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Tax Evasion, Endogenous Spending and the Design of Optimal Tax Codes

by

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Abstract

In this paper I show that, in economies characterized by tax evasion and endogeneity of government spending, the qualitative characterization of the optimal factor income taxes depends on both these factors.

In particular, the well known Chamley - Judd result that, in the long run, tax rates on capital income should be zero is, in many cases, overturned. It is shown that the nature of tax evasion matters: if a reproducible factor --a capital stock-- or a whole sector evades taxes, then the long run tax code still displays no taxation of capital, but increased taxation of labor. If, on the other hand, it is a non-reproducible factor --e.g. labor-- that is evading tax payments, then the optimal tax code requires that capital be either taxed or subsidized depending on the elasticity of substitution between capital and the evading factor.

Next, I discuss the role of endogenous spending. I consider two types of public goods: utility enhancing --e.g. parks, arts-- and productivity enhancing --e.g. infrastructure, education. I show that suboptimal settings for the spending variables can affect the long run taxation of capital. For example, in the case of productive public goods, I show that excessive --relative to the first best-- levels of spending require taxes on capital income, while low levels of spending result in subsidies to capital.

Overall, our results point to an interaction between tax evasion, spending and the design of the optimal tax code. Policy makers engaged in tax reforms need to take into account tax evasion not only for its revenue effects, which are secondary in our story, but also for efficiency considerations associated with the relative returns to all factors. Similarly, policy makers need to make assumptions about the structure of spending. Suboptimality of spending plays no role in the Ramsey result. However, when the suboptimal level is endogenous --a more realistic setting-- the optimal tax code needs to "correct" for the inefficiencies in the level and composition of publicly provided goods.

1. Introduction

One of the most interesting and relevant topics in the economics of taxation is the optimal choice of a tax code. The seminal work on this topic goes back to Ramsey (1927). In that paper, Ramsey characterized the optimal levels for a system of excise taxes on consumption goods. He assumed that the government's goal was to choose these taxes to maximize social welfare subject to the constraints it faced. These constraints were assumed to be of two types. First, a given amount of revenue --independent of the state of the economy-- was to be raised. Second, Ramsey understood that whatever tax system the government adopted, consumers and firms in the economy would react in their own interest through a system of (assumed competitive) markets. This observation gives rise to a second type of constraint on the behavior of the government--it must take into account the equilibrium reactions by firms and consumers to the chosen tax policies. The problem of choosing the optimal tax rates subject to these two implementability constraints is what has become known as a "Ramsey Problem."

Ramsey's insights have been developed extensively in the last few years [see the excellent survey in Auerbach (1985)] as applied to optimal commodity taxation. At the same time, the macro literature has emphasized the optimal choice of factor income --both labor and capital-- taxes. Contributions to this literature include Atkinson and Sandmo (1980), Chamley (1985) and (1986), Judd (1985) and (1990), Stiglitz (1985), (1987), Barro (1990), King (1990), Lucas (1990), Yuen (1990), Chari, Christiano and Kehoe (1994), Zhu (1991), Bull (1992) and Jones, Manuelli and Rossi (1993), (1997).

The most surprising finding of the literature on factor income taxation is that the optimal tax rate on capital income --a stock-- is zero in the long run, while the optimal tax rate on labor --a pure flow-- is positive. This was first exposited in Chamley and Judd in the context of simple single sector models of exogenous growth and has been shown to hold in cases with steady state, endogenous growth as well. Thus the zero limiting tax rate on capital is a "robust" result from the optimal taxation literature. Refinements to the stochastic case have been explored in King (1990), Chari, Christiano and Kehoe (1994), and Zhu (1991), while the nature of optimal tax codes in the presence of human capital is the topic of Judd (1997) and Jones and Manuelli (1999).

Most of these analyses --Jones, Manuelli and Rossi (1997) and Jones and Manuelli (1999) are exceptions-- rely on two basic assumptions. First, that the set of available taxes is incomplete but still fairly generous. In particular, the original Ramsey analysis assumed that all goods but one can be taxed. Second, the standard experiment is one in which changes in the tax code that give rise to changes in endogenous variables, e.g. output, do not trigger changes in spending; that is, spending is truly exogenous[2].

Even though standard, these assumptions are not very realistic, especially in the case of developing countries. I many less developed countries --and quite of few developed as well-- tax evasion is a serious problem. In addition, the allocation of government expenditures need not be efficient and, in many case, is endogenous; that is, the amount spent on individual projects depends on tax revenue or, more generally, on aggregate economic conditions. In this paper, I consider variations of the Ramsey problem to accommodate both tax evasion and endogenous --and potentially inefficient-- levels of government spending. The main objective is to understand how these deviations from the original Ramsey problem affect the qualitative features of optimal tax codes and, in particular, the no taxation of capital result.

From a formal point of view, tax evasion could be considered a restriction on the tax rates that the Ramsey planner can choose. Specifically, if a factor can completely evade paying taxes, this corresponds to a situation in which the tax authority is restricted to choose a 0% tax on that

[2] An exception is Chisari and Navajas (1992) where they study the effects of tax evasion on the optimal choice of consumption taxes.

factor. This is a restriction on the tax code of the type studied by Jones, Manuelli and Rossi (1997). In this setting, I show that the nature of tax evasion matters: if a reproducible factor --a capital stock-- or a whole sector evades taxes, then the long run tax code still displays no taxation of capital, but increased taxation of labor. If on the other hand, it is a non-reproducible factor --e.g. labor-- that is evading tax payments, then the optimal tax code requires that capital be either taxed or subsidized depending on the elasticity of substitution between capital and the evading factor.

Next, I discuss the role of endogenous government spending. I consider two types of public goods: utility enhancing --e.g. parks, arts-- and productivity enhancing --e.g. infrastructure, education. I show that suboptimal settings for the spending variables can affect the long run taxation of capital. For example, in the case of productive public goods, I show that excessive --relative to the first best-- levels of spending require taxes on capital income, while low levels of spending result in subsidies to capital.

