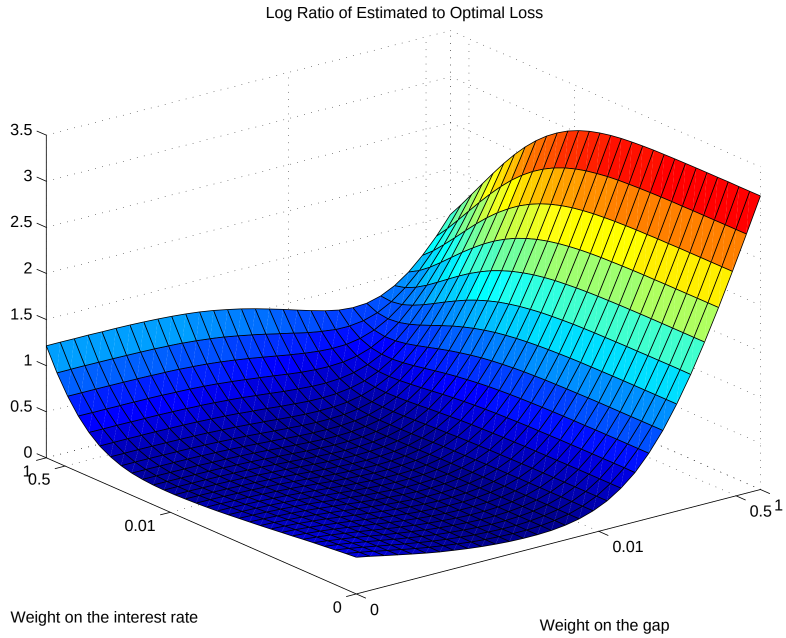

Figure 12: Log of the ratio of the loss under the estimated policy rule to the loss under the optimal rule.

Loss Function	Coefficient on:							
	ϵ^b	ϵ^I	ϵ^a	ϵ^L	ϵ^G	η^Q	η^p	η^w
$\bar{\Lambda}$	0.1888	-0.1339	-0.1175	-0.0108	0.0254	0.0051	-0.0109	0.0588
Λ^*	0.1171	-0.0669	-0.0644	-0.0064	0.0148	0.0028	-0.0072	0.0257

Table 5: Coefficients of the optimal policy rules corresponding to the shocks, for two loss functions.

stay roughly constant for any weights in the loss function. According to our calculations, the inflation component of the optimal loss is 4.78 for Λ^*, which puts very low weight on the output gap and interest rate variability. It is equal to 5.56, only about 16% larger, when instead the loss function is $\bar{\Lambda}$, which places equal weights on all terms. This suggests that, at least for the range of policies which are optimal for the losses we consider, policymakers can do relatively little to stabilize inflation in this model.

As we noted above, it is difficult to directly interpret the coefficients of an optimal policy rule, as the rule may have many different implementations. The most direct representation, which we give in (14) above, expresses the policy as a function of the predetermined variables and can be written: $i_t = F_{21,i}x_{1t} + F_{22,i}\lambda_{2t}$. In our formulation of the SW model, x_{1t} is a 16×1 vector composed of the lagged endogenous variables and current shocks, while λ_{2t} is a 15×1 vector of Lagrange multipliers. So it is difficult to gain much insight by looking at the policy function directly, particularly since the evolution of the multipliers λ_{2t} will vary with the specification of the loss function. In Table 5, we list the coefficients of the optimal policy rules corresponding to the shocks for the two loss functions $\bar{\Lambda}$ and Λ^*. Here we see that the coefficients of the rule for Λ^* are uniformly lower than for $\bar{\Lambda}$, suggesting that putting less weight on the output gap and interest rate movements leads to less aggressive responses to exogenous shocks, at least initially.

