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## MICROECONOMIC HANDOUT (Part 2)

V.

TWO SECTOR ECONOMY: PRODUCTION SIDE
(1) Case of one input: L (Labor)

- Assume 2 produced goods: M \& F
- Fixed amount of labor: $L$ (Needs to be allocated to the 2 sectors)
- Firms maximize profits taking prices $\left(\mathrm{P}_{\mathrm{M}} \& \mathrm{P}_{\mathrm{F}}\right)$ and wages (w) as given.
(a) Linear Production Functions

Example:
Assume: $\mathrm{q}_{\mathrm{M}}=1 / 2 \quad \mathrm{~L}_{\mathrm{M}}, \mathrm{q}_{\mathrm{F}}=2 \mathrm{~L}_{\mathrm{F}}, \mathrm{L}=20$
Here $\mathrm{MPL}_{\mathrm{M}}=1 / 2$

$$
\mathrm{MPL}_{\mathrm{F}}=2
$$

Unit labor coeff: $\mathrm{a}_{\mathrm{LM}}=2$
$\mathrm{a}_{\mathrm{LF}}=1 / 2$



Then we can draw the PPF (Production Possibility Frontier) (combinations of $\mathrm{q}_{\mathrm{M}} \& \mathrm{q}_{\mathrm{F}}$ that can be produced using the 20 units of labor available: $\left.20=\mathrm{a}_{\mathrm{LM}} \cdot \mathrm{q}_{\mathrm{M}}+\mathrm{a}_{\mathrm{LF}} \cdot \mathrm{q}_{\mathrm{F}}=2 \cdot \mathrm{q}_{\mathrm{M}}+1 / 2 \cdot \mathrm{q}_{\mathrm{F}}\right)$ :


Definition:
$M R T=\mid$ slope of PPF $\mid$

Also true that $\mid$ slope of $P P F \left\lvert\,=\frac{M P L_{M}}{M P L_{F}}\right.$
We need to figtte grit:
(i) - How many units of $\mathrm{q}_{\mathrm{F}} \& \mathrm{q}_{\mathrm{M}}$ will profit max. firms produce?
(ii) - How is labor allocated across sectors?

Remark 1: If you know the answers to one question, you can answer the other.
(i) It is a good idea in this case to think of an economy with only one firm. If the firm knows how to produce both goods, profit maximization is equivalent to maximize the value of production subject to the technology constraint (PPF) i.e.: maximize

$$
\mathrm{Y}=\mathrm{p}_{\mathrm{M}} \cdot \mathrm{q}_{\mathrm{M}}+\mathrm{p}_{\mathrm{F}} \cdot \mathrm{q}_{\mathrm{F}} \quad \text { Subject to PPF in Diagram } 1
$$

Value production
We can also write the previous equation as: $q_{M}=\frac{Y}{p_{M}}-\frac{p_{F}}{p_{M}} q_{F}$
Isovalue lines: Lines along which the value of output is constant ( Y is a constant).

$$
q_{M}=\frac{\bar{Y}}{p_{M}}-\frac{p_{F}}{p_{M}} q_{F}
$$

The slope of the isovalue line is the price ratio.
Suppose $\quad p_{F}=4 \quad p_{M}=2$
Then it is optimal to specialize: Produce only $\mathrm{q}_{\mathrm{F}}$ :


$$
\begin{aligned}
& \mathrm{q}_{\mathrm{F}}^{*}=40 \\
& \mathrm{q}_{\mathrm{M}}^{*}=0
\end{aligned}
$$

Therefore $\mathrm{L}_{\mathrm{F}}{ }^{*}=20$ and $\mathrm{L}_{\mathrm{M}}{ }^{*}=0$
(ii) Suppose we want to look at labor allocation first. If both sectors produce positive amounts of output, profit maximization will require:
(+) $\quad \mathrm{MPL}_{\mathrm{F}} \cdot \mathrm{p}_{\mathrm{F}}=\mathrm{w}$

$$
\} \Rightarrow \mathrm{MPL}_{\mathrm{F}} \cdot \mathrm{p}_{\mathrm{F}}=\mathrm{MPL}_{\mathrm{M}} \cdot \mathrm{p}_{\mathrm{M}}(*)
$$

$(++) \quad \mathrm{MPL}_{\mathrm{M}} \cdot \mathrm{p}_{\mathrm{M}}=\mathrm{w}$
We will see that $\left({ }^{*}\right)$ does not hold in our case $\left(p_{\mathrm{F}}=4, \mathrm{p}_{\mathrm{M}}=2\right)$ :

$$
\mathrm{MPL}_{\mathrm{F}} \cdot \mathrm{p}_{\mathrm{F}}=2 \times 4=8 \quad \neq \quad \mathrm{MPL}_{\mathrm{M}} \cdot \mathrm{p}_{\mathrm{M}}=1 / 2 \times 2=1
$$

Since the left side is larger:
$\mathrm{L}^{*}{ }_{\mathrm{F}}=20, \mathrm{~L}^{*}{ }_{\mathrm{M}}=0 \quad$ And as a consequence:

$$
\mathrm{q}_{\mathrm{F}}^{*}=2 \cdot \mathrm{~L}_{\mathrm{F}}^{*}=2 \times 20=40
$$

$$
\mathrm{q}_{\mathrm{M}}=1 / 2 \cdot \mathrm{~L}_{\mathrm{M}}=0
$$

## Remark 2:

Only one type of good will be produced except when the slope of the PPF coincides with the scope of the Isovalue lines i.e.:

$$
\frac{M P L_{M}}{M P L_{F}}=\frac{p_{F}}{p_{M}}
$$

(a) Production functions with decreasing MPL Example:

(i)



$$
\text { Assume } \begin{aligned}
\mathrm{p}_{\mathrm{F}} & =4 \\
\mathrm{p}_{\mathrm{M}} & =2
\end{aligned}
$$

We get $L^{*}{ }_{F} \& L^{*}{ }_{M}$ From Production functions.
(ii) Looking first at labor allocation (take w as given) from production functions we get


OR

(2) Case of 2 inputs
(a) Linear prod. Functions [Similar to (1) (a)]
(b) "Nice Isoquants" [Similar to (1) (b)]