Overall, our results point to an interaction between tax evasion, spending and the design of the optimal tax code. Policy makers engaged in tax reforms need to take into account tax evasion not only for its revenue effects, which are secondary in our story, but also for efficiency considerations associated with the relative returns to all factors. Similarly, policy makers need to make assumptions about the structure of spending. Suboptimality of spending plays no role in the Ramsey result. However, when the suboptimal level is endogenous --a more realistic setting-- the optimal tax code needs to "correct" for the inefficiencies in the level and composition of publicly provided goods.

The remainder of the paper is organized as follows. In section 2, I present the basic Chamley-Judd about asymptotic tax rates on labor and capital income. In section 3, I discuss tax evasion, while section 4 studies endogenous spending. Finally, section 5 contains concluding comments.

2. The Basic Model

I start by describing a generalized version of the model analyzed by Judd (1985) and Chamley (1986). To explore the question of the role played by public spending, it is necessary to expand the model. I do this by adding two forms of government goods: utility enhancing goods, and productive public goods. An example of the former is national parks, while roads and infrastructure are examples of the latter. The main point of this section is to show that, when spending is exogenous, then the qualitative results of Judd (1985) and Chamley (1986) remain unchanged: in the long run the tax rate on capital is zero, while the tax rate on labor is positive.

I consider the simplest infinitely lived agent model consistent with the presence of both human and physical capital. I assume that there is one representative family that takes prices and tax rates as given. Their utility maximization problem is given by:

(2.1) $\sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_t, g_{ct})$.

This family faces a sequence of budget constraints and capital accumulation constraints given by,

(2.2) $c_t + x_t + b_{t+1} \leq (1-\tau_t^h)w_t n_t + (1-\tau_t^k)r_t k_t + (1+(1-\tau_t^k)R_t)b_t$, $\quad t=0,1,...$

(2.3) $k_{t+1} \leq (1-\delta)k_t + x_t$, $\quad t=0,1,...$

and the transversality condition to prevent Ponzi games $\lim_{T\to\infty} \beta^T u'(c_T) b_{T+1} = 0$. In this formulation c_t, n_t, g_{ct}, x_t and k_t stand for consumption, employment, public goods provided by the government, investment and capital at time t. I normalize the price of consumption at t (and new capital) at one, and let w_t and r_t denote the rental prices of labor and capital in terms of contemporaneous consumption. The tax code includes taxes on capital income, τ_t^k, and labor income, τ_t^h. Finally, b_t is the stock of government bonds held by the private sector. I take both b_0 and k_0 as given.

An alternative --and it turns out more convenient-- representation of (2.2) and (2.3) is given by the present value formulation. In this case the present value formulation is given by,

(2.4) $\sum_{t=0}^{\infty} p_t c_t = \sum_{t=0}^{\infty} p_t(1-\tau_t^h)w_t n_t + p_0[(1-\delta)+(1-\tau_0^k)r_0]k_0+q_0(1+(1-\tau_0^k)R_0)b_0,$

and the no-arbitrage conditions,

(2.5) $p_t = p_{t+1}[1-\delta+(1-\tau_{t+1}^k)r_{t+1}]$ $\quad t = 0,1,...$

(2.6) $1-\delta+(1-\tau_{t+1}^k)r_{t+1} = 1+(1-\tau_{t+1}^k)R_{t+1}$ $\quad t = 0,1,...$

The first order conditions for the consumer's problem (using (2.4) as the budget constraint) are just,

(2.7) $\beta^t u_c(c_t,1-n_t) = \lambda p_t,$ $\quad t = 0,1,...$

(2.8) $\beta^t u_\ell(c_t,1-n_t) = \lambda p_t(1-\tau_t^h)w_t,$ $\quad t = 0,1,...$

and the budget constraint (2.4).

At this point it is easy to argue that, given any allocation, equation (2.7) can be used to pin down the sequence of prices (and the Lagrange multiplier, λ, is set equal to one), while (2.8) can be used to determine τ_t^h given the wage rate; then, equation (2.5) can be used to determine τ_{t+1}^k given the rental rate on capital, and (2.6) defines the rate of return on government bonds, given the after tax rate of return on real investments. Thus, the only condition that an allocation has to satisfy in order to be supportable as an equilibrium --at least from the representative consumer's point of view-- is the budget constraint (2.4). Using (2.7) and (2.8) in the budget constraint yields the following formulation,[3]

(2.9) $\sum_{t=0}^{\infty} \beta^t[u_c(c_t,1-n_t,g_{ct})c_t - u_\ell(c_t,1-n_t,g_{ct})n_t] = u_c(c_0,1-n_0,g_{c0})[(1-\delta)+(1-\tau_0^k)r_0]k_0+q_0(1+(1-\tau_0^k)R_0)b_0.$

As is standard in macro models, the supply side of the model is characterized by a large number of firms that behave competitively. In addition, I will assume that the technology

[3] This "primal" formulation follows the strategy of Lucas and Stokey (1983)

displays constant returns to scale in privately purchased factors[4]. It is then convenient, but not necessary, to write the production function for firm i as,

$$y_i \leq f(k_i,n_i)z(g_p),$$

where f is homogeneous of degree one, and $z(g_p)$ captures the productivity enhancing effects of public goods. I assume that private firms rent both capital and labor in spot markets. In this case profit maximization implies,

$$(2.10)\quad f_k(k_t,n_t)z(g_{pt}) = r_t,$$

$$(2.11)\quad f_n(k_t,n_t)z(g_{pt}) = w_t,$$

where g_{pt} is the amount of productive public goods supplied at time t. Note that, given an allocation, (2.10) and (2.11) determine r_t and w_t. Finally, feasibility for this economy is given by,

$$(2.12)\quad f(k_t,n_t)z(g_{pt}) + (1-\delta)k_t - c_t - k_{t+1} - g_{ct} - g_{pt} \geq 0.$$