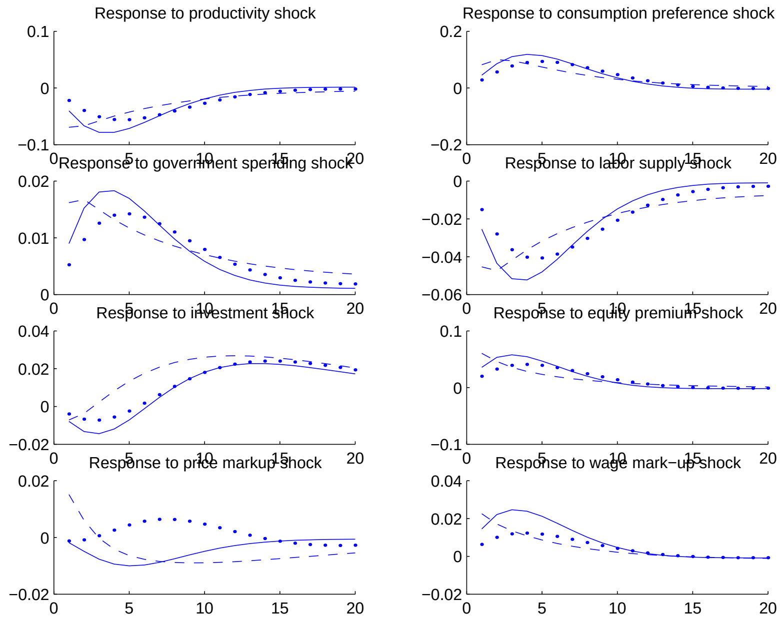

Figure 13: Impulse responses of the interest rate under the estimated policy rule (dashed lines) and the optimal policies for loss functions $\bar{\Lambda}$ (solid lines) which puts equal weights on all terms and Λ^* (dotted lines) which puts low weight on output gap and interest rate variability.

In order to further distinguish the policy effects, it is interesting to compare the impulse responses of the interest rate to different exogenous shocks under the optimal and the estimated policy rules. This gives an indication of the relative weight the different rules put on different sources of variability in the economy. The impulses for the rules which are optimal with losses $\bar{\Lambda}$ and Λ^* are reported in Figure 13 along with the impulses for the estimated policy rule. We see that with the exception of the response to price markup shock, all estimated impulse responses are similar to the optimal ones. Moreover, again apart from the price markup shock, the impulses for the optimal policies are qualitatively similar for the two different loss functions. However the optimal policy for $\bar{\Lambda}$ is more aggressive, as the interest rate reaction is larger in absolute value for nearly all of the shocks. We also note that the estimated impulses are somewhat closer to the optimal responses for $\bar{\Lambda}$, even though the estimated rule performs better in terms of losses for Λ^*. This suggests that although the estimated rule and optimal rule for Λ^* are close in terms of losses, they may result in fairly different dynamics. We will see this further below.

Figure 14 reports the impulse responses of inflation and output to the exogenous shocks. Again we consider the dynamics under the estimated rule and under the rules which are optimal for $\bar{\Lambda}$ and Λ^*. There are several interesting observations to be made. First, the estimated responses of inflation to the shocks seem to be larger than the optimal responses under both almost zero weights on the gap and interest rate stabilization and equal weights on inflation, the gap, and the interest rate. This lends further support to the view that the "near optimality" of the estimated rule for Λ^* does not tell anything about actual weights on the gap and interest rate stabilization that policy makers might have. Second, smaller output fluctuations under the optimal policy corresponding to $\bar{\Lambda}$ are achieved by making

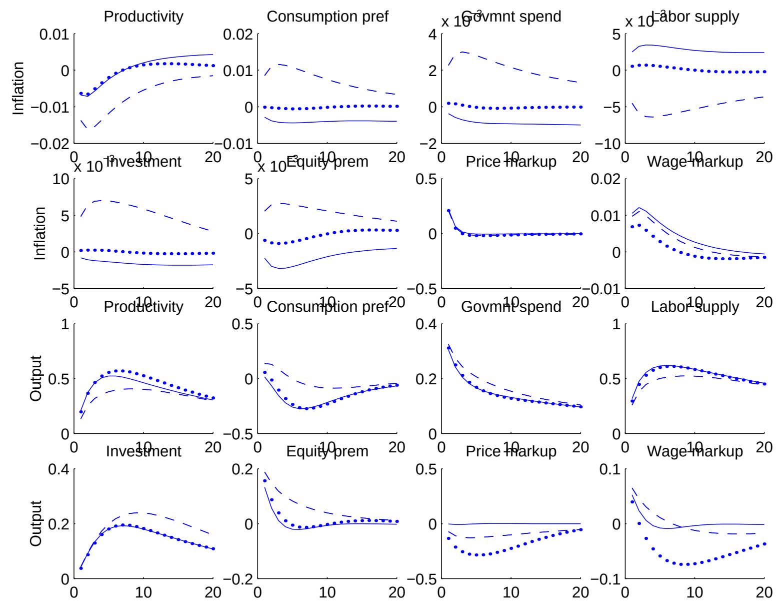

Figure 14: Impulse responses of output and inflation under the estimated policy rule (dashed lines) and the optimal policies for loss functions $\bar{\Lambda}$ (solid lines) which puts equal weights on all terms and Λ^* (dotted lines) which puts low weight on output gap and interest rate variability.

