Economics 302
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The Keynesian Model of Income Determination (revised)

This set of notes outlines the Keynesian model of national income determination in closed and open economy. It then shows how to solve for multipliers.

1. An Expanded Model and Equilibrium

Eq.No.	Equation	Description
(1)	$Y = Z$	Output equals aggregate demand, an equilibrium condition
(2)	$Z \equiv C + I + G + X - IM$	Definition of aggregate demand
(3)	$C = c_o + c_1 Y_D$	Consumption function, c_1 is the mpc
(4)	$Y_D \equiv Y - T$	Definition of disposable income
(5)	$T = t_0 + t_1 Y$	Tax function; t_0 is lump sum taxes, t_1 is marginal tax rate.
(6)	$I = b_0$	Investment function, exogenous
(7)	$G = GO_0$	Government spending on goods and services, exoagenous
(8)	$X = x_0$	Exports, exogenous
(9)	$IM = m_0$	Imports, exogenous

Note: the marginal propensity to consume out of disposable income is $c_1 = \dfrac{\partial C}{\partial (Y - T)}$.

To simplify matters, for the moment, "close" the economy so that there are no imports and exports, $x_0 = m_0 = 0$. Then (8) and (9) are irrelevant.

Substitute (3)-(7) into (2), and substitute (2) into (1):

(10) $$Y = Z = (c_0 + c_1 Y_D) + (b_0) + (GO_o)$$
$$Y = Z = (c_0 + c_1 (Y - (t_0 + t_1 Y))) + (b_0) + (GO_o)$$

Collect up terms:

(11) $Y = c_1(1 - t_1)Y + \Lambda_0$ where $\Lambda_0 \equiv c_0 - c_1(t_0) + b_0 + GO_o$

Shift "*Y*" terms to the left hand side:

(12) $Y - c_1(1 - t_1)Y = \Lambda_0 \rightarrow Y[1 - c_1(1 - t_1)] = \Lambda_0$

Divide both sides by the term in the square bracket to obtain equilibrium income, Y_0:

(13) $\boxed{Y_0 = \bar{\gamma}\Lambda_0}$ let $\bar{\gamma} = \dfrac{1}{[1 - c_1(1 - t_1)]}$

Where the "0" subscript denotes the equilibrium value of output. Interpretation of (13): equilibrium income is a multiple of the amounts of "autonomous" spending. The higher the level of autonomous spending, the higher the equilibrium level of income. Notice also that lump sum taxes enter in negatively, so the higher lump sum taxes, the lower equilibrium income is.

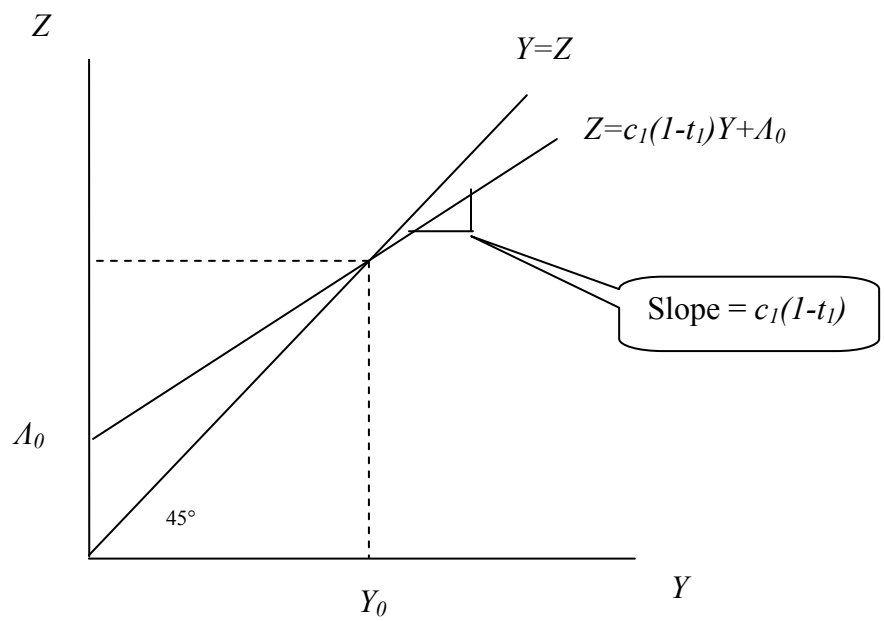

Figure 1: Equilibrium in the Keynesian Cross

2. Effects of Changes in Autonomous Spending and Multipliers

To think about how changes in autonomous spending – the constant in consumption (c_0), government spending (GO_0), investment spending (b_0),– affect equilibrium income, consider a change of income (ΔY) as being attributable to changes in each of those autonomous spending components. Take equation (13):

(14) $\Delta Y = \bar{\gamma}\Delta\Lambda$

(holding constant the tax *rate*). Remember, however, that $\Lambda_0 \equiv c_0 - c_1(t_0) + b_0 + GO_o$. So if, for instance, the only autonomous spending component that changes is government spending (so) $\Delta\Lambda = \Delta GO$, then:

(15) $\Delta Y = \bar{\gamma}\Delta GO$

Notice that a similar expression occurs if one holds government spending, lump sum taxes, exports and imports constant, then:

(16) $\Delta Y = \bar{\gamma}\Delta b_0$

Returning to (15), the increase in government spending can interpreted in the following figure.