In the absence of heterogeneity among agents, the government's objective is to maximize the utility of the representative agent subject to the constraints imposed by the behavior of both firms and families. It turns out that the "best" tax code is for the government to raise all its revenue needs (in present value) from taxing initial capital. The intuition underlying this result is straightforward: at t = 0 capital is in fixed supply; thus a tax on capital is equivalent to a lump sum tax. Since this is both unrealistic and uninteresting from the point of view of the design of optimal tax codes, I will restrict the government not to tax capital at t=0. It turns out that to prevent the government from taxing away capital at t=1, it is convenient to impose an upper bound on the tax rate on capital income. If this upper bound is $\bar{\tau}$, the condition $1-\tau^k_{t+1} \geq 1-\bar{\tau}$ is equivalent (see (2.5) and (2.7)) to the restriction,

$$(2.13)\quad [u_c(t)/(\beta u_c(t+1)) - (1-\delta)]/f_k(t+1)z(t+1) - (1-\bar{\tau}) \geq 0 \qquad t= 0,1,...$$

[4] This is not an "innocent" assumption. Jones, Manuelli and Rossi (1997) show that if the presence of public goods generates pure profits, the optimal tax code displays positive taxation of capital.

The benevolent's planner problem then maximizes the utility of the representative consumer subject to the budget constraint (2.9), the feasibility constraint (2.12) and the limit on the tax rate on capital income given by (2.13). As the preceding discussion shows, any allocation that solves this constrained optimization problem can be supported as a competitive allocation using as a price system the prices that satisfy the consumers' and firms' first order conditions.

The basic results on the structure of optimal tax codes in the long run are summarized in the following proposition:

Proposition 1. Consider the unrestricted optimal taxation problem. Assume that government spending is constant. Then,

i) If the solution converges to a steady state, the steady state tax rate on capital income is zero.

ii) If the utility function is separable in all three arguments and exhibits increasing relative risk aversion (i.e. $\sigma(c) \equiv -u_{cc}(c)c/u_c(c)$ is increasing in c), and the initial capital labor ratio falls short of the steady state capital labor ratio, the tax rate on capital is positive.

iii) If the utility function is separable in all three arguments and exhibits constant relative risk aversion (i.e. $\sigma(c) \equiv -u_{cc}(c)c/u_c(c)$ is constant in c), the optimal tax on capital income is either at its upper bound or zero.

iv) Under the assumptions in iii) and that the utility of leisure is given by the function $\ell^{(1-\phi)}/(1-\phi)$, the steady state tax rate on labor income is positive. Increases in the initial level of government debt as well as increases in the (constant) level of government spending decrease steady state output and increase the tax rate on labor income.

Proof: See Appendix.

What is this result saying? In the long run, the marginal rate of substitution between consumption in two consecutive periods is just the discount factor β^{-1}. At the same time, the marginal rate of transformation is $[1-\delta + f_k z]$. If the limiting tax rate on capital is positive, the equilibrium condition is $\beta^{-1} = [1-\delta +(1-\tau^k)f_k z]$ and, hence, it involves a wedge between marginal rate of substitution and transformation. Thus, in the long run, the efficiency costs of distorting this margin exceed those of generating larger levels of distortion in the short run.

This result, does not depend on the presence of public goods, but it relies critically on the existence of a sufficiently generous tax code. In the next section I show how tax evasion overturns this result.

3. Tax Evasion and Optimal Taxation

In this section I begin the analysis of the effects of tax evasion on the optimal tax code. In this paper I do not model the decision to evade or the detection technology. I simply assume that some factors (or sectors) cannot be taxed.[5] This seems like a reasonable first step in the analysis of the interaction between tax evasion and tax design.

The first case that I study is the situation when some capital-like factor cannot be taxed. Formally, this is equivalent to a restriction on the set of available taxes, namely a specification that the tax rate on the factor that evades be equal to zero. The effects of these restrictions upon optimal tax codes is one of the topics in Jones, Manuelli and Rossi (1997). However, the case that they study --equality of taxes across goods-- cannot be directly applied to the tax evasion problem.

Consider a two capital stock model. The technology is a simple extension of the technology in section 2, and it is given by,

(3.1) $f(k_t,s_t,n_t) + (1-\delta_k)k_t + (1-\delta_s)s_t - c_t - k_{t+1} - s_{t+1} - g_t \geq 0,$

[5] I can easily accommodate cases in which only a fraction of a given factor income is subject to taxation, or only a fraction of say, sales, is subject to taxation. The results for these more realistic cases are qualitatively the same. However , this partial tax evasion would require additional notation, with no added insights.

where I omit the level of government spending from both preferences and technology. This is without loss of generality, as shown in section 2.[6] The interpretation of this technology is simple: it assumes that there are two types of capital stock, k and s, that are inputs into the production function. Each of them is accumulated using standard linear laws of motion with potentially different depreciation.

To model tax evasion, assume that the income accrued to the stock s_t cannot be taxed. In this economy, the Ramsey planner's problem is similar to the one studied in section 3, with the additional requirement that the tax on s_t income be zero. What are the long run effects?

First, it is useful to discuss what would happen in the absence of tax evasion. In this setting, arguments similar to those of section 2 can be used to show that the results in this case mirror those of the one capital case: In the long run there is no taxation of either type of capital, and, in the case of separable utility that has constant relative risk aversion (CRRA), there is no taxation after a finite number of periods.

Consider now the modified problem. It is just the original set up with an additional restriction, namely that the tax rate on s_t be equal to zero. From the previous discussion, it follows that the additional constraint --the no taxation of s_t-- is not binding in the long run, and, in the case of CRRA preferences, not binding after finitely many periods. Thus, in terms of the qualitative features of the tax code, tax evasion does not change the results: in the unconstrained problem the optimal long run tax rate on s_t is zero and, hence, adding this constraint does not change the qualitative outcome.

Even though tax evasion does not affect the long run tax rate, it does affect the solution to the problem. The reason for this is simple: the additional restriction are binding in the first few periods. Thus, the value of the problem is lower and, consequently, the marginal welfare cost of

[6] Formally, I assume that g_t is wasted.

taxation is higher. This, has some implication for the long run tax rate on labor income. Formally I have,

Proposition 2. Assume that the tax rate on income produced by s_t is set equal to zero. Then,

i) In the long run, the tax rate on capital (k) is also zero.

ii)If utility is separable and displays weakly increasing relative risk aversion, tax evasion increases the long run tax rate on labor, and results in lower GDP.

iii) If preferences are separable and of the CRRA variety, then the tax rate on k_t-income becomes zero in finite time.

