4•2 The Golden Rule Level of Capital

Now that we have examined the link between the rate of saving and the steady-state levels of capital and income, we can discuss what amount of capital accumulation is optimal. Later, in Section 4•5, we describe how government policies alter the saving rate. But first, in this section, we present the theory behind these policy decisions. To keep our analysis as simple as possible, we assume that a policymaker can set the economy's saving rate at any level. By setting the saving rate, the policymaker determines the economy's steady state. We consider what steady state the policymaker should choose.

Comparing Steady States

When choosing a steady state, the policymaker's goal is to maximize the well-being of the individuals who make up the society. Individuals themselves do not care about the amount of capital in the economy, or even the amount of output. They care about the amount of goods and services they can consume. Thus, a benevolent policymaker would want to choose the steady state with the highest level of consumption. The steady state with the highest consumption is called the **Golden Rule level of capital accumulation** and is denoted k^*_{gold}.[3]

How can we tell whether an economy is at the Golden Rule level? To answer this question, we must first determine steady-state consumption per worker. Then we can see which steady state provides the most consumption.

To find steady-state consumption per worker, we begin with the national income accounts identity

$$y = c + i,$$

and rearrange it as

$$c = y - i.$$

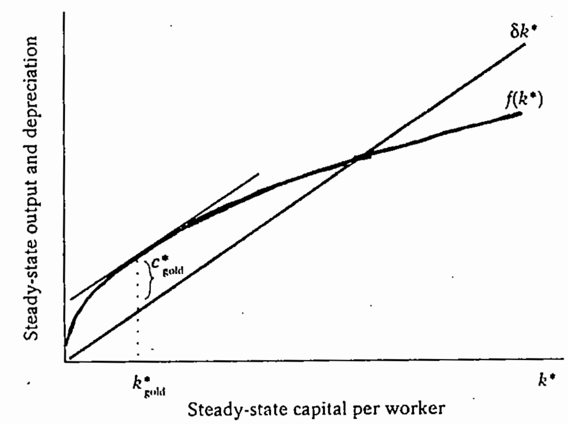
Consumption is simply output minus investment. Since we want to find steady-state consumption, we substitute steady-state values for output and investment. Steady-state output per worker is $f(k^*)$, where k^* is the steady-state capital stock per worker. Furthermore, since the capital stock is not changing in the steady state, investment is equal to depreciation, δk^*. Substituting $f(k^*)$ for y and δk^* for i, we can write steady-state consumption per worker as

$$c^* = f(k^*) - \delta k^*.$$

This equation states that steady-state consumption is the difference between steady-state output and steady-state depreciation. It shows that increased capital has two effects on steady-state consumption: it causes greater output, but more output must be used to replace depreciating capital.

[3] Edmund Phelps, "The Golden Rule of Accumulation: A Fable for Growthmen," *American Economic Review* 51 (September 1961): 638–643.

Figure 4-7



Steady-State Consumption The econ my's output is used for consumption or investment. In the steady state, investment equals deprec ation. Therefore, steady-state consumption is the difference between outpu $f(k^*)$ and depreciation δk The steady state that maximizes steady-state consumption is called the Golden Rule. The Golde Rule capital stock is denoted k^*_{gold}, and the Golden Rule consumptio is denoted c^*_{gold}.

Figure 4-7 graphs steady-state output and steady-state depreciation as a function of the steady-state capital stock. Steady-state consumption is the gap between output and depreciation. This figure shows that there is one level of the capital stock—the Golden Rule level k^*_{gold}—that maximizes consumption.

When we compare steady states, we must take into account the effects of higher capital on both output and depreciation. If the capital stock is below the Golden Rule level, an increase in the capital stock raises output more than depreciation, so that consumption rises. In this case, the production function is steeper than the δk^* line, so the gap between these two curves—which equals consumption—grows as k^* rises. By contrast, if the capital stock is above the Golden Rule level, an increase in the capital stock reduces consumption, since the increase in output is smaller than the increase in depreciation. In this case, the production function is flatter than the δk^* line, so the gap between the curves—consumption—shrinks as k^* rises. At the Golden Rule level of capital, the production function and the δk^* line have the same slope, and consumption is at its greatest level.

