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Midterm 1

(I) Solow Model without technological change.

Consider the following version of the Solow model.
The production function is:

(1) $Y = B K^{\alpha} L^{1-\alpha}$, where α is a parameter between zero and one, B is a positive constant, K is capital and L the number of workers (population)

Assume that raw labor (or population) grows at a rate **n**, that physical capital depreciates at a rate δ (between zero and one) and that the investment rate in physical capital (or savings rate) is s (also between zero and one). Therefore physical capital is accumulated according to:

(2) $\dot{K} = sY - \delta K$

and consumption is: C =(1-s) Y.

(1) **(12 pts)** What are the conditions that need to be satisfied along a Balanced Growth Path (BGP)? Use these conditions and equation (1) to calculate the growth rates of Y and K along a BGP.
To solve the model it is convenient to divide (1) by **L**, so that we work with per worker variables output per worker (y = Y/ L) , capital per worker (k = K / L), and similarly.
(2) **(8 pts)** Write the production function in per worker terms (i.e. y = f(k)) and derive the equation for the law of motion of capital per worker .

BGP or Steady State in the per worker variables:
(3) **(6 pts)** Find the steady state level of k using a diagram.
(4) **(6 pts)** What is the growth rate of capital per worker , output per worker and consumption per worker at the steady state?

Comparative Statics: decrease in B (negative shock to technology)

Assume that at time $\bar{t}$, the parameter B decreases (i.e., there is a negative shock to the technology or in other words : now the same amount of inputs results in less output) .
Answer questions (5) and (6) :

(5) **(8 pts)** What happens to the steady state level of k ? To the growth rate of output per worker (y) at the new steady state? Justify using diagram/diagrams.
(6) **(12 pts)** Adjustment paths of k and y. Draw a diagram showing how these variables move over time (use the horizontal axis for the variable t and the vertical axis for the relevant variable (k(t) , y(t)).

Golden Rule Savings Rate

Let B=1, n=0, δ =0.10**.** In this case the golden rule savings rate (s^G) is equal to 0.5 and the level of capital per worker at the BGP (k^{*G}) is 25 .
Assume the economy's savings rate has been with a savings rate s^G = 0.5 for a long time and all the relevant per worker variables are at their steady state values (capital per worker is at k^{*G} **= 25** , etc).

Suppose now that at time $\bar{t}$, the savings rate **decreases**.
Answer questions (7) and (8)

(7) (6 pts) Draw a diagram showing how capital per worker and output per worker (k and y)evolve over time.

(8) (14 pts) What happens to consumption per worker at time $\bar{t}$? At the steady state? Justify. Draw a diagram showing how consumption per worker evolves over time.

(II) Solow Model with Labor Augmenting Technological Change

Consider the following version of the Solow model. The production function is:

(1) $Y = K^{\alpha} (AL)^{1-\alpha}$

where α is a parameter between zero and one, A is a positive parameter, K is capital and L the number of workers (population). We assume that A grows at a rate **g** (i.e. there is labor augmenting technological change), raw labor (or population) grows at a rate **n**, physical capital depreciates at a rate δ (between zero and one) , and that the investment rate in physical capital (or savings rate) is **s** (also between zero and one). Therefore physical capital is accumulated according to:

(2) $\dot{K} = sY - \delta K$,

and consumption is: C =(1-s) Y.

To solve the model in this case it is convenient to divide (1) by **A L**, so that the modified variables are:

$\tilde{y} = Y/(AL)$, $\tilde{k} = K/(AL)$ and similarly.

The law of motion for $\tilde{k}$ is in this case is:

(5) $\dot{\tilde{k}} = s\tilde{k}^{\alpha} - (\delta + n + g)\tilde{k}$

Comparative Statics: decrease in n (growth rate of population)

Assume that at time $\bar{t}$, the parameter n decreases (i.e., the population starts growing at a lower rate). Answer questions (6)- (9) :

(6) **(4 pts)** What happens to the steady state level of $\tilde{k}$? Justify using a graph.

(7) **(6 pts)** Adjustment path of $\tilde{k}$: Draw a diagram showing how this variable moves over time . (you may want to draw first a diagram that shows how the growth rate of this variable moves over time).

(8) **(9 pts)** Draw a diagram showing how the **growth rate of capital per worker** moves over time.

(9) **(9 pts)** Draw a diagram showing how the **log of capital per worker** (log k) moves over time.