Proof: See Appendix

Not all forms of tax evasion can be fit into this framework. In some cases, it seems reasonable assume that there are some activities that cannot be taxed. For example, it is possible that there are some firms that operate in the formal economy, while other firms transact exclusively in black markets. To capture this idea I allow for a second production technology to produce the same good, and I designate the factors used in this technology as different inputs. Formally, the economy's feasibility constraint is,

$$(3.2) \quad f^1(k_t,n_{1t}) + f^2(s_t,n_{2t}) + (1-\delta_k)k_t + (1-\delta_s)s_t - c_t - k_{t+1} - s_{t+1} - g_t \geq 0.$$

In this case, I assume that the government cannot tax capital income generated in the second sector. Arguments paralleling those in the proof of Proposition 2 show that, in this case, the tax rate on capital income is still zero in the long run.

What happens if a form of labor income can be hidden from the tax authority? Formally consider an economy that uses two types of labor, n and v, and capital to produce a single consumption good. The feasibility constraint for this economy is, (3.3) $f(k_t,n_t,v_t) + (1-\delta_k)k_t +- c_t - k_{t+1} - g_t \geq 0.$

Assume that labor of type v cannot be taxed --it completely evades taxes. In this case, the same arguments used in section 2 show that the Ramsey planner's problem is to maximize the utility of the representative agent, subject to the standard implementability constraint (the budget constraint given by (2.9)), the feasibility constraints, constraints on upper bounds in taxes and,

(3.4) $u_s(t)/[u_c(t)f_v(t)] \geq 1,$

where this constraint simply formalizes the idea that income from v cannot be taxed. It is clear that this constraint (at equality) must be binding in the steady state (otherwise both forms of labor taxation and capital taxes would be zero), and this distorts the payoff to the government of an additional unit of capital. In particular, if f_{vk} is positive --the evading factor is complementary to capital-- then the limiting tax rate on capital income is positive. I formalize this in the following results,

Proposition 3. Consider a one sector economy with two types of labor, n and v. Assume that labor of type v can evade income taxes. Then, the optimal long run tax rate on capital income is positive, zero or negative depending on whether the evading factor is complement, independent or substitutable with capital (i.e depending on whether $f_{vk} \gtreqless 0$).

Proof: see Appendix.

What are the implications of these results? First, they point out that the nature of tax evasion matters. More precisely, the implication for capital income taxation depends on whether the evading factor is a capital-like object, or a labor-like object. Tax evasion from capital-like objects do not change the qualitative implications of the Ramsey model of optimal taxation: in the long run taxes on capital income are zero. On the other hand, tax evasion from some form of labor is substantially more difficult to handle. The basic intuition is that if the evading factor and capital are complements, this is if $f_{vk} > 0$, then taxation of capital reduces the capital stock and this, in turn. reduces the marginal product of labor of type v; that is f_v is lower. Since direct taxation of type v labor also has the same effect, it is clear that capital taxation, which is not desirable per se, is just an indirect way of restricting the supply of type v labor. The reason why this is desirable is because, if the tax rate on capital were zero, too many people would be supplying type v labor, as its relative return is high. By taxing capital --a complementary factor-- the optimal tax code reduces the private incentives to supply labor in the black market, since it lowers its marginal product.

The picture that emerges is a complex one. Tax evasion has two effects: first, it has a revenue effect which is not the main point of this analysis;[7] it also has an efficiency effect in that it can change the qualitative nature of the "third best" tax code. Any process of tax reform should take into account the existing distortions to optimally choose tax rates for the taxable factors and sectors.

4. Endogenous Determination of Spending

So far, following a long tradition in public finance, I have taken the spending side of the model as fixed. In other words, the agents responsible for designing tax policy pretend that whatever changes in the economy occur because of the tax reforms, there will be no changes on the spending side. Although a useful abstraction to study the structure of tax codes, this is probably an unrealistic assumption. For example, increases in output may require increases in the supply of roads or infrastructure. In this section, I relax the exogeneity assumption and I allow the

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[7] However, the revenue effect does have an impact on the quantitative predictions of the model. In particular, the more restrictions faced by the Ramsey planner, the higher the marginal welfare cost of taxation, the Lagrange multiplier Φ in the proof of Proposition 1, and this, in turn. results in changes in the asymptotic tax rates.

planner to chose tax rates, understanding that spending is endogenous. In a "first best" world, the same planner would choose taxation and spending. However, in many political systems, decisions about taxation and spending are taken by different agencies. In this section, I explore the impact on the optimal tax code of a given spending policy. The class of policies that I study are those that keep the ratio of spending to output constant. These policies are realistic, and it is relatively common to see arguments in favor of increasing a given spending category based on the current level being below its historical average. Our purpose in conducting this exercise is to understand how the existence of this type of responsive government spending will affect the design of the optimal tax code.

To keep the presentation simple I will consider two types of publicly provided goods: those that enter in the utility function (parks, art, recreation) and those that affect the technology (infrastructure, educational spending). First, I study the impact of endogeneizing the supply of utility enhancing public goods. I study the effects of productive public goods later.

I consider a very simple setting. Some governmental agency has decided that government spending on utility enhancing goods will be a <u>fixed fraction</u> of output. Let this fraction be denoted υ. The agency in charge of designing the tax code takes υ as given but understands that any increases in output that would result as a consequence of changes in the tax code will be allocated --in proportion υ-- to the purchase of more government goods.

