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Econ 464

MICROECONOMIC HANDOUT (Part 2)

V. TWO SECTOR ECONOMY: PRODUCTION SIDE

(1) Case of one input: L (Labor)

- Assume 2 produced goods: M & F
- Fixed amount of labor: L (Needs to be allocated to the 2 sectors)
- Firms maximize profits taking prices (P_M & P_F) and wages (w) as given.

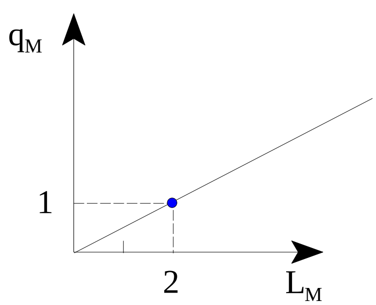
(a) Linear Production Functions

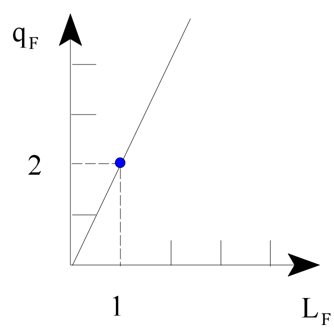
Example:

Assume: $q_M = ½ \; L_M$, $q_F = 2 \, L_F$, $L = 20$

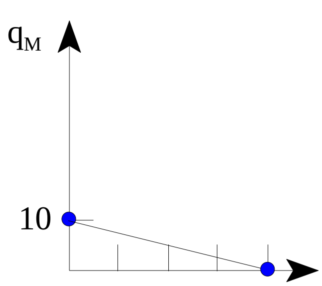
Here $MPL_M = ½$ $MPL_F = 2$

Unit labor coeff: $a_{LM} = 2$ $a_{LF} = ½$





Then we can draw the PPF (Production Possibility Frontier) (combinations of q_M & q_F that can be produced using the 20 units of labor available : $20 = a_{LM} \,.\, q_M + a_{LF} \,.\, q_F = 2 \,.\, q_M + 1/2 \,.\, q_F$):



Definition:

$$MRT = \left| \, slope \;\; of \;\; PPF \, \right|$$

Also true that $\left| \, slope \;\; of \;\; PPF \, \right| = \dfrac{MPL_M}{MPL_F}$

We need to figure out:

(i) - How many units of q_F & q_M will profit max. firms produce?
(ii) - How is labor allocated across sectors?
Remark 1: If you know the answers to one question, you can answer the other.

(i) It is a good idea in this case to think of an economy with only one firm. If the firm knows how to produce both goods, profit maximization is equivalent to maximize the value of production subject to the technology constraint (PPF) i.e.: maximize

$Y = p_M \cdot q_M + p_F \cdot q_F$ Subject to PPF in Diagram 1

Value of production

We can also write the previous equation as: $q_M = \frac{Y}{p_M} - \frac{p_F}{p_M} q_F$

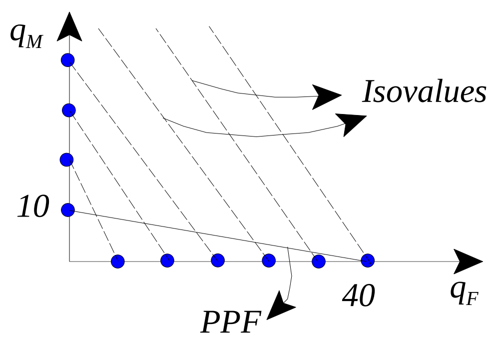
Isovalue lines: Lines along which the value of output is constant (Y is a constant).

$$q_M = \frac{\bar{Y}}{p_M} - \frac{p_F}{p_M} q_F$$

The slope of the isovalue line is the price ratio.