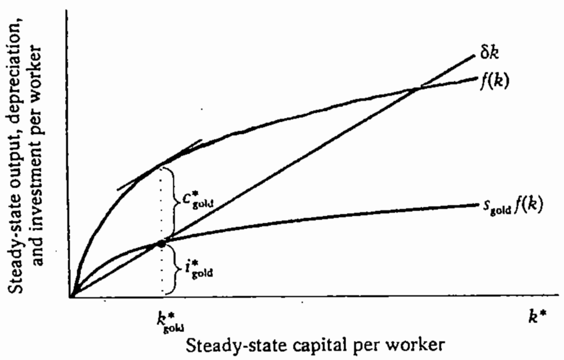
To make the point somewhat differently, suppose that the economy starts at some capital stock k^* and that the policymaker is considering increasing the capital stock to $k^* + 1$. The amount of extra output would then be $f(k^* + 1) - f(k^*)$, which is the marginal product of capital MPK. The amount of extra depreciation from having one more unit of capital is the depreciation rate δ. The net effect of this extra unit of capital on consumption is then $MPK - \delta$, which is the marginal product of capital less the depreciation rate. If the steady-state capital stock is below the Golden Rule level, increases in capital increase consumption because the marginal product of capital is greater than the depreciation rate. If the steady-state capital stock exceeds the Golden Rule level, increases in capital reduce consumption because the marginal product of capital is less than the depreciation rate. Therefore, the following condition describes the Golden Rule:

$$MPK = \delta.$$

At the Golden Rule level of capital, the marginal product of capital equals the rate of depreciation. In other words, at the Golden Rule, the marginal product net of depreciation, $MPK - \delta$, equals zero.

Keep in mind that the economy does not automatically gravitate toward the Golden Rule steady state. The choice of a particular steady-state capital stock, such as the Golden Rule, implies a choice of a particular saving rate. Figure 4-8 shows the steady-state if the saving rate is set to produce the Golden Rule level of capital. If the saving rate is higher than the one used in this figure, the steady-state capital stock will be too high. If the saving rate is lower, the steady-state capital stock will be too low.

Figure 4-8



The Saving Rate and the Golden Rule There is one saving rate that produces the Golden Rule level of capital k^*_{gold}. A change in the saving rate would shift the $sf(k)$ curve, which would move the economy to a steady state with a lower level of consumption.

Comparing Steady States: A Numerical Example

Consider the decision of a policymaker choosing a steady state in the following economy. The production function is the same as in our earlier example:

$$y = \sqrt{k}.$$

Output per worker is the square root of capital per worker. Depreciation is again 10 percent of capital. This time, the policymaker chooses the saving rate s and thus the economy's steady state.

To see the outcomes available to the policymaker, recall that the following equation holds in the steady state:

$$\frac{k^*}{f(k^*)} = \frac{s}{\delta}.$$

In this economy, this equation becomes

$$\frac{k^*}{\sqrt{k^*}} = \frac{s}{0.1}.$$

Squaring both sides of this equation yields a solution for the steady-state capital stock. We find

$$k^* = 100s^2.$$

Using this result, we can compute the steady-state capital stock for any saving rate.

Table 4-3 presents calculations showing the steady states that result from various saving rates. We see that higher saving leads to higher capital, which in turn leads to higher output and higher depreciation. Steady-state consumption, the difference between output and depreciation, first rises with higher saving rates and then declines. Consumption is highest when the saving rate is 0.5. Hence, a saving rate of 0.5 produces the Golden Rule steady state.

Comparing Steady States: A Numerical Example

Assumptions: $y = \sqrt{k}$ $\delta = 0.1$

s	k^*	y^*	δk^*	c^*	MPK	$MPK -$
0.0	0.0	0.0	0.0	0.0	∞	∞
0.1	1.0	1.0	0.1	0.9	0.500	0.40
0.2	4.0	2.0	0.4	1.6	0.250	0.15
0.3	9.0	3.0	0.9	2.1	0.167	0.06
0.4	16.0	4.0	1.6	2.4	0.125	0.02
0.5	25.0	5.0	2.5	2.5	0.100	0.00
0.6	36.0	6.0	3.6	2.4	0.083	−0.01
0.7	49.0	7.0	4.9	2.1	0.071	−0.02
0.8	64.0	8.0	6.4	1.6	0.062	−0.03
0.9	81.0	9.0	8.1	0.9	0.056	−0.04
1.0	100.0	10.0	10.0	0.0	0.050	−0.05

Another way to identify the Golden Rule steady state is from the marginal product of capital. For this production function, the marginal product is[4]

$$MPK = \frac{1}{2\sqrt{k}}.$$

Using this formula, the last two columns of Table 4-3 present the value of $MPK - \delta$ in the different steady states. Note again that in the Golden Rule steady state, the marginal product of capital net of depreciation equals zero.