To illustrate the effects of this policy on the choice of optimal taxes, let's consider the simple model of section 2. It follows that the planner's problem is to maximize the utility of the representative agent given by,

$$(4.1) \quad \sum_{t=0}^{\infty} \beta^t \{u(c_t,1-n_t,g_t) ,$$

subject to the standard implementability constraint (equation (2.9)) which I repeat,

$$(4.2) \quad \sum_{t=0}^{\infty} \beta^t[u_c(c_t,1-n_t,g_t)c_t - u_\ell(c_t,1-n_t,g_t)n_t] = u_c(c_0,1-n_0,g_0)[(1-\delta)+(1-\tau_0^k)r_0]k_0+q_0(1+(1-$$

$\tau_0^k)R_0)b_0$,

the feasibility constraint,

(4.3) $f(k_t,n_t) + (1-\delta)k_t - c_t - k_{t+1} - g_t \geq 0$,

and the "fiscal policy" constraint,

(4.4) $g_t = \upsilon f(k_t,n_t)$.

It is this last constraint that makes the Ramsey problem slightly different. It is clear that the first best level of government spending satisfies $u_c = u_g$, that is, the marginal rate of substitution between public and private goods is one. How does the existence of an arbitrary rule like (4.4) affect the optimal long run tax on capital? It turns out that the answer depends on how spending responds to changes in the quantity of public goods. In general, it is not possible to give a sharp characterization. The results are summarized in,

Proposition 4 Suppose that the economy converges to the steady state.

i) In general, the optimal tax rate can be either positive or negative.

ii) If the utility is separable in all three factors, then the optimal long run tax rate on capital satisfies $\tau_\infty^k = \upsilon[u_c-u_g+\Phi u_c(1-\sigma(c))]/[u_c(1+\Phi(1-\sigma(c))]$, where Φ is the (positive) marginal welfare cost of taxation, $u_c(1+\Phi(1-\sigma(c)) > 0$, and $\sigma(c) \equiv -u_{cc}c/u_c$.

iii) If utility is separable and spending is chosen at its first best level, $u_c = u_g$, the tax rate on capital will be positive if preferences display high intertemporal elasticity of substitution ($\sigma(c) < 1$), and negative if the degree of intertemporal substitution is low ($\sigma(c) > 1$).

Proof: see Appendix.

What is the economic intuition for this finding? Note that the Ramsey planner understands that any increases in output associated with "improvements" in the tax code are not fully captured as increases in consumption. The reason is simple: equation (4.4) shows that a fraction of any increase in output goes to providing public consumption goods. This acts as a "tax" on the payoff to an additional unit of capital. On the other hand, the "proceeds" of this tax also increase welfare through higher quantities of public goods.

To illustrate the main point suppose that utility is separable and close to logarithmic (this corresponds to $\sigma(c) = 1$); then the proposition shows that if government spending falls short of the optimum (we take this to be the case whenever $u_g > u_c$ then capital should be subsidized, while in the opposite case ($u_g < u_c$) it should be taxed. Thus, when government spending is excessive, taxation of capital reduces the level of output and, through the impact of (4.4), the excessive level of government spending.

The result also indicates that even when government spending is chosen at its first best,[8] it is the intertemporal elasticity of substitution that determines whether capital is taxed or subsidized. If preferences display small degree of substitution, $\sigma(c) > 1$, small changes in the after tax rate of return do not change the ratio of future to present consumption. Thus, the planner subsidizes the capital stock to induce the appropriate amount of saving.

It is possible to use the results in the proposition to determine how increases in initial government debt will affect the long run tax rates on capital. Standard arguments, see Jones, Manuelli and Rossi (1993), can be used to show that an increase in government debt increases the marginal welfare cost of taxation as measured by the Lagrange multiplier Φ. It follows that, in the separable case,

$$d\tau_\infty^k/d\Phi = \upsilon u_{gc}(1-\sigma(c))/[u_c(1+\Phi(1-\sigma(c))]^2.$$

Thus, if the steady state involves taxation and $\sigma(c) < 1$, then higher debt levels result in even higher tax rates. Of course, it is possible that $\tau_\infty^k > 0$, and $\sigma(c) < 1$ (high intertemporal

[8] Note that this will be the outcome whenever "first best" prices are used to evaluate projects using cost benefit analysis, ignoring the fact that, at the margin, government spending and the tax code interact in complicated ways

substitution). In this case, higher initial debt results in lower taxes. Thus, the effects of public debt on taxation are theoretically ambiguous, but they depend in a simple way of features of preferences and levels of spending. Thus, in quantitative implementations of the Ramsey program, it is possible to determine the sign of the total effect.

How do the results depend on the specification of preferences over public goods? One alternative to government goods in the utility function is to assume that g is pure waste; that is, it does not appear in the utility function. In this case, the proof of Proposition 4.1 shows that capital should be taxed, and that the tax rate is just υ. This result is not surprising. As indicated above, the Ramsey planner knows that each additional unit of output produces only $1-\upsilon$ units of potentially consumable goods. Thus, to signal to the private sector this lower social productivity, it chooses a tax equal to the degree of waste.

Our results are related to Turnovsky (1996). The main difference is that he considers a linear technology (in capital) and, hence, cannot tax labor. In addition, he studies a special case of preferences that implies that if spending is chosen optimally and there are no congestion effects, then the optimal tax is zero. Note that our result shows that the tax rate on capital maybe positive or negative even if spending is at the optimal (steady state) level, but determined as a fixed fraction of output.

We next study the case of productive government spending. Let output, y, be given by,[9]

(4.5) $y = f(k,n)g^{\phi}, \qquad 0 < \phi < 1,$

and,

[9] This specification is different from Turnovsky's (1996) analysis in that it ignores congestion effects, but allows for non-linear technologies. It is also related to Corsetti and Roubini (1996). The main difference is that although they allow government spending to enter in several production functions, their specifications always imply that the private sector faces decreasing returns to scale in the private factors. In this case, it is well known (see Jones, Manuelli and Rossi (1993) and (1997)) that the optimal tax code is different from the Ramsey code because of the existence of pure profits. Since this is a well understood result, our emphasis is to try to understand the effect of spending in as close a setting to the original Ramsey problem as possible. From a practical point of view, the cases studied by Turnovsky and Corsetti and Roubini, as well as our framework, deserve further analysis.

(4.6) $g = \upsilon y$.