output relatively insensitive to price markup and wage markup shocks. This result is not surprising, as these shocks only affect the output gap, which get near zero weight under Λ^*. The estimated responses of output to the price markup and wage markup impulses are close to the optimal ones for $\bar{\Lambda}$. This suggests that the policy makers might care about the output after all and the "near optimality" of the estimated rule for Λ^* may be spurious.

4.4 Results for Smets and Wouters's Estimates

As we discussed in our empirical analysis, based on different prior assumptions and a slightly different estimation method, we arrived at rather different parameter estimates than Smets and Wouters (2003). We now study the implications of these different estimates for policy. We thus recompute the optimal equilibrium under the SW estimates and compare it to what we found above.

The optimal loss surface for the SW estimates on the space of the loss parameters $\Lambda \in [0,1] \times [0,1]$ is qualitatively very similar to that in Figure 11. Quantitatively, the optimal loss under the SW estimates is about the same as the optimal loss under our estimates when $\Lambda \approx (0,0)$ but is much larger (91 vs. 32) when $\Lambda = (1,1)$. The ratio of the optimal loss to that under the estimated policy rule is very similar to that shown in Figure 12. The minimum of the ratio on our grid is achieved at $\Lambda = (0.0012, 0.0189)$ (compared to $\Lambda^* = (0.0017, 0.0039)$ for our estimates), which still gives inflation the predominant weight. Figure 15 shows the optimal impulse responses of inflation and the output gap under our estimates (solid line) and the SW estimates (dashed line). In both cases the parameters of the loss function were taken to be $\Lambda = (0.5, 0.5)$. We see that many of the responses are similar, with the exception of the equity premium shock, which recall has much larger volatility under our estimates.

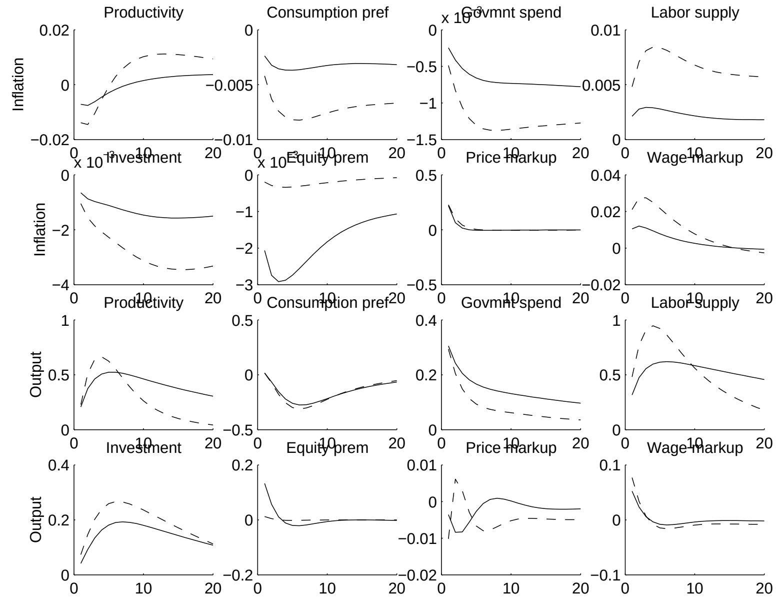

Figure 15: Optimal impulse responses of inflation and the output gap under our estimates (solid line) and the SW estimates (dashed line).

We see that the response of output to productivity and labor supply shocks is dampened initially but drawn out under our estimates.

Nonetheless, the differences in dynamics between the optimal policies under our and SW's estimates are smaller than between either set and the estimated policy rule. We saw above the relative robustness of the model in accounting for the qualitative dynamics, which were largely insensitive to the different structural estimates. This same robustness is preserved in the optimal equilibrium dynamics.