Suppose $p_F = 4$ $p_M = 2$

Then it is optimal to specialize: Produce only q_F:



$q_F^* = 40$

$q_M^* = 0$

Therefore $L_F^* = 20$ and $L_M^* = 0$

(ii) Suppose we want to look at labor allocation first. If both sectors produce positive amounts of output, profit maximization will require:

(+) $MPL_F \cdot p_F = w$

(++) $MPL_M \cdot p_M = w$

$\Rightarrow MPL_F \cdot p_F = MPL_M \cdot p_M$ (*)

We will see that (*) does not hold in our case ($p_F = 4$, $p_M = 2$) :

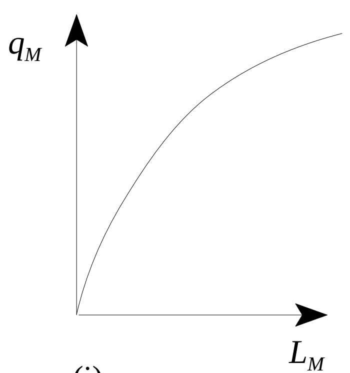
$MPL_F \cdot p_F = 2 \times 4 = 8 \quad \neq \quad MPL_M \cdot p_M = ½ \times 2 = 1$

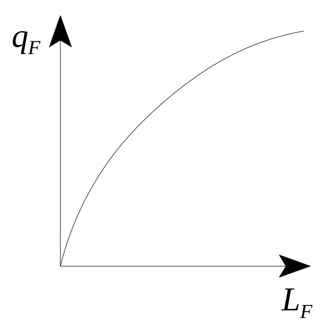
Since the left side is larger:
$L^*_F = 20$, $L^*_M = 0$ And as a consequence:
$q^*_F = 2 \cdot L^*_F = 2 \times 20 = 40$
$q^*_M = ½ \cdot L^*_M = 0$

<u>Remark 2</u>:

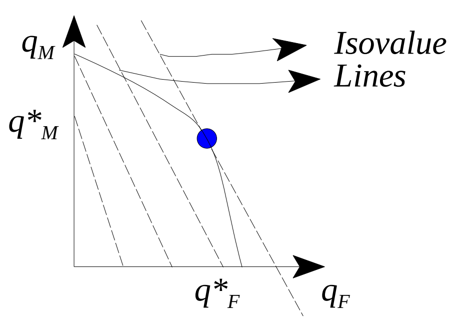
Only one type of good will be produced except when the slope of the PPF coincides with the scope of the Isovalue lines i.e.: $\frac{MPL_M}{MPL_F} = \frac{p_F}{p_M}$

(b) Production functions with decreasing MPL
Example:





(i)