Thus, the government spends fraction υ of output on productive government goods, and the average productivity of government spending is ϕ. Solving for g using (4.5) and (4.6), it follows that g is given by,

(4.7) $g = \upsilon^{1/(1-\phi)}f(k,n)^{1/(1-\phi)}$,

output is just,

(4.8) $y = \upsilon^{\phi/(1-\phi)}f(k,n)^{1/(1-\phi)}$,

and output net of government spending, y^N, is,

(4.9) $y^N = (1-\upsilon)\ \upsilon^{\varphi/(1-\varphi)}f(k,n)^{1/(1-\varphi)}$,

Following the arguments in the proof of Proposition 1, if the solution converges to a steady state, the steady state condition is that the discounted marginal product of *net* output be equal to one. Formally,

(4.10) $1 = \beta[(1-\delta) + (1-\upsilon)f(k,n)^{\varphi/(1-\varphi)}\upsilon^{\varphi/(1-\varphi)}f_k(k,n)/(1-\varphi)]$,

while the private equality of marginal benefit and marginal cost is,

(4.11) $1 = \beta[(1-\delta) + (1-\tau^k)f_k(k,n)g^{\varphi)}]$.

Evaluating this latter condition at the endogenously determined g, I get,

(4.12) $1-\tau^k = (1-\upsilon)/(1-\varphi)$.

This proves:

Proposition 5 In the case of productive public goods, the steady state tax rate on capital income is positive (negative) if the share of output spent on productive inputs exceeds (falls short) of their average product.

Thus, the optimal tax code takes into account inefficiencies in the allocation of public spending. To understand the economic forces at work, note that the first best level of public spending, simply maximizes net output; that is, the optimal g solves,

$$\max_g f(k,n)g^{\varphi} - g.$$

It is easy to check that the solution is $g = \varphi^{1/(1-\varphi)} f(k,n)^{1/(1-\varphi)}$, while (4.7) implies that the actual g satisfies, $\upsilon^{1/(1-\varphi)} f(k,n)^{1/(1-\varphi)}$. Comparison of these two expressions reveals the economics of the result. If $\varphi > \upsilon$, then government spending is excessive relative to the first best. In that case, the only way that the tax authority can reduce g is through reductions in output, and this is accomplished through taxation of capital. Note that in this case, (4.12) shows that the optimal tax rate on capital is positive.

Comparison of the two cases, utility enhancing and productivity enhancing spending, shows that they have different effects for the qualitative features of the tax code. The contrast is particularly clear when considering situations in which the spending levels are at their (steady state) first best. If government spending is productivity enhancing, this requires no taxes on capital, while if government spending is utility enhancing, capital may be taxed or subsidized depending on the intertemporal elasticity of substitution.

5. Conclusions

In this paper I have studied the effects of two realistic deviations from the basic Ramsey problem --tax evasion and endogenous government spending-- for the design of optimal factor income tax codes.

A major finding is that both tax evasion and endogenous spending have an impact upon the optimal tax code. Moreover, the qualitative impact is difficult to determine without detailed knowledge of the economy.

In the case of tax evasion the key determinant is whether the evading factor behaves more like capital (in the sense that it can be accumulated) or it behaves like labor (in the sense that it cannot be accumulated). In the former case, tax evasion has no effects on the long run tax rate of capital. It is zero, as in the Ramsey problem. Under reasonable assumptions, the tax rate on labor like inputs increases. In the second case, the tax rate on capital income is, in general, non-zero. It will be positive if the evading factor and capital are complements, and capital will be subsidized if the two factors are substitutes.

The presence of endogenous government spending also changes the limiting tax rates on capital income. I consider cases in which some other agency --say Congress-- determines a rule for allocating public goods. The rules that I consider are fixed, but arbitrary, fractions of output. In this case, I show that if public goods are utility enhancing, endogenous spending may require taxation or subsidization of capital depending on details of preferences and other taxes. If, on the other hand, public goods are productivity enhancing, capital maybe taxed or subsidized in the long run. If the publicly provided good is supplied in quantities exceeding its first best proportions, then capital is taxed. If, instead, the supply of productive public goods falls short of the optimum, then capital should be subsidized.

The main message that emerges from this paper is that fiscal reforms --or the design of optimal tax codes-- is a complicated issue. Even abstracting from pressing revenue needs --in the model of this paper governments have access to perfect capital markets-- the choice and level of tax rates is influenced by the set of distortions in the economy. Even for “simple” distortions --like those modeled here-- the nature of the optimal tax code is not obvious. Even though we have identified departures from the Ramsey rule, theory cannot provide a sense of their quantitative importance. This can only be done by solving the actual Ramsey problems for the relevant parameter values. Thus, the next step --and one that will take the theory closer to providing concrete policy recommendations-- is to conduct numerical exercises, along the lines of Jones, Manuelli and Rossi (1993),(1997), to assess the quantitative importance of these departures from

Ramsey taxation.

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Appendix

Proof of Proposition 1: The benevolent planners' problem can be described by the following Lagrangean,

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \{u(c_t,1-n_t,g_{ct}) + \Phi[u_c(t)c_t - u_\ell(t)n_t] + \mu_t[f(k_t,n_t)z(g_{pt}) + (1-\delta)k_t - c_t - k_{t+1} - g_{ct} - g_{pt}]$$
$$+ \theta_t[u_c(t)/(\beta u_c(t+1)f_k(t+1)) - (1-\delta)/f_k(t+1) - (1-\bar{\tau})]\} - \Phi[u_c(0)(1-\delta + f_k(0))k_0 + b_o].$$

The first order conditions for the planner's problem are,

(A.1.1) c_t: $u_c(t)\,[1+ \Phi(1+u_{cc}(t)c_t/u_c(t) - u_{c\ell}(t)n_t/u_c(t))] - \mu_t + \theta_t[u_{cc}(t)/(\beta u_c(t+1)f_k(t+1)z(t+1))] - \theta_{t-1}[u_c(t-1)u_{cc}(t)/(\beta u_c(t+1))^2 f_k(t+1)z(t+1))] = 0,$ $t = 1,2,...$