4.5 Simple Rules and Robustness

While we have focused on the optimal equilibrium dynamics, we have not focused on the implementation of this equilibrium. It turns out that optimal policy rules in the model are quite complex and difficult to summarize, and may not be robust to parameter variations. In particular, in an earlier version of the paper we documented one form of this robustness. Using two different representations of optimal rules, we found that the rules which implemented the optimal equilibrium for our parameter estimates led to indeterminacy when applied to an economy governed by the SW estimates (and vice versa). Of course this is sensitive to particular implementation, and alternative representations may not suffer from this indeterminacy problem. But the fact that we had difficulty finding a representation of the optimal rule which resulted in a determinate equilibrium under plausible parameter uncertainty suggests that an alternative approach may be warranted. In many cases, simple policy rules which react to only a few current variables are nearly as efficient as fully optimal rules in terms of performance and are more robust.[15] Hence this is a natural approach to consider

[15] Many of the contributions in Taylor (1999) arrived at this conclusion.

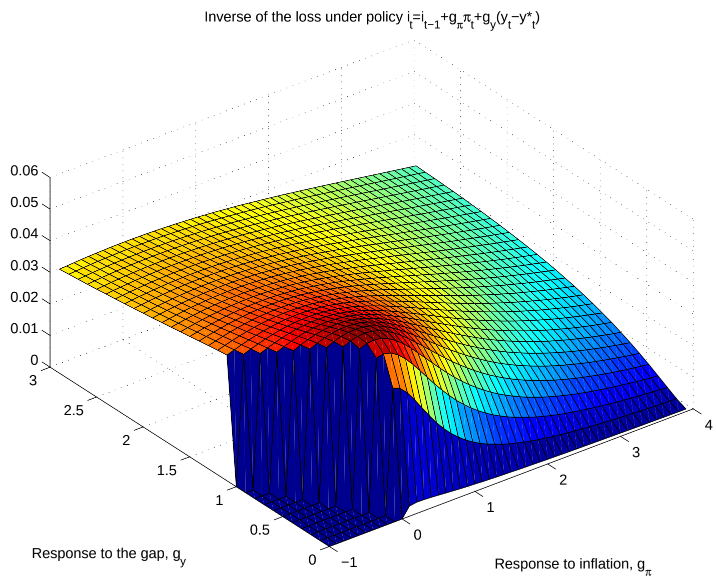

Figure 16: Inverse of the loss function for simple rules.

here.

For simplicity, we focus on the loss function (11) with coefficients of 0.5 each on the output gap and the interest rate, and we analyze simple Taylor-type policy rules of the form:

$$i_t = \rho i_{t-1} + r_\pi \pi_t + r_y(Y_t - Y_t^*).$$

We analyzed these rules over a grid on $\rho \in [0.5, 1.5]$, $r_\pi \in [-1 : 3]$, and $r_y \in [0, 2]$. We experimented with the grid and chose this one because it includes the rule minimizing the loss.

For our estimates of the model, the best rule on the grid was:

$$i_t = i_{t-1} + 0.4(Y_t - Y_t^*).$$

A surface plot of the *inverse* of the loss corresponding to the rules with $\rho = 1$ and r_π, r_y running through the grid is given in Figure 16. Interestingly, we find that the optimal simple rule is formulated in differences, which is also consistent with the findings in the utility-based analysis of this model (with different estimates) by Levin, Onatski, Williams, and Williams (2006). Surprisingly, the optimal rule here does not respond to inflation at all, setting changes in interest rates solely in response to the output gap. (This differs from Levin, Onatski, Williams, and Williams (2006), as we discuss below.) The nominal anchor in the policy rule is entirely the nominal interest rate. Of equal importance, the loss under this rule is equal to 19.51, which is only 6% higher than the optimal loss of 18.34. Thus even though the model is sufficiently complex and has significant dynamics, simple rules are able to adequately stabilize inflation, the output gap, and interest rates.

Interestingly, this exact same optimal simple rule remains optimal (on our grid) when we adopt Smets and Wouters' estimates of the model's coefficients. In such a case, the corresponding loss is equal to 52.33, which is again about 6% larger than the optimal loss of 49.59. Thus simple rules are truly robust here. We can think of the two sets of parameter estimates as very particular competing reference models in the sense of Levin and Williams (2003). The fact that the optimal rule is the same under each case means that it is also the minimax rule for this type of model uncertainty. Thus the model is "highly tolerant" to different parameter estimates when using simple rules. It was rather surprising to us to find out that such a simple rule performs only marginally worse than the optimal rule, which is necessarily very complicated merely because of the size of the model. It was even more surprising that the same rule is the best for two rather different sets of parameter estimates.

