Econ 871: Lecture Notes, Part 6

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Reading

- Bernard, Eaton, Jensen and Kortum (2003)

BEJK (2003)

- Build on Eaton and Kortum (2002)

- Introduce imperfect competition and get

  - firm level heterogeneity in measured productivity ($\approx$profitability)

  - not all firms export, and those which do, still sell mostly at home

- Alternative to Melitz, with the feature of ‘head-to-head’ competition within the same variety

Firm Level Evidence

- Document characteristics of an ‘exporting plant’ (*relative to* ‘non-exporting plant’)

- Data from 1992 US Census of Manufactures: 200,000 plants in the sample

- See figures in the paper

Exporter Facts

- Very few plants report exporting anything – about 21%

- Those that do, still sell mostly at home – 2/3 of plants in the sample export less than 10% of their total output

- Exporting firms are on average larger (ship 5.6 more output), and more productive (more profitable)

  – productivity measured by value added to total payroll bill of production workers (controlling for capital/skill intensity of the plant)

Formalization

- World comprised of N countries

- Continuum of goods and Dixit-Stiglitz preferences

$$U = u(C)$$

where

$$C = \left[\int_0^1 q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}$$

- Exogenous endowment of labor L

Standard Dixit-Stiglitz Demand

- Expenditures on good j in country n given by

$$x_n(j) = X_n \left(\frac{p_n(j)}{P_n} \right)^{1-\sigma},$$

where X_n are total expenditures, P_n is the price index, $p_n(j)$ price of good j.

Production

- Each country has multiple potential producers of each variety with varying levels of technical efficiency

- The k'the most efficient producer of good j in country i can convert local bundle of inputs into quantity $z_{ki}(j)$ of good j at constant returns to scale, and deliver a unit of the good to country n at cost

$$c_{kni}(j) = \frac{w_i}{z_{ki}(j)} d_{ni}$$

Market Structure and Ownership

- BEJK'03 assume Bertrand competition between the producers within same variety (each producer owns one technology)

- Implications: similarly to EK'02, each market is captured by the low cost supplier, but unlike in EK'02 the markup can be positive:

  – lowest cost supplier is constrained not to charge less than the cost of the second-lowest cost supplier

$$c_{2n}(j) = \min\left\{c_{2nl}(j), \min_{i \neq l}\{c_{1nl}(j)\right\}$$

  where l is the country of origin of the lowest-cost supplier.

Prices

- The price of good j in country n is

$$P_n(j) = \text{min}\{c_{2n}(j), \frac{\sigma}{\sigma - 1} c_{1n}(j)\}.$$

- The markup is the maximal feasible markup as long as it is not higher than the MC optimal markup

Probabilistic Formulation of Technology

- To cover all possibilities, need to know the highest and second-highest efficiency draw $z_{1i}(j), z_{2i}(j)$ in each country

- Similarly to EK'02 these productivities are realizations of a random variable drawn from a carefully chosen distribution

$$F_i(z_1, z_2) = \Pr\left[Z_{1i} \leq z_1, Z_{2i} \leq z_2\right] = \left[1 + T_i(z_2^{-\theta} - z_1^{-\theta})\right] e^{-T_i z_2^{-\theta}},$$

for $0 \leq z_2 \leq z_1$, drawn independently across countries i and goods j.

Justification From Endogenous Innovation

- Let $R_i(t)$ be research effort at location i to discover a new technique to produce a good

- Technique is characterized by the efficiency z in producing it

  – cost of producing is then w_i/z

- Given measure 1 of goods, let good j be a representative good with research effort $R_i(t)$ devoted in each instance of time to discover a new technique to produce it

- Let the arrival of new techniques be governed by a Poisson process with intensity $\bar{a}R_i(t)$ and efficiency draw from a Pareto distribution $F(Z > z) = (\frac{z}{\bar{z}})^{-\theta}$, conditional on $z \geq \bar{z}$.

- Assume arrival of a draw $Z \geq z$ is $R_i(t)z^{-\theta}$ by assuming $\bar{z}^{\theta}\bar{a} = 1$.

- Clearly, at time t the number of techniques with efficiency higher than $\hat{z}$ is distributed Poisson with parameter $T_i\hat{z}^{-\theta}$, where

$$T_i = \int_0^t R_i(t)dt.$$

Refresher on Poisson Process

- Let the number of counts up to time t be denoted by $N(t)$ (which is total number of occurence of certain "events" up to time t)
  - e.g. arrival of phone calls to a customer service center

- The counting process $N(t)$ is Poisson iff the inter-arrival times are exponentially distributed with parameter λ

$$P(T < t) = 1 - e^{-\lambda t}.$$

- The distribution of the number of 'counts' up to time t is

$$P(N = n) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}.$$

Best and Second-Best Technology

- First derive the distribution of efficiency for a fixed number of techniques. Suppose there are n techniques discovered up to time t. What is the probability that exactly 2 are above z_1 and $n-2$ are below z_2, where $z_1 \geq z_2$?