(A.1.2) n_t: $-u_\ell(t) + \Phi[-u_{c\ell}(t)c_t - u_\ell(t) + u_{\ell\ell}(t)n_t] + \mu_t f_n(t)z(t) - \theta_t[u_{c\ell}(t)/(\beta u_c(t+1)f_k(t+1)z(t+1))] + \theta_{t-1}[u_c(t-1)(-u_{c\ell}(t)f_k(t) + u_c(t)f_{kn}(t))/(\beta u_c(t)f_k(t))^2 - (1-\delta)f_{kn}(t)/\beta(f_k(t))^2] = 0,$ $t = 0,1,...$

(A.1.3) k_{t+1}: $-\mu_t + \beta\mu_{t+1}[(1-\delta) + f_k(t+1)z(t+1)] + \theta_t[(u_c(t)/(\beta u_c(t+1)) - (1-\delta))(f_{kk}(t+1)/f_k(t+1))] = 0,$ $t = 0,1,...$

and the constraints. To proceed with the analysis, recall that the tax rate on capital income satisfies (see (2.5) and (2.7))

(A.1.4) $u_c(t) = \beta u_c(t+1)[1-\delta + (1-\tau^k_{t+1})f_k(t+1)z(t+1)]$ $t = 0,1,...$

To prove i) simply note that at the steady state (and if the tax rate is less than the upper bound) $\theta = 0$. Then, with $\mu_t = \mu_{t+1} = \mu$, equation (A.1.3) implies that $\beta[1-\delta + f_k z]=1$. This condition and (A.1.4) evaluated at the steady state, implies that $\tau^k = 0$.

To prove ii) first note that if the upper bound on the tax rate of capital income is binding, then there nothing to be proved. Thus assume that θ_t is equal to zero. Let $m(c) \equiv 1+\Phi(1-\sigma(c))$. Then the relevant version of (A.1.1) is

(A.1.5) $$u_c(c_t)m(c_t) = \mu_t,$$

where it is easy to check that under the stated assumptions m(c) and $u_c(c)m(c)$ are decreasing in c. Note that from (A.1.3) it follows that $\mu_t > \mu_{t+1}$ since the capital labor ratio κ_{t+1} falls short of the steady state capital labor ratio κ. It then follows that $c_{t+1} > c_t$. Next, using (A.1.5) in (A.1.3) and assuming that the upper bound on the tax rate on capital is not binding (if not it is clear that tax rates are constant), it follows that,

$$u_c(c_t)m(c_t) = \beta\, u_c(c_{t+1})m(c_{t+1})[1-\delta + f_k(t+1)z(t+1)],$$

while from the consumer's problem (A.1.4) implies that $(1-\tau^k_{t+1}) < 1$ if and only if $m(c_{t+1}) < m(c_t)$. This, in turn, follows from the monotonicity of m and $c_{t+1} > c_t$.

Next consider iii). Under the assumptions of the proposition and if θ_t=0, (A.1.5) is just, $u_c(t)\,[1+ \Phi(1- \sigma)] = \mu_t$. Using this in (A.1.3) (again when θ_t=0) implies that $u_c(t) - \beta u_c(t+1)[1-\delta + f_k(t+1)] = 0$. This condition and (A.1.4) imply that the tax rate on capital income is zero.

To prove iv), note that the steady state feasibility constraint implies that,

(A.1.6) $$c = f(\kappa,1)zn - \delta\kappa n - g_c - g_p,$$

where κ is the steady state capital labor ratio. Combining the steady state versions of (A.1.1) and (A.1.2) I get,

(A.1.7) $$c^\sigma = [(1+\Phi(1-\sigma))f_n(\kappa,1)z](1-n)^\varphi/[1+\Phi(1+\varphi n/(1-n))] \equiv h(n,\Phi).$$

The steady state values of (c,n) are the solution to (A.1.6) and (A.1.7). Since h(n,Φ) is decreasing in both arguments, it follows that an increase in Φ decreases n, and hence, output. Standard arguments (see Judd (1986), Chamley (1986) and Jones, Manuelli and Rossi (1997)) show that Φ is the marginal welfare cost of distortionary taxation and that it increases with the stock of initial debt as well as the level of government spending. Thus, increases in debt or the size of the government decrease steady state output and, in all cases, increase the relative size of the government as given by g/y, where y is per capita output. Finally, note that the steady state tax rate on labor income is,

$$(1-\tau^h) = [(f(\kappa,1) - \delta\kappa)n - g]^{\sigma}/[(1-n)^{\varphi}f_n(\kappa,1)].$$

Thus, an increase in Φ which decreases employment must, of necessity, increase the tax rate on labor income. ■

Proof of Proposition 2: Most of the proof parallels that of Proposition 2.1. The argument that the steady state tax rate on capital is zero is identical, and it will not be repeated here. At the steady state it can be verified that,

$$h^1(n) = h^2(n,\Phi),$$

where,

$$h^1(n) = v'(1-n)/[u'((f(\kappa,\upsilon,1) - \delta_k\kappa - \delta_s\upsilon)n - g)\, f_n(\kappa,\upsilon,1)],$$

$$h^2(n,\Phi) = [1+\Phi(1-\sigma((f(\kappa,\upsilon,1) - \delta_k\kappa - \delta_s\upsilon)n - g)))]/[1+\Phi(1+\sigma(1-n)n/(1-n))],$$

where, as in the proof of Proposition 1, $\sigma(x)$ is the elasticity of the marginal utility of x (its coefficient of relative risk aversion), κ and υ are the steady state k/n and s/n ratios that are completely determined by the efficiency conditions on capital. For simplicity, I denote the utility of leisure v, while u corresponds to the utility of consumption. Under our assumptions h^1 is an increasing function of n, while h^2 decreases in both n and Φ.

