More on Intertemporal Trade

This appendix contains a more detailed examination of the two-period intertemporal trade model described in the chapter. The concepts used are the same as those used in Chapter 5 to analyze international exchanges of different consumption goods at a single point in time. In the present setting, however, the trade model explains international patterns of investment and borrowing and the determination of the intertemporal terms of trade (that is, the real interest rate).

First consider Home, whose intertemporal production possibility frontier is shown in Figure 7A-1. Recall that the quantities of present and future consumption goods produced at Home depend on the amount of present consumption goods invested to produce future goods. As currently available resources are diverted from present consumption to investment, production of present consumption, \( Q_p \), falls and production of future consumption, \( Q_f \), rises. Increased investment therefore shifts the economy up and to the left along the intertemporal production possibility frontier.

The chapter showed that the price of future consumption in terms of present consumption is \( 1(1 + r) \), where \( r \) is the real interest rate. Measured in terms of present consumption, the value of the economy’s total production over the two periods of its existence is therefore

\[
V = Q_p + Q_f (1 + r).
\]

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FiguRE 7A-1 Determining Home’s Intertemporal Production Pattern

At a world real interest rate of \( r \), Home’s investment level maximizes the value of production over the two periods that the economy exists.

Future consumption

In value lines with slope \( -1(1 + r) \)

\( Q_f \)

Intertemporal production possibility frontier

\( Q_p \)

Present consumption

Investment
Figure 7A-1 shows the isovalue lines corresponding to the relative price 1/(1 + r) for different values of \( \nu \). These are straight lines with slope \(-1/(1 + r)\) (because future consumption is on the vertical axis). As in the standard trade model, firms' decisions lead to a production pattern that maximizes the value of production at market prices, \( Q_0 + Q_1/(1 + r) \). Production therefore occurs at point \( Q \). The economy invests the amount shown, leaving \( Q_1 \) available for present consumption and producing an amount \( Q_2 \) of future consumption when the first-period investment pays off.

Notice that at point \( Q \), the extra future consumption that would result from investing an additional unit of present consumption just equals \((1 + r)\). It would be inefficient to push investment beyond point \( Q \) because the economy could do better by lending additional present consumption to foreigners instead. Figure 7A-1 implies that a rise in the world real interest rate \( r \) which steepens the isovalue lines, causes investment to fall.

Figure 7A-2 shows how Home's consumption pattern is determined for a given world interest rate. Let \( D_0 \) and \( D_1 \) represent the demands for present and future consumption goods, respectively. Since production is at point \( Q \), the economy's consumption possibilities over the two periods are limited by the intertemporal budget constraint:

\[
D_0 + D_1/(1 + r) = Q_0 + Q_1/(1 + r)
\]

This constraint states that the value of Home's consumption over the two periods (measured in terms of present consumption) equals the value of consumption goods produced in the two periods (also measured in present consumption units). Put another way, production and consumption must lie on the same isovalue line.

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**Exhibit 7A.2** Home's Intertemporal Consumption Pattern

![Diagram showing Home's consumption pattern with isovalue lines, indifference curves, and budget constraint](image)

Home's consumption places it on the highest indifference curve touching its intertemporal budget constraint. The economy exports \( Q_0 - D_0 \) units of present consumption and imports \( D_1 - Q_0 = (1 + r) \times (Q_0 - D_0) \) units of future consumption.
Point B, where Home's budget constraint touches the highest attainable indifference curve, shows the present and future consumption levels chosen by the economy. Home's demand for present consumption, \(D_p\), is smaller than its production of present consumption, \(Q_p\), so it exports (that is, lends) \(Q_p - B_p\) units of present consumption to Foreigners. Correspondingly, Home imports \(D_f - Q_f\) units of future consumption from abroad when its first-period loans are repaid to it with interest. The intertemporal budget constraint implies that \(D_f - Q_f = (1 + r) \times (Q_p - D_p)\), so that trade is intertemporally balanced.

Figure 7A-3 shows how investment and consumption are determined in Foreign. Foreign is assumed to have a comparative advantage in producing future consumption goods. The diagram shows that at a real interest rate of \(r\), Foreign borrows consumption goods in the first period and repays this loan using consumption goods produced in the second period. Because of its relatively rich domestic investment opportunities and its relative preference for present consumption, Foreign is an importer of present consumption and an exporter of future consumption.

As in Chapter 5 (appendix), international equilibrium can be portrayed by an offer curve diagram. Recall that a country's offer curve is the result of plotting its desired exports against its desired imports. Now, however, the exchanges plotted involve present and future consumption. Figure 7A-4 shows that the equilibrium real interest rate is determined by the intersection of the Home and Foreign offer curves \(OP\) and \(OF\) at point \(E\). The ray \(OE\) has slope \((1 + r')\), where \(r'\) is the equilibrium world interest rate. At point \(E\), Home's desired export of present consumption equals Foreign's desired import of present consumption. Put another way, at point \(E\), Home's desired first-period lending equals Foreign's desired first-period borrowing. Supply and demand are therefore equal in both periods.