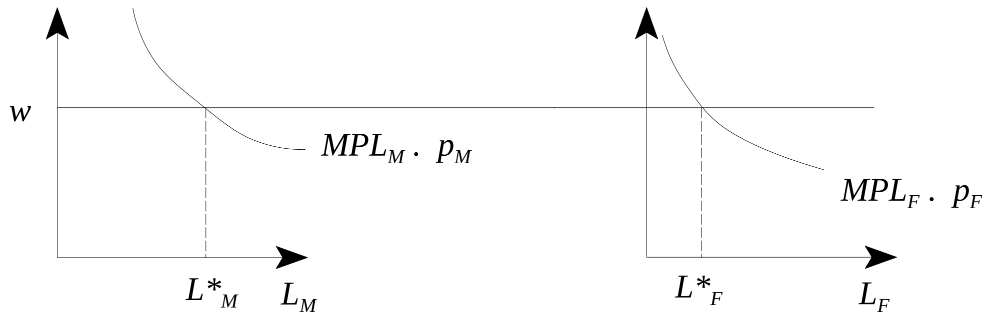


Assume $p_F = 4$
$p_M = 2$

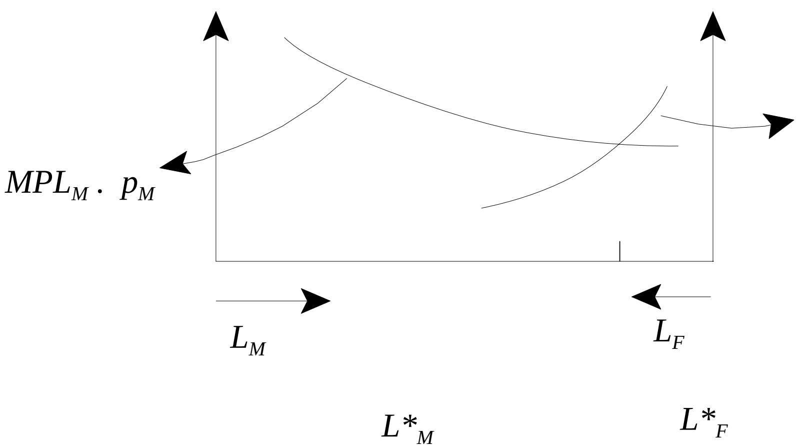
We get L^*_F & L^*_M From Production functions.

(ii) Looking first at labor allocation (take w as given) from production functions we get

MPL curves and then use the profit maximizing conditions (+) & (++).



<u>OR</u>



(Not in Scale)

Things to do:

- What happens if $P_F \uparrow$?

- What happens if $P_M \uparrow$?

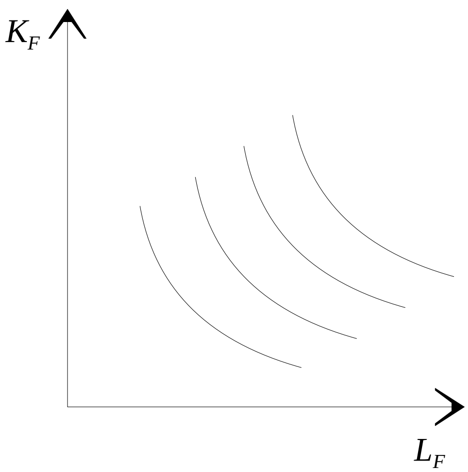
- What happens if $MPL_F \downarrow$?

(2) Case of Two Inputs: $\bar{L}$ (Labor) and K (capital)

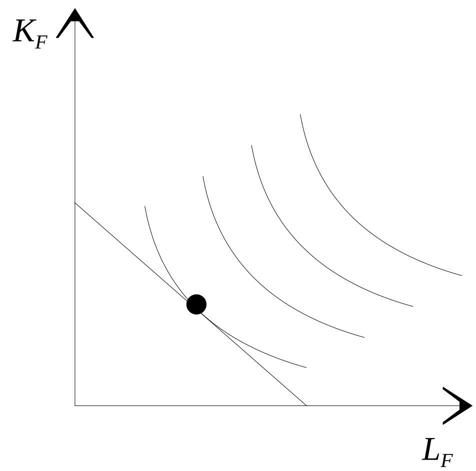
- Assume 2 produced goods: M (machines) and F (food).
- Both inputs are used in the production of both goods and the *isoquants* are "nice".
- The production of F is *labor-intensive* and the production of M is *capital-intensive* (i.e., for any given input prices the capital-labor ratio is higher in the M sector than in the F sector).
- There are fixed amounts of labor ($\bar{L}$) and capital (K) that have to be allocated among the two productive sectors.
- There are constant returns to scale in both sectors (i.e., the average cost is constant for all levels of output and equals the marginal cost).

– Firms maximize profits taking output prices (p_M and p_F) and input prices (w and r) as given.