Consider now the comparison between two economies with the same $\{g_t\}$ sequence but different degrees of tax evasion. To simplify consider either no tax evasion --both k and s can be taxed-- or complete evasion of s-income. The latter planner's problem mimics the former with some additional constraints. Since the constraint is binding, it must be that the maximized value of welfare is lower. It then follows that in the economy with tax evasion, the marginal welfare cost of taxation --the increase in utility associated with the ability to use lump-sum taxes to raise one a dollar of revenue-- must be higher. It then follows that the higher the level of Φ the lower the optimal level of n. This, of course, implies that τ^h must be higher in the economy with tax evasion, and output lower. ■

Proof of Proposition 3: The essence of the proof follows the strategy of the proof of Proposition 1. The key difference is that, from the point of view of the consumer, the relevant marginal conditions are

(A.3.1) $\beta^t u_\ell(c_t,1-n_t,1-v_t) = \lambda p_t(1-\tau^h_t)w_{nt}$,

(A.3.2) $\beta^t u_v(c_t,1-n_t,1-v_t) = \lambda p_t w_{vt}$,

(A.3.3) $\beta^t u_c(c_t,1-n_t,1-v_t) = \lambda p_t$,

where u_ℓ and u_v indicate the marginal utility of leisure of type n and v, respectively, while w_{jt} is the market wage --in units of contemporaneous consumption-- of labor of type j = n,v. Using arguments similar to those presented in section 2, it follows that,

(A.3.4) $f_j(k,n,v) = w_j \qquad j = n,v.$

It follows that any candidate allocation must satisfy these equations. In particular, it implies that the following restriction must hold,

(A.3.5) $u_v(c_t,1-n_t,1-v_t) = u_c(c_t,1-n_t,1-v_t)f_j(k_t,n_t,v_t)$.

It can be checked that the relevant version of (2.9) is,

$$\sum_{t=0}^{\infty} \beta^t[u_c(c_t,1-n_t,1-v_t)c_t - u_\ell(c_t,1-n_t,1-v_t)n_t - u_v(c_t,1-n_t,1-v_t)v_t] = u_c(c_0,1-n_0,1-v_0)[(1-\delta)+(1-\tau^k_0)r_0]k_0+q_0(1+(1-\tau^k_0)R_0)b_0.$$

Imposing (A.3.5) to guarantee that the allocation is implementable, implies that the relevant budget constraint is,

(A.3.6) $$\sum_{t=0}^{\infty} \beta^t[u_c(c_t,1-n_t,1-v_t)c_t - u_\ell(c_t,1-n_t,1-v_t)n_t - u_c(c_t,1-n_t,1-v_t)f_v v_t] = u_c(c_0,1-n_0,1-v_0)[(1-\delta)+(1-\tau^k_0)r_0]k_0+q_0(1+(1-\tau^k_0)R_0)b_0.$$

The Ramsey problem (for details see the proof of Proposition 1) is associated with the following Lagrangean,

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \{u(t) + \Phi[u_c(t)c_t - u_\ell(t)n_t - u_c(t)v_t f_v(t)] + \mu_t[f(k_t,n_t)z(g_{pt}) + (1-\delta)k_t - c_t - k_{t+1} - g_{ct} - g_{pt}] + \theta_t[u_c(t)/(\beta u_c(t+1)f_k(t+1)) - (1-\delta)/f_k(t+1) - (1-\bar{\tau})]\} - \Phi[u_c(0)(1-\delta + f_k(0))k_0 + b_o].$$

The relevant steady state version of the marginal condition for the optimal choice of capital (the analog of equation (A.1.3)) implies,

(A.3.7) $$1 = \beta(f_k+1-\delta) - \Phi u_c v f_{vk}.$$

Comparing this with (A.1.4) at the steady state shows that the tax rate satisfies,

(A.3.8) $$\tau^k = \Phi u_c v f_{vk}/(\beta f_k).$$

Since Φ is the marginal welfare cost of distortionary taxation (and, hence, positive) the sign of τ^k is the same as the sign of f_{vk}. This completes the proof. ■

Proof of Proposition 4: The proof strategy is similar to that of Proposition 1. Thus, only a sketch is presented here. For notational simplicity, let $V(c,n,k,\Phi) = u(c,1-n,\upsilon f(k,n)) + \Phi[u_c c - u_\ell n]$. Then, the Ramsey problem is associated with the following Lagrangean:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \{V(c_t,n_t,k_t,\Phi)+ \mu_t[(1-\upsilon)f(k_t,n_t) + (1-\delta)k_t - c_t - k_{t+1}] + \theta_t[u_c(t)/(\beta u_c(t+1)f_k(t+1)) - (1-\delta)/f_k(t+1) - (1-\bar{\tau})]\} - \Phi[u_c(0)(1-\delta + f_k(0))k_0 + b_o].$$

Simple calculations show that --at the steady state-- the optimality condition for the capital stock --the analog of (A.1.3)-- is just,

$$1 = \beta(1-\delta + f_k) + \beta(V_k/V_c)-\upsilon f_k].$$

Since in the steady state the tax rate on capital income satisfies, $\beta(1-\delta + (1-\tau^k_\infty)f_k) = 1$, it follows that the tax rate satisfies,

(A.4.1) $$\tau^k_\infty = \upsilon - (V_k/V_c f_k) = \upsilon[u_c - u_g + \Phi(\partial q/\partial c - \partial q/\partial g)]/u_c m(c),$$

where $q \equiv u_c(c,1\text{-}n,g)c - u_\ell(c,1\text{-}n,g)n$ is the value, using equilibrium quantities, of the excess of consumption expenditures over labor income, and m(c) is defined in the proof of Proposition 1. In general, theory does not restrict the sign of the terms $\partial q/\partial c - \partial q/\partial g$ and, hence, it is not possible to determine the sign of τ^k_∞.

In the special case of separable utility functions, (A.4.1) is,

$$\text{(A.4.2)} \qquad \tau^k_\infty = \upsilon[u_c - u_g + u_c\Phi(1-\sigma(c))]/[u_c(1+\Phi(1-\sigma(c))].$$

Since the denominator is always positive and Φ, the marginal welfare cost of distortionary taxation, is also positive (see Jones, Manuelli and Rossi (1993)), the results of the proposition follow. ■