The Transition to the Golden Rule Steady State

Let's now make our policymaker's problem more realistic. So far, we have been assuming that the policymaker can simply choose the economy's steady state. In this case, the policymaker would choose the steady state with highest consumption—the Golden Rule steady state. But now suppose that the economy has reached a steady state other than the Golden Rule. What happens to consumption, investment, and capital when the economy makes the transition between steady states? Might the impact of the transition deter the policymaker from trying to achieve the Golden Rule?

We must consider two cases: the economy might begin with more capital than in the Golden Rule steady state, or with less. The second case—too little capital—presents far greater difficulties; it forces the policymaker to evaluate the benefits of current consumption relative to future consumption. As we see in Section 4•5, this situation describes actual economies, including that of the United States.

Starting With More Capital Than in the Golden Rule We first consider the case in which the economy begins with more capital than it would have in the Golden Rule steady state. In this case, the policymaker should pursue policies aimed at reducing the rate of saving in order to reduce the steady-state capital stock. Suppose that these policies succeed and that, at some point in time—call it t_0—the saving rate falls to the level that will eventually lead to the Golden Rule steady state.

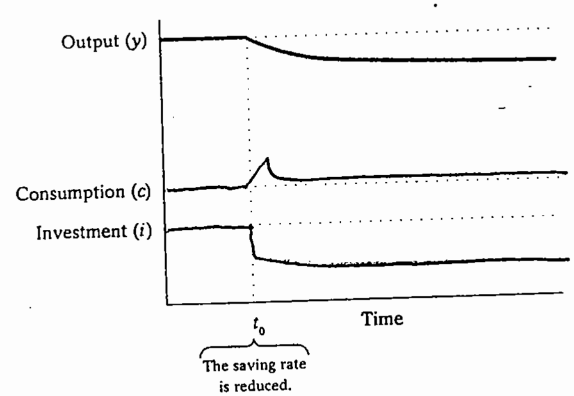
Figure 4-9 shows what happens to output, consumption, and investment when the saving rate falls. The reduction in the saving rate causes an immediate increase in the level of consumption and a decrease in the level of investment. Investment is now less than depreciation, so the economy is no longer in a steady state. Gradually, as the capital stock falls, output, consumption, and investment also fall to the new steady state. Because the new steady state is the Golden Rule steady state, we know that the level of consumption is now higher than it was before the change in the saving rate, even though output and investment are lower.

[4] *Mathematical note*: To prove this formula, note that the marginal product of capital is the derivative of the production function with respect to k.

Note that, compared to the old steady state, consumption is higher not just in the new steady state but also along the entire path to it. When the capital stock exceeds the Golden Rule level, reducing saving is clearly a good policy, for it increases consumption at every point in time.

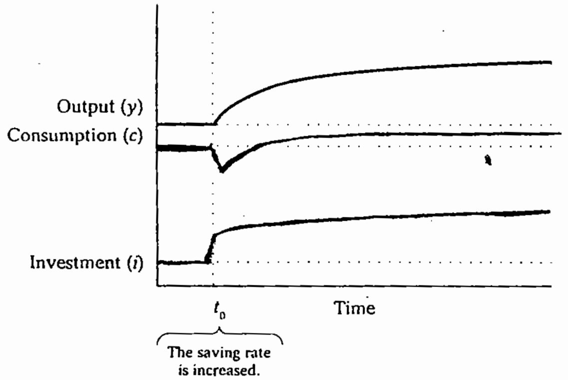
Starting With Less Capital Than in the Golden Rule When the economy begins with less capital than in the Golden Rule steady state, the policymaker must raise the saving rate to reach the Golden Rule. Figure 4-10 shows what happens. The increase in the saving rate at time t_0 causes an immediate fall in consumption and a rise in investment. Over time, higher investment causes the capital stock to rise. As capital accumulates, output, consumption, and investment gradually increase, eventually approaching the new steady-state levels. Because the initial steady state was below the Golden Rule, the increase in saving eventually leads to a higher level of consumption than that which prevailed initially.

