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Econ 364

Microeconomics Handout (Part 1)

I. TECHNOLOGY : Production Function, Marginal Productivity of Inputs, Isoquants

(1) Case of One Input: L (Labor): $q = f(L)$

- Let q equal output so the production function relates L to q. (How much output can be produced with a given amount of labor?)

- Marginal productivity of labor = MPL is defined as

$$\left(\frac{\Delta q}{\Delta L}\right)_{\text{Small changes}} = \text{Slope of prod. Function}$$

i.e. The change in output if we change the amount of labor used by a very small amount.

- How to find total output (q) if we only have information about the MPL: "In general" q is equal to the area under the MPL curve when there is only one input.

Examples:

(a) Linear production functions.
Possible forms:

$q = 10\,L \Rightarrow MPL = 10$
$q = ½\,L \Rightarrow MPL = ½$
$q = 4\,L \Rightarrow MPL = 4$

The production function q = 4L is graphed below.

Diagram 1

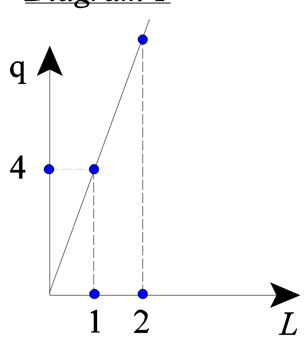
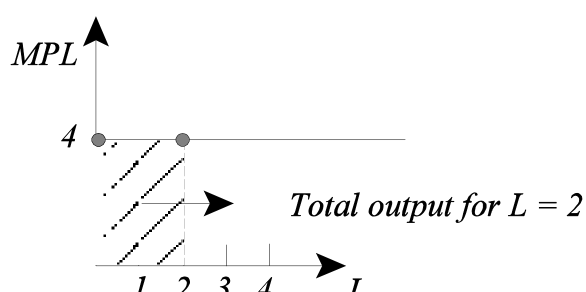
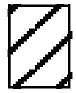


Diagram 2

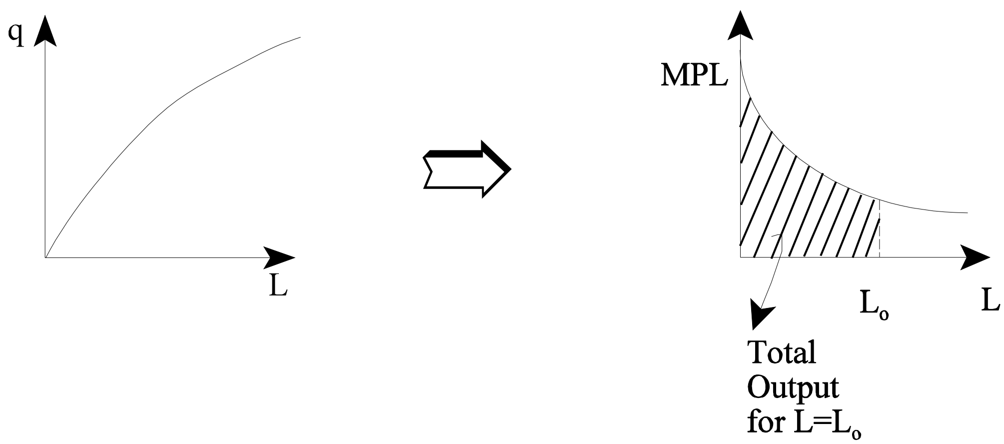


Notice that if we only have diagram 2, we can calculate output for different amounts of labor as the area under MPL:

If L = 2 ⇒ q = Area below MPL for L Less or equal to 2 = 8 =  in Diagram 2

<u>Remark</u>: In all the examples in (a) MPL is <u>constant</u>.

(b) Production Functions With Decreasing MPL.



Remark: Often this is thought as the case of one variable input (Labor = L) and a fixed factor (land or entrepreneurial ability)

(2) <u>Case of Two Variable Inputs</u>: q = f (L, K)

L (Labor), K (Capital)

- Production function relates L & K to q (total output)
- Isoquant: Combinations of L & K that can achieve the same q

