

Econ 871: Homework 1
Traditional Trade Theory

You have 1 week to complete this homework.

Problem 1 (Preliminaries: Ideal Price Index)

Consider the following expenditure minimization problem:

$$E(C) = \min_{(x_i)_{1..N}} \sum_{i=1}^N p_i x_i$$

subject to

$$\sum_{i=1}^N (\alpha_i^{\frac{1}{\sigma}} x_i^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} = C,$$
$$x_i \geq 0, \text{ all } i = 1..N,$$

where p_i 's denote prices, $E(C)$ are total expenditures, α_i 's are the preference weights, σ is the elasticity of substitution, C is the 'composite good' consumption level. Assume that p_i 's, α_i 's, σ and C are all strictly positive.

a. Show that $E(C)$ takes the form $P * C$ where P is some positive scalar.

HINT: You might want to first establish that the solution to any problem in which both the objective function and the constraint set are h.o.d. 1 is h.o.d. 1 w.r.t. C . Given that the objective function is h.o.d. 1 it will immediately establish $E(C) = PC$. Then, to proceed further in point *b* below, you might want to set up a Lagrangian, and prove the envelope theorem, i.e. $E'(C) = \lambda$, where λ is the Lagrange multiplier on the constraint set.

b. Using the results from point *a* (specifically: $E'(C) = P = \lambda$), show that

$$E(C) = \left(\sum_{i=1}^N \alpha_i p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} C.$$

c. Suppose the household's problem is to maximize the utility

$$\max_{C^s, (x_i)_{i=1..N}} \left\{ (1-\gamma) \ln C^s + \gamma \ln \sum_{i=1}^N (\alpha_i^{\frac{1}{\sigma}} x_i^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} \right\}$$

subject to

$$P^s C^s + \sum_{i=1}^N p_i x_i \leq \text{Income},$$
$$x_i \geq 0, \text{ all } i = 1..N.$$

where *Income* and all prices are strictly positive.

Using the results from *a* and *b* show that this problem can be equivalently represented by:

1. $P^s C^s = (1 - \gamma) \text{Income}$
2. $PC = \gamma \text{Income}$ where P and C solves

$$PC = \min_{(x_i)_{1..N}} \sum_{i=1}^N p_i x_i$$

subject to

$$\sum_{i=1}^N (\alpha_i^{\frac{1}{\sigma}} x_i^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} = C,$$
$$x_i \geq 0, \text{ all } i = 1..N,$$

HINT: Recall the duality between the utility maximization problem and the expenditure minimization problem (first year micro).

NOTE for later use that if $\gamma = 1$ we can easily compute welfare by exploiting the fact that $U = \ln \frac{\text{Income}}{P}$.

Problem 2 (Preliminaries: Cost and Factor Demand)

Consider the Cobb-Douglas production function $K^\alpha L^{1-a}$. Denote the wage rate by w , the rental price of capital by r , and output by Y .

Using the results from problem 1, we know that the cost function $C(Y)$ takes the form cY . Derive the unit production cost (=marginal cost) $c(w, r)$ and factor demand functions $K(Y; w/r), L(Y; w/r)$ by solving the following problem:

$$cY = \min_{K, L} wL + rK$$

subject to

$$Y = K^\alpha L^{1-a}.$$

Problem 3 (Armington Model)

Consider the setup from lecture notes 1 (course website) with $N = 100$, $\alpha = 1/N$, $\sigma = 11$, $L_i = L_j = 100/N$, all $i, j = 1..N$, $\gamma = 1/2$. Call ‘countries’ 1 through N regions, and assume that they are located in two large countries: US and Canada. Assume 10% of regions are in Canada and 90% are in the US (roughly corresponds to the ratio of Canadian to US GDP). Assume the transportation cost between the regions within the same country is zero, i.e. $d_{ni} = 1$ whenever $i, n \in US$, or $i, n \in CAN$, and assume that the transportation cost between the regions within two different countries is equivalent to a 20% iceberg transportation cost, i.e. $d_{ni} = 1.2$ whenever $i \in US, n \in CAN$ or $i \in CAN, n \in US$.

a. Use the system of equation from slide 19 in lecture notes 1 and the outline of the numerical algorithm to solve the model in MATLAB. Compute the overall price level of a representative US region and Canadian region, and prices of their corresponding goods.

HINT: The algorithm will be unstable unless you slow down the updates a bit. Use the updating rule that puts .9 weight on the old value and only .1 weight on the newly solved value: $p_{i+1} = .1p' + .9p_i$, where i is the iteration number, and p_i is used to solve for the vector p' in iteration i using the system from slide 19. Don’t forget to evaluate the convergence, and the residuals of equilibrium conditions at the end. Remember that p_{NN} is the numeraire.

b. What is the home-bias from the US side (report $X_{US,US}/X_{US,CAN}$ and interpret) and from the Canadian side (report $X_{CAN,CAN}/X_{CAN,US}$ and interpret).

c. Using data generated by the model, suppose you run the following regression of trade flows on the border dummