

Solow Model with Human Capital and technological change.

The production function in this case is:

$$(1) Y = K^\alpha (AH)^{1-\alpha}, \text{ where } H \text{ is the stock of skilled labor, } \alpha \text{ is a parameter between zero and one, and } A \text{ is the labor augmenting technology that grows exogenously at a rate } g.$$

Let L be the total amount of labor (raw labor or number of workers) used in production. The variable H can be written as :

$$(2) H = h L$$

so that h is defined as the ratio of skilled labor to total amount of labor. You can think of h as a parameter that depends on the fraction of time individuals spend learning skills (u): $h = e^{\psi u}$, where ψ is a positive constant.

Equation (1) can then be written as:

$$(3) Y = K^\alpha (A h L)^{1-\alpha}$$

Assume that raw labor (or population) grows at a rate n (i.e. $\dot{L} = nL$), that physical capital depreciates at a rate δ (between zero and one) and that the investment rate in physical capital (or savings rate) is s_k (also between zero and one). Therefore physical capital is accumulated according to:

$$(4) \dot{K} = s_k Y - \delta K$$

To solve the model in this case it is convenient to divide (3) by $A h L$, so that the modified variables are:

$$y_h = Y / (A h L), k_h = K / (A h L) \text{ and similarly.}$$

The law of motion for k is in this case is:

$$(5) \dot{k}_h = s_k k_h^\alpha - (\delta + n + g) k_h$$

Steady State:

- (1) (8 pts) Find the steady state level of k_h using a diagram.
- (2) (6 pts) What is the growth rate of capital per worker ($k = K/L$) at the steady state? Justify.

Comparative Statics: decrease in s_k

Assume that at time \bar{t} , the parameter s_k decreases (i.e., the savings/investment rate decreases). Answer the following questions :

- (3) (5 pts) What happens to the steady state level of k_h ? Justify
- (4) (9 pts) Adjustment path of k_h and k : Draw separate diagrams that show how the growth rates of these variables move over time.