- Marginal Productivities

$$MPL = \left(\frac{\Delta q}{\Delta L}\right)$$ Small changes
K constant

$$MPK = \left(\frac{\Delta q}{\Delta K}\right)$$ Small changes
L constant

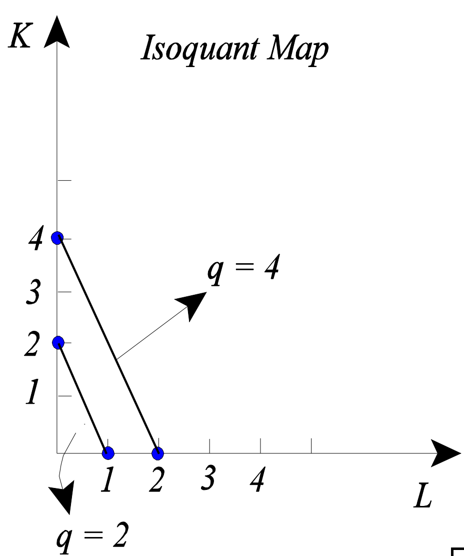
- MRTS = - Slope of Isoquant = Absolute value of $\frac{\Delta K}{\Delta L}$ Along Isoquant

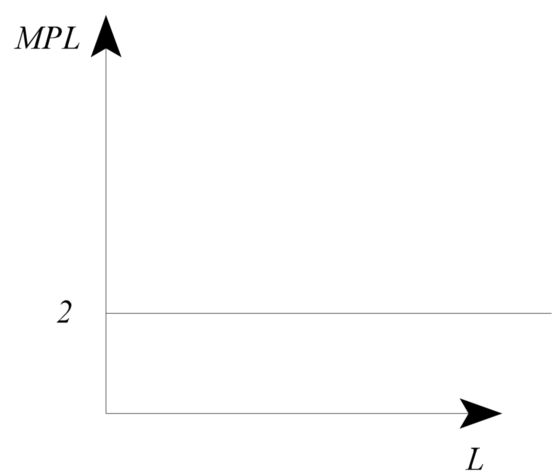
<u>Examples</u>

(a) Linear (L & K are perfect substitutes)
Possible forms:

q = 10 L + 5 K ⇒ MPL = 10 MPK = 5
q = L + K ⇒ MPL = 1 MPK = 1
q = 2L + K ⇒ MPL = 2 MPK = 1

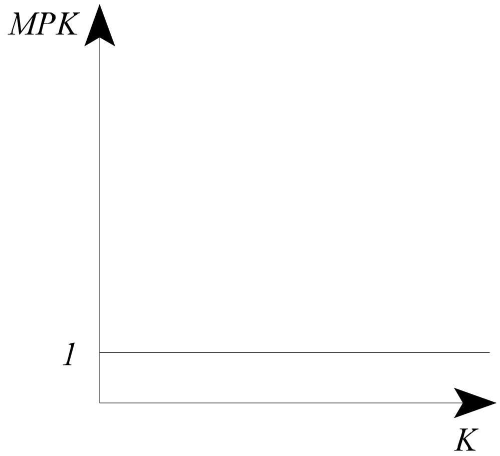
- The production function q = 2 L + K is graphed below.





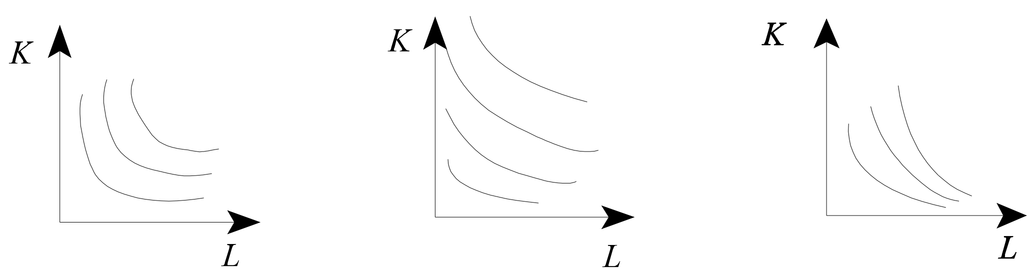
MRTS = 2

<u>Marginal Productivities</u> are constant in all these examples, then MRTS = constant



(b) "Nice Isoquants"

Possible Forms:



Slope of Isoquants in absolute value ↓ as L↑ (i.e. ↓MRTS as L↑, as L↑it becomes more and more costly to replace capital.)

II. **COST MINIMIZATION**

How to choose inputs to minimize cost:

(1) One input: Trivial: Price of labor
Total Cost = w. L , where w is the price of labor

(2) Two variable inputs
(a) Fixed coefficients: No substitution (still easy)
Let r be the price of capital, then the cost of producing one unit of output (Average cost: AC) is:

$$AC = a_L \cdot w + a_K \cdot r$$

Notice that this does not change for different levels of output:

The AC is constant.