Figure 4-9



Reducing Saving When Starting With More Capital Than in the Golden Rule Steady State This figure shows what happens over time to output, consumption, and investment when the economy begins with more capital than the Golden Rule and the saving rate is reduced. The reduction in the saving rate (at time t_0) causes an immediate increase in consumption and an equal decrease in investment. Over time, as the capital stock falls, output, consumption, and investment fall together. Since the economy began with too much capital, the new steady state has a higher level of consumption than the initial steady state.

Figure 4-10



Increasing Saving When Starting With Less Capital Than in the Golden Rule Steady State This figure shows what happens over time to output, consumption, and investment when the economy begins with less capital than the Golden Rule, and the saving rate is increased. The increase in the saving rate (at time t_0) causes an immediate drop in consumption and an equal jump in investment. Over time, as the capital stock grows, output, consumption, and investment increase together. Since the economy began with less capital than the Golden Rule, the new steady state has a higher level of consumption than the initial steady state.

Does the increase in saving that leads to the Golden Rule steady state raise economic welfare? Eventually it does, because the steady-state level of consumption is higher. But achieving that new steady state requires an initial period of reduced consumption. Note the contrast to the case in which the economy begins above the Golden Rule. *When the economy begins above the Golden Rule, reaching the Golden Rule produces higher consumption at all points in time. When the economy begins below the Golden Rule, reaching the Golden Rule requires reducing consumption today to increase consumption in the future.*

Deciding whether to try to reach the Golden Rule steady state is especially difficult because the population of consumers changes over time. Reaching the Golden Rule achieves the highest steady-state level of consumption and thus benefits future generations. But, when the economy is below the Golden Rule, reaching the Golden Rule requires raising investment and thus lowering the consumption of current generations.

When choosing whether to increase capital accumulation, the policymaker must compare the welfare of different generations. A policymaker who cares more about current generations than about future generations may decide not to pursue policies to reach the Golden Rule steady state. By contrast, a policymaker who cares about all generations equally will choose to reach the Golden Rule. Even though current generations will consume less, an infinite number of future generations will benefit by moving to the Golden Rule.

Thus, optimal capital accumulation depends crucially on how we weigh the interests of current and future generations. The biblical Golden Rule tells us to "do unto others as you would have them do unto you." If we heed this advice, we give all generations equal weight. In this case, it is optimal to reach the Golden Rule level of capital—which is why it is called the "Golden Rule."

4•3 Population Growth

The basic Solow model shows that capital accumulation, by itself, cannot explain sustained economic growth. High rates of saving lead to high growth temporarily, but the economy eventually approaches a steady state in which capital and output are constant. To explain the sustained economic growth that we observe in most parts of the world, we must expand the Solow model to incorporate the other two sources of economic growth: population growth and technological progress. In this section we add population growth to the model.

Instead of assuming that the population is fixed, as we did in Sections 4•1 and 4•2, we now suppose that the population and the labor force grow at a constant rate n. For example, in the United States, the population grows about 1 percent per year, so $n = 0.01$. This means that if 150 million people are working one year, then 151.5 million (1.01×150) are working the next year, and 153.015 million (1.01×151.5) the year after that, and so on.

The Steady State With Population Growth

How does population growth affect the steady state? To answer this question, we must discuss how population growth, along with investment and depreciation, influences the accumulation of capital per worker. As we noted before, investment raises the capital stock, and depreciation reduces it. But now there is a third force acting to change the amount of capital per worker: the growth in the number of workers causes capital per worker to fall.

We continue to let lowercase letters stand for quantities per worker. Thus, $k = K/L$ is capital per worker, and $y = Y/L$ is output per worker. Keep in mind, however, that the number of workers is growing over time.

The change in the capital stock per worker is

$$\Delta k = i - (\delta + n)k.$$

This equation shows how new investment, depreciation, and population growth influence the per-worker capital stock. New investment increases k, whereas depreciation and population growth decrease k. We have seen this equation before in the special case of a constant population ($n = 0$).

We can think of the term $(\delta + n)k$ as ***break-even investment***: the amount of investment necessary to keep the capital stock per worker constant. Break-even investment includes the depreciation of existing capital, which equals δk.