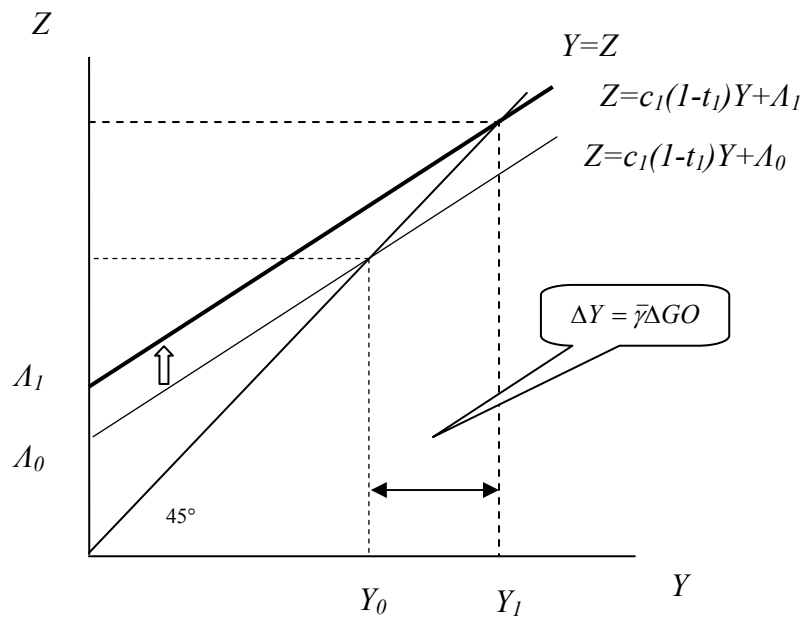

Figure 2: Change in income due to a change in autonomous spending

Note that $Y_1 = Y_0 + \Delta Y$. You should also understand that $\Lambda_1 = \Lambda_0 + \Delta\Lambda$.

Returning to (15), consider what the change in income for a change in government spending is. That can be obtained by dividing both sides by ΔGO:

(17) $$\frac{\Delta Y}{\Delta GO} = \bar{\gamma} \equiv \frac{1}{1 - c_1(1 - t_1)}$$

Why is the change in income greater than the change in government spending? Consider a \$1 increase in government spending, and for simplicity set the marginal tax rate (t_1) to zero. The \$1 in spending is income for others; of that \$1, \$$c_1$ is spent. That spending becomes income for somebody else, of which (c_1)×(\$$c_1$). Adding up the entire sequence of spending, one obtains:

$1 + c_1 + c_1^2 + c_1^3 + ... + c_1^n = \dfrac{1}{1 - c_1}$ for $n = \infty$.

See if you can solve for the lump sum tax multiplier. How does it differ from the government spending multiplier?

3. An Alternative Approach

(18) $S \equiv Y_D - C \equiv Y - T - C$

In a closed economy:

(19) $Y \equiv C + I + G$

Bringing C to the left hand side, and subtracting T from both sides:

(20) $Y - T - C \equiv I + G - T$

Notice the left hand side of (20) is the same as (18)

(21a) $S \equiv I + G - T$
(21b) $I \equiv S + (T - G)$

The term in the parentheses is the government budget balance. (21b) indicates investment equals the sum of private saving and public saving.

An interesting aspect of the equilibrium condition that saving equals investment is "Paradox of Thrift", which states that individual consumers' attempts to increase saving will end up decreasing overall saving. To see this, assume no government sector (so T, G both equal zero), and restate (18):

(22) $S \equiv Y - C = Y - (c_0 + c_1 Y)$

(23) $\Delta S = \Delta Y - \Delta c_0 - c_1 \Delta Y = -\Delta c_0 + (1 - c_1)\Delta Y$

An increase in saving is represented by a decrease in autonomous consumption $\Delta c_0 < 0$. Recall, we know (15) or (16) that $\Delta Y = \bar{\gamma}\Delta c_0$. Substituting this in:

(24) $\Delta S = -\Delta c_0 + (1 - c_1)\bar{\gamma}\Delta c_0$

In this economy, with no government, the marginal tax rate is zero (t_1=0), so $\bar{\gamma} = \dfrac{1}{[1 - c_1]}$. Then (24) becomes:

(25) $\Delta S = -\Delta c_0 + \Delta c_0 = 0$

In other words, aggregate saving remains unchanged, despite the fact that individual households attempt to save more. At the same time, output is lower.

4. The Open Economy Multiplier

One important point to realize is that there are different multipliers corresponding to different variables changing (consider the government spending vs. tax lump multipliers). More importantly, the multiplier for the same variable will be different if the *model* changes. In the previous section, net exports depended upon an exogenously determined amount. Suppose we assume net exports, the difference between exports and imports, depend upon income. In particular, suppose when income rises, consumption rises, and some of that consumption falls on imported goods. Hence, imports (which enter in negatively in the calculation of net exports) rise with income. This assumption can be incorporated into the export equation thus:

(9') $IM = m_0 + m_1 Y$

Note that $m_1 = \dfrac{\partial imports}{\partial Y}$. If one goes through the same steps as in Section 1, one obtains:

(26) $Y = Z = (c_0 + c_1 Y_D) + (b_0) + (GO_o) + (x_0) - (m_0 + m_1 Y)$

(27) $Y = Z = (c_0 + c_1(Y - (t_0 + t_1 Y))) + m_1 Y + (b_0) + (GO_o) + (x_0) - (m_0)$

Solving for equilibrium, one obtains:

(28) $Y_0 = \left(\dfrac{1}{1 - c_1(1 - t_1) + m_1} \right) \Lambda_0$ where $\Lambda_0 \equiv c_0 - c_1(t_0) + b_0 + GO_o + x_0 - m_0$

In this case the, government spending multiplier is:

(29) $\dfrac{\Delta Y}{\Delta GO} = \dfrac{1}{1 - c_1(1 - t_1) + m_1}$

In this open economy model, the government spending multiplier is less than that in the closed economy.

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