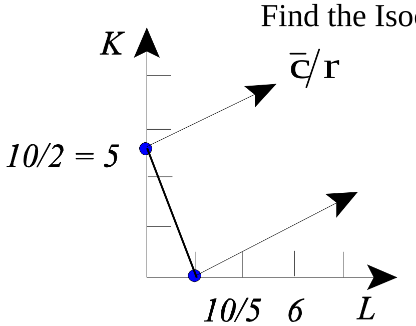
If either w or r change, the AC changes but the firm still uses a_L units of L and a_K units of K to produce one unit of output.

(b) Linear Isoquants (perfect substitutes) (Hard - we will skip this)

(c) "Nice Isoquants"
- Define Isocost lines
- Find tangency between Isoquant and Isocost lines

Isocost = $\bar{c}$ = w.L + r.k (Combinations of L & K that cost same amount of money)

Example: w = 5 , r = 2
Find the Isocost for $10



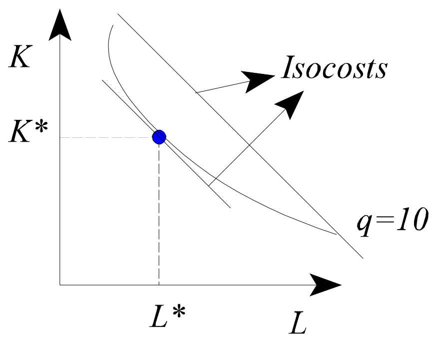
$$\left| Slope\ of\ Isocost \right| = \frac{5}{2}$$

In General:

$$\left| Slope\ of\ Isocost \right| = \frac{w}{r}$$

Example:

Cost minimizing combination of K & L to produce q = 10



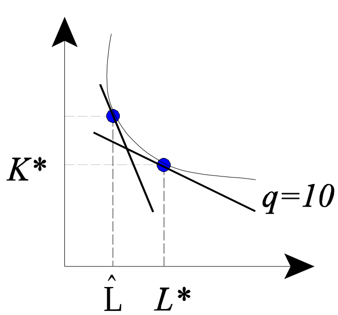
$$|\,slope\,| = \frac{w}{r}$$

Effect of changes in prices:

- ↑ w, r constant (i.e. ↑ w/r)

  ⇒ ↓ L* & ↑ K*

New Combination: $\hat{K}, \hat{L}$



- ↑ r w constant (i.e. ↓ w/r)

  ⇒ ↑ L* & ↓ K*

III.

PROFIT MAXIMIZING FACTOR DEMANDS

We assume that each firm maximizes profits
- taking the product price as given (P) and
- taking input prices as given (w, r)

How many units of L & K should the firm hire?

(1) Case of One Variable Input: Labor (L)

A profit maximizing firm will hire L up to the point at which:

Marginal product x output price = input price

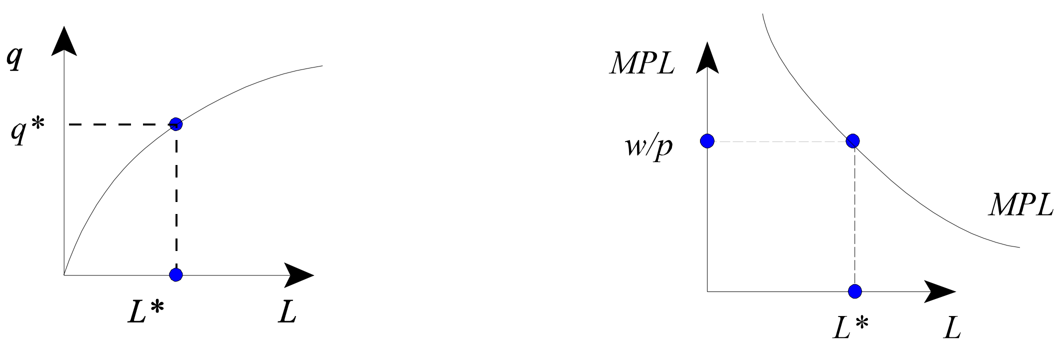
In our case:

MPL . p = w

Marginal benefit for the firm — Marginal cost of hiring an extra unit of labor

$\Longrightarrow$ **MPL = w/p**

Example: Production function with decreasing MPL (Think about a fixed factor).



(2) Case of Two Variable Inputs: L & K

"Nice Isoquants"