You are not allowed to consult any notes or books. You have to write your answers in the blue book that the persons proctoring the exam give you. Your answers should reflect only your knowledge. Attempting to consult with others or to look at someone else's work constitutes academic misconduct.
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Econ 475 Spring 2007
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Exam 2

(I) Jones Ch 5 Model

Consider the following version of the model studied in class.
The production function is:

(1) $Y = K^{\alpha} (A L_Y)^{1-\alpha}$, where A is the stock of ideas, L_Y is the amount of labor used in the production of the final good , K the stock of capital and, α is a parameter between zero and one. At the aggregate level , the law of motion of A is given by equation (2).

(2) $\dot{A} = \delta L_A$

where δ is a parameter between zero and one and L_A is the amount of labor devoted to the discovery of new ideas (R&D).

The resource constraint of the economy is: $L_Y + L_A = L$ and the proportion of labor devoted to both activities is constant (i.e $L_A / L = s_R$ and $L_Y / L = s_Y = 1 - s_R$ are constant).

Assume that the labor force labor (or population) grows at a rate **n** (i.e $\hat{L} = n$) and as a consequence, the rates of growth of L_A and L_Y are also n.

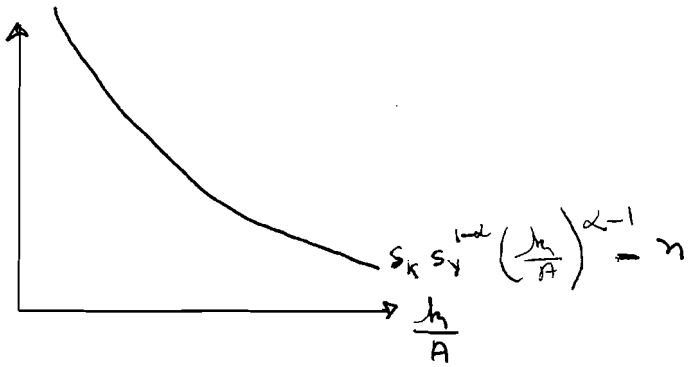
Physical capital evolves according to the equation given below (where the investment rate in physical capital or savings rate s_K is between zero and one).

(3) $\dot{K} = s_K Y$

Notice that in this version of the Jones model we assume that the capital stock does not depreciate.

In this case the growth rate of capital per worker is:

(4) $\hat{k} = s_K \; s_Y^{1-\alpha} (k/A)^{\alpha-1} - n$



Notice that equation (4) holds everywhere and that the approximate shape of the function is the one shown in the diagram.

(1) **(12 pts)** Derive the rate of growth of A , of output per worker (y=Y/L) and of capital per worker (k=K/L) along a BGP.

(2) **(4 pts)** Draw a diagram to identify the ratio L_A/A along the BGP . To answer this question you will need to use the equation for the growth rate of A derived from equation (2) and your results in question (1).

(3) **(6 pts)** Draw a diagram to identify the ratio k/A along the BGP . To answer this question you will need to use equation (4) and your results in question (1)

Assume we are on the BGP and suddenly at time t=100 , the growth rate of population (n) **increases to n'**. Answer questions (4) and (5).

(4) **(9 points)** Use the diagram from question (2) to answer the following questions:
(a) What happens to L_A/A over time?
(b) Draw a diagram that shows what happens to the growth rate of A over time.
(c) What is the growth rate of output per worker and of capital per worker once all the adjustments have taken place and the economy reaches the BGP ?

(5) **(9 points)** Use the diagram from question (3) to answer the following questions:
(a) What happens to k/A over time ?
(b) Draw a diagram that shows what happens to the growth rate of k over time.

(II) Questions on Microfoundations From the Jones Ch. 5 Model

(1) **(8 points)** Briefly describe the final-goods sector (be sure to write down the production function and the profit maximizing problem the typical firm solves).
(2) **(6 points)** Consider now the intermediate goods sector (i.e. specialized capital goods sector).
What is the marginal cost of production?
Is the marginal cost equal to the price? Justify fully.
(3) **(6 points)** Consider now the R&D sector.
Give the intuition behind the following arbitrage condition:

$$r P_A = \pi + \dot{P}_A$$

where r is the real interest , P_A is the price of a design and π are the production profits of a producer of intermediate goods .

(III) Model from Jones Ch. 6
The production function is:

$$(1)\ Y = L^{1-\alpha} \int_0^h x_j^{\alpha}$$

where each x_j is a differentiated intermediate good (capital good), h is the number of intermediates that the country knows how to use, L is the labor force and α is a parameter between zero and one. In this case the aggregate stock of capital K can be defined as:

$$(2)\ K = \int_0^h x_j\, dj$$

Using (2) , equation (1) can be rewritten as:

$$(3)\ Y = K^{\alpha} (h L)^{1-\alpha}$$

The law of motion of h is given by:

$$(4)\ \dot{h} = \mu e^{\psi u} A$$

where A is the number of intermediates available in the advanced country (technological leader), μ and ψ are positive parameters and u represents the time devoted to skill accumulation (measured for example as years of schooling).

Assume that the number of intermediates available in the advanced country (A) grows at a constant rate g, that raw labor (or population) grows at a rate **n** and that the law of motion of physical capital is:

$$(5)\ \dot{K} = s_K Y - dK$$

(1) **(6 pts)** How is the parameter μ in equation (4) related to the ideas presented by Prescott and Parente in "Barriers to Riches"?

(2) **(12 pts)** Derive the growth rate of output , output per worker and h along a BGP (balanced growth path).

(3) **(12 pts)** Analyze the effects of a **decrease in μ at time** $\bar{t}$. Use a diagram that has A/h in the horizontal axis to answer the following questions:

(a) What happens to A/h over time?

(b) Draw a diagram that shows what happens to the growth rate of h over time.

(IV) Romer Mauritius Model (10 points)

Explain why in this model the wage after the EPZ was established is higher than the autarky wage (a precise and complete explanation or a diagram is required).