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Name:

Econ 475  
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### Midterm 2 (100 points)

#### (I) Solow Model with Human Capital and Technological Change

The production function in this case is:

$$(1) Y = K^\alpha (A h L)^{1-\alpha},$$

where  $\alpha$  is a parameter between zero and one,  $A$  is the labor augmenting technology that grows exogenously at a rate  $g$ ,  $L$  is the total amount of labor (raw labor or number of workers) used in production. The variable  $h$  is a measure of the skill level of the labor force. You can think of  $h$  as a parameter that depends on the fraction of time individuals spend learning skills ( $u$ ):  $h = e^{\psi u}$ , where  $\psi$  is a positive constant.

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Assume that raw labor (or population) grows at a rate  $n$  (i.e.  $\dot{L} = nL$ ), that physical capital depreciates at a rate  $\delta$  (between zero and one) and that the investment rate in physical capital (or savings rate) is  $s_K$  (also between zero and one). Therefore physical capital is accumulated according to:

$$(2) \dot{K} = s_K Y - \delta K$$

To analyze this model it is convenient to work with the modified variables:

$$\tilde{y} = Y / (A h L), \tilde{k} = K / (A h L) \text{ and similarly.}$$

The law of motion for  $\tilde{k}$  is in this case is:

$$\dot{\tilde{k}} = s_K \tilde{k}^\alpha - (\delta + n + g) \tilde{k}$$

(1) (10 pts) Derive the growth rate of output per worker ( $Y/L$ ) along the BGP.

#### Comparative Statics: decrease in $s_K$

Assume that at time  $t = 10$ , the parameter  $s_K$  decreases (i.e., the investment rate decreases). Answer the following questions :

(2) (8 pts) Draw diagrams to show what happens to the growth rate and the level of  $\tilde{k}$  over time.

(3) (8 pts) Draw two diagrams to show what happens to the growth rate and level of  $\tilde{k} = K/L$  over time (you may want to draw  $\ln \tilde{k}$ ).

#### (II) Jones Ch 5

The production function is:

$$(1) Y = K^\alpha (A L_Y)^{1-\alpha}, \text{ where } A \text{ is the stock of ideas, } L_Y \text{ is the amount of labor used in the}$$

production of the final good,  $K$  the stock of capital and,  $\alpha$  is a parameter between zero and one. At the aggregate level, the law of motion of  $A$  is given by equation (2).

$$(2) \dot{A} = \delta A^\phi L_A^\lambda$$

where  $\lambda$ ,  $\phi$  and  $\delta$  are parameters between zero and one and  $L_A$  is the amount of labor devoted to the discovery of new ideas (R&D).

The resource constraint of the economy is:  $L_Y + L_A = L$  and the proportion of labor devoted to both activities is constant (i.e.  $L_A / L = s_R$  and  $L_Y / L = 1 - s_R$  are constant).

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Assume that the labor force labor (or population) grows at a rate  $n$  (i.e.  $\dot{L} = n$ ) and as a consequence, the rates of growth of  $L_A$  and  $L_Y$  are also  $n$ .

Physical capital evolves according to the equation given below (where the investment rate in physical capital or savings rate  $s_K$  is between zero and one) and  $d$  (also between zero and one) is the depreciation rate.

$$(3) \dot{K} = s_K Y - d K$$

(1) (12 pts) Derive the rate of growth of  $A$  and of output per worker ( $Y/L$ ) along a BGP.

**Assume that  $\lambda = 1$  and  $\phi = 0$ . Answer questions (2) and (3).**

(2) (12 points) Assume we are on the BGP and suddenly at time  $t=100$ , the growth rate of population ( $n$ ) decreases.

Use the relevant diagrams to answer the following questions:

What happens to the growth rate of  $A$  over time?

What is the growth rate of output per worker ( $Y/L$ ) once all the adjustments have taken place and the economy reaches the BGP?

(3) (12 points) Assume we are on the BGP and suddenly at time  $t=100$ , a deadly virus kills 25% of the population (notice that the proportion of labor devoted to production and R&D is still constant).

What happens to the growth rate of  $A$  over time?

What is the growth rate of output per worker ( $Y/L$ ) once all the adjustments have taken place and the economy reaches the BGP?

**Assume now that  $\lambda = 1$  and  $\phi = 1$ . Answer questions (4) and (5).**

(4) (6 pts) Is there a BGP in this case? Justify fully.

(5) (4 pts) Do you think that a permanent increase in the size of the R&D sector (because of a government subsidy to R&D) will affect the growth rate of output per worker? Justify (no diagram or math derivation needed).

### (III) Jones Chapter 5 Model: Microfoundations

(1) (6 pts) Explain why it is necessary to assume that firms in the intermediate goods sector do not behave in a perfectly competitive fashion.

(2) (6 pts) Explain why an increase in the number of intermediate goods available increases the amount of the final good produced (assuming that the amount of labor used in production stays constant)

### (IV) Dahlin: Education

(1) (6 pts) Briefly enumerate the different ways by which education affects economic growth.

(2) (4 pts) State two of the findings he presents on the rate of return to education throughout the world.

### (V) Acemoglu, Johnson, Robinson: The Colonial Origins of Comparative development

(6 pts) What is the main result provided by the authors?