As is to be expected, such results do depend heavily on our choice of a simple loss function which is independent of the parameter estimates. In Levin, Onatski, Williams, and Williams (2006) we also find that simple difference rules perform relatively well with a utility-based loss. However the rule, which reacts strongly to wage inflation alone, is substantially different. While Levin, Onatski, Williams, and Williams (2006) do show that the model is reasonably robust to their estimated parameter uncertainty, it is unlikely that our strong robustness result here would be preserved with a utility-based loss as our two sets of estimates imply a greater divergence in values than they analyzed.

5 Conclusion

The basic goal of this project was to study the usefulness of a fully specified forward looking model for monetary policy purposes. We were impressed by the empirical performance of Smets and Wouters (2003) but were unsure of the importance of their empirical success for policy. The answer to this question depends on the exact uses of models in policy analysis. As a description of the data, we find that the model is a qualified success. By re-estimating the model imposing less prior information, we arrived at different point estimates, but many of the qualitative features of the model remained unchanged. However as a practical guide for policy, the results are less reassuring. Optimal policy in the model results in significant dynamics which may be difficult to implement, and may not be robust to alternative parameter estimates. Promising results were found by sacrificing full optimality in favor of simple rules. While empirical dynamic stochastic general equilibrium models have indeed come a long way, they are not yet able to deliver sharply identified and interpretable structural parameter estimates which are robust to different prior assumptions. However our findings do suggest that the current models are indeed able to robustly capture many qualitative features of the data, and to provide policy guidance for stabilizing output and inflation.

Appendix

A Correcting the Equilibrium Condition

In this Appendix we derive the corrected version of the goods market equilibrium condition we discussed above. We start with the goods market equilibrium condition given in equation (27) of Smets and Wouters (2003):

$$Y_t = C_t + G_t + I_t + \psi(z_t)K_{t-1} \tag{16}$$

Linearizing this gives (suppressing time subscripts and using bars for steady states and hats for log deviations):

$$\bar{Y}\hat{Y} = \bar{C}\hat{C} + \bar{G}\hat{G} + \bar{I}\hat{I} + \psi'\bar{K}\hat{z}.$$

In a steady state everybody is identical so from the production function given in Smets and Wouters (2003) equation (21) we also have the relation:

$$Y_t = \epsilon_t^a (z_t K_{t-1})^\alpha L_t^{1-\alpha} - \Phi$$

Define $F(x, L) = x^\alpha L^{1-\alpha}$, where $x = zK$. Note that F is CRS and in steady state $\bar{Y} + \Phi = F$. Then linearizing gives:

$$\begin{aligned} \bar{Y}\hat{Y} &= F\hat{\epsilon}^a + F_x\bar{x}\left(\hat{K} + \hat{z}\right) + F_L\bar{L}\hat{L} \\ &= (\bar{Y} + \Phi)\left(\hat{\epsilon}^a + \alpha\hat{K} + \alpha\hat{z} + (1-\alpha)\hat{L}\right) \end{aligned}$$

Equating the two expressions for $\hat{Y}$ then gives:

$$\hat{Y} = \frac{\bar{C}}{\bar{Y}}\hat{C} + \frac{\bar{G}}{\bar{Y}}\hat{G} + \frac{\bar{I}}{\bar{Y}}\hat{I} + \psi'\frac{\bar{K}}{\bar{Y}}\hat{z} = \frac{(\bar{Y} + \Phi)}{\bar{Y}}\left(\hat{\epsilon}^a + \alpha\hat{K} + \alpha\hat{z} + (1-\alpha)\hat{L}\right).$$

Now using the relations $\psi' = \bar{r}^k$, $\hat{z} = \psi\hat{r}^k$, $\hat{G} = \epsilon^G$ and $\bar{I} = \tau\bar{K}$, and defining c_y, k_y and g_y as in the paper this simplifies to:

$$\hat{Y} = c_y\hat{C} + g_y\epsilon^G + \tau k_y\hat{I} + \bar{r}^k k_y\psi\hat{r}^k = \phi\left(\hat{\epsilon}^a + \alpha\hat{K} + \alpha\psi\hat{r}^k + (1-\alpha)\hat{L}\right).$$

The right side agrees with Smets and Wouters (2003), and the left side agrees with Christiano, Eichenbaum, and Evans (2005).

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