Econ 475
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Problem on Jones Ch 5

Jones version of the Romer Model (Ch 5)

Consider the following model.
The production function is:

(1) $Y = K^{\alpha} (A L_Y)^{1-\alpha}$, where A is the stock of ideas, L_Y is the amount of labor used in the production of the final good , K the stock of capital and, α is a parameter between zero and one.

Individual agents believe that the law of motion of A is given by the equation (2) . At the aggregate , the law of motion of A is instead given by equation (4).

(2) $\dot{A} = \tilde{\delta} L_A$ $\qquad \tilde{\delta} > 0$

(4) $\dot{A} = \delta A^{\phi} L_A^{\lambda}$

where λ, ϕ and δ are parameters between zero and one and L_A is the amount of labor devoted to the discovery of new ideas.
The resource constraint of the economy is: $L_Y + L_A = L$
Assume that raw labor (or population) grows at a rate **n** (i.e $\hat{L} = n$), that physical capital does not depreciate and that the investment rate in physical capital (or savings rate) is s_K (between zero and one). Therefore physical capital is accumulated according to:

(3) $\dot{K} = s_K Y$

(1) **(8 pts)** Give an intuitive explanation of what the parameters λ, ϕ between zero and one imply.

Assume now that $\lambda = 1$ and $\phi = 0$ and proceed to answer the questions below
(2) **(8 pts)** Write down the equation for the growth rate of A at the aggregate level, the growth rate of $k = K/L$ and the growth rate of $y = Y/L$.
(3) **(4 pts)** Define a balanced growth path (BGP) in this model.

Assume that along a BGP the growth rates of L_A and L_Y equal the rate of population growth (n).
(4) **(8 pts)** Derive the equation for the growth rate of A along the BGP .
(5) **(10 pts)** Derive the equation for the growth rate of y along the BGP.

Comparative Statics

(6) Assume we are on the BGP and suddenly at time t=0 , δ (the productivity of research) decreases. Use the relevant diagrams to answer the following questions:

(i) **(6 pts)** What happens to the growth rate of A over time?

(ii) **(6 pts)** What happens to the growth rate of y over time?

(7) **(25 pts)**

Assume that we are on a BGP and that suddenly at time t=0, the investment rate in physical capital (s_K) increases.

(i)What happens with the value of k/A at the steady state?

(ii) What happens to the growth rate of y over time?

Hint:

Here are the steps you need to follow.

(a) Write the production function per worker (i.e. divide equation (1) by L). To simplify the notation use : $L_Y /L = s_Y$.

(b) Write down the equation for the growth rate of capital per worker as a function of k/A, s_K , s_Y and n.

Draw this function together with the rate of growth of capital per worker at the steady state.

(c) Use the diagram and additional elements to answer the question.