- Since these are disjoint events, the probability is given by the Bernoulli trial formula:

$$\binom{n}{2}(1-F(z_1))^2 F(z_2)^{n-2}.$$

Given the distribution F, we can calculate the density function by taking the negative of its derivative w.r.t. z_1 and then w.r.t. z_2. This way we can the obtain the joint density conditional on n

$$g(z_1, z_2|n) = \frac{n!(1-F(z_1))^{2-1}F(z_2)^{n-2-1}}{(2-1)!(n-2-1)!}F'(z_1)F'(z_2) \text{ for } n \geq 3$$

$$0 \text{ otherwise}$$

- Given that at time t we know the distribution of n (Poisson with parameter $T_i\hat{z}^{-\theta}$), the conditional distribution F (conditional on $z \geq \hat{z}$), we can calculate the unconditional joint density function $g(z_1, z_2) =$

$\sum_{n=0}^{\infty} g(z_1, z_2, n) P(n; Z \geq \bar{z})$

$$g(z_1, z_2) = \sum_{n=0}^{\infty} g(z_1, z_2, n) \frac{(T\bar{z}^{-\theta})^n}{n!} \exp(-T\bar{z}^{-\theta}) =$$

$$\frac{(1 - F(z_1))(T\hat{z}^{-\theta})^{2+1} F'(z_1) F'(z_2) \exp(-T\hat{z}^{-\theta}(1 - F(z_2)))}{(2-1)!} \times$$

$$\times \sum_{n=2+1}^{\infty} \frac{(T\bar{z}^{-\theta} F(z_2))^{n-2-1} \exp(-T\hat{z}^{-\theta} F(z_2))}{(n-2-1)!}$$

$$\frac{(1 - F(z_1))(T\hat{z}^{-\theta})^{2+1} F'(z_1) F'(z_2) \exp(-T\hat{z}^{-\theta}(1 - F(z_2)))}{(2-1)!}$$

$$\frac{\theta^2 (\frac{z_1}{\hat{z}})^{-\theta} (T\hat{z}^{-\theta})^{2+1} (\frac{z_1}{\hat{z}})^{-\theta-1} (\frac{z_2}{\hat{z}})^{-\theta-1} (\frac{1}{\hat{z}})^2 e^{-T\bar{z}^{-\theta}(\frac{z_2}{\bar{z}})^{-\theta}}}{(2-1)!} =$$

$$\frac{\theta^2 (\frac{z_1}{\hat{z}})^{-2\theta-1} T^3 \hat{z}^{-3\theta} (\frac{z_2}{\hat{z}})^{-\theta-1} (\frac{1}{\hat{z}})^2 e^{-Tz_2^{-\theta}}}{(2-1)!}$$

- Finally, assuming $\hat{z} \to 0$ (note: $\hat{z}$ canceled out, so taking this limit has no bearing on the result), we obtain

$$g_k(z_1, z_2) = \theta^2 {z_1}^{-2\theta-1} T_i^3 {z_2}^{-\theta-1} e^{-T_i z_2^{-\theta}}$$

- Given the 'conditional' cumulative density, we can integrate twice to obtain the 'unconditional' distribution function:

$$G(Z_1 \leq z_1, Z_2 \leq z_2) = \left[1 + T_i(z_2^{-\theta} - z_1^{-\theta})\right] e^{-T_i z_2^{-\theta}},$$

for $0 \leq z_2 \leq z_1$, drawn independently across countries i and goods j.

Cost Functions

- The cost is a realization of the following two random variables:

$$
\begin{aligned}
c_{1ni}(j) &= \frac{w_i}{Z_{1i}(j)} d_{ni} \\
c_{2ni}(j) &= \frac{w_i}{Z_{2i}(j)} d_{ni}
\end{aligned}
$$

- Given distribution of efficiency can obtain distribution of first and second lowest cost:

$$
\begin{aligned}
G^c_{ni}(C_1 &> c_1, C_2 > c_2) = \Pr\left[C_1 \leq z_1, C_2 \leq c_2\right] \\
&= \Pr\left[Z_1 \leq \frac{w_i d_{ni}}{c_1}, Z_2 \leq \frac{w_i d_{ni}}{c_2}\right] \\
&= G(\frac{w_i d_{ni}}{c_1}, \frac{w_i d_{ni}}{c_2}).
\end{aligned}
$$

- It solves to

$$
G^c_{ni}(c_1, c_2) = \left[1 + T_i[w_i d_{ni}]^{-\theta}(c_2^\theta - c_1^\theta)\right] e^{-T_i[w_i d_{ni}]^{-\theta} c_2^\theta}
$$

- The complementary distribution of the the lowest and second-lowest cost regardless the source (the probability that the the lowest and second-lowest cost in all countries is above c_2+ probability that in one of the countries the lowest cost is between c_1 and c_2, second-lowest is above c_2 and in all other countries both lowest and second-lowest is above c_2)

$$\begin{aligned} G_n^c(c_1, c_2) &= \Pi_{i=1}^N G_{ni}^c(c_2, c_2) + \\ &+ \sum_{i=1}^N [G_{ni}^c(c_1, c_2) - G_{ni}^c(c_2, c_2)] \Pi_{k \neq i} G_{nk}^c(c_2, c_2) \\ &= \left[1 + \Phi_n(c_2^\theta - c_1^\theta)\right] e^{-\Phi_n c_2^\theta} \end{aligned}$$

where

$$\Phi_n = \sum_i T_i [w_i d_{ni}]^{-\theta}.$$

- The distribution of the lowest cost regardless second lowest cost can be obtained by taking the limit $c_2 \to \infty$ of the following expression

$$G_n(c_1, c_2) = 1 - G_n^c(0, c_2) - G_n^c(c_1, 0) + G_n^c(c_1, c_2)$$

- The remaining steps are almost identical to EK'02, but here need to take into account distribution of markups when deriving price indices:
  - see Appendix to the paper