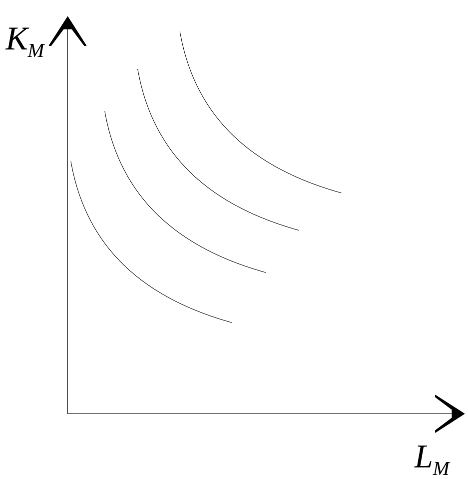
Food Isoquants



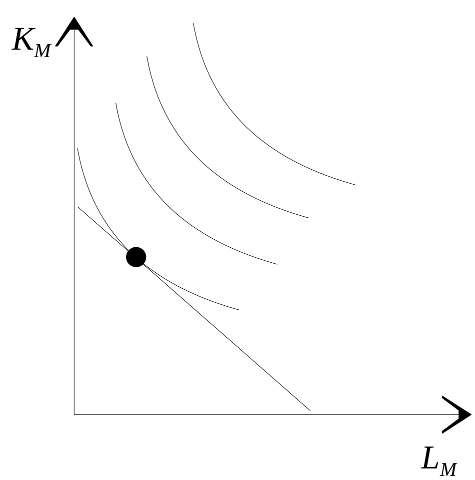
Cost minimization in food



Machines Isoquants



Cost minimization in machines

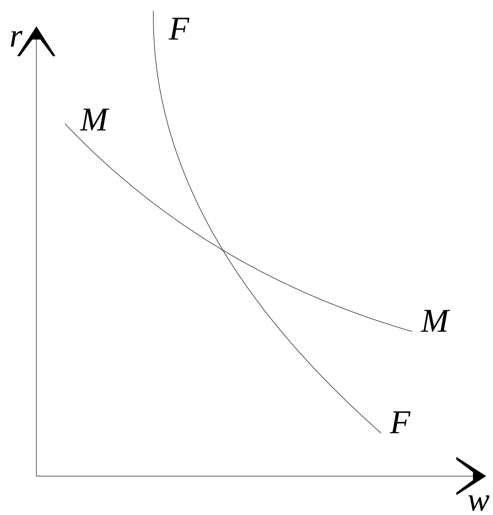


With constant returns to scale, the price of a good is equal to its average cost. For any given price of a good, this condition defines a set of possible input prices. For example, given the price of food, the higher the wage (w), the lower must be the rental price of capital (r).

Curve FF represents all the combinations of w and r for which the price of food equals its average cost, while MM represents all combinations for which price and average cost are equal for machines.

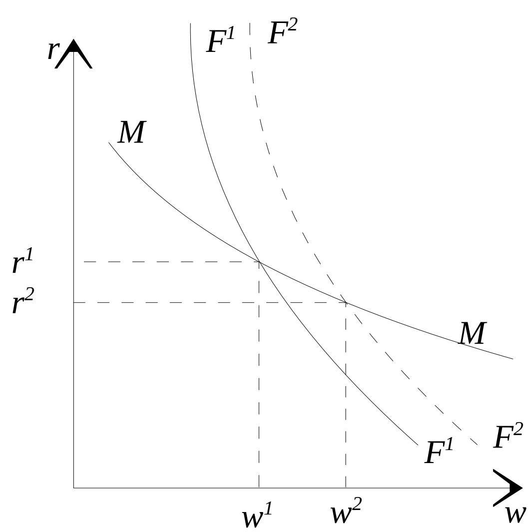
Since food production is more labor-intensive than machine production, the wage has relatively more effect on the cost of food production, and the rental price of capital less effect. As a result, to offset the effect of a higher wage on the average cost of food, r must fall more than is true for machines; thus FF is steeper than MM.

The equilibrium input prices (w and r) must be such that the average cost equals the price in both food and machines. This occurs at the intersection of FF and MM.



Changes in prices

Suppose the price of food rises, then the food industry is able to pay a higher wage, a higher rent on capital, or both: FF shifts from F^1F^1 to F^2F^2. This raises the wage rate from w^1 to w^2, while lowering r from r^1 to r^2. The rise in w is more than proportional to the increase in the price of food. So changes in prices have strong effects on input prices (i.e., factor retributions).



Allocation of Resources in

General Equilibrium.

To allocate the total stocks of labor and capital between food and machines, given goods prices there are 3 steps to follow: (1) Use goods prices to determine input prices. (2) Use factor prices to determine the K/L ratio in each sector. (3) Use the assumption that labor and capital are fully employed to determine the resource allocation.
Step (1) was developed above. Step (2) is a direct consequence of cost minimization (see Part 1 of the handout). We will not study how step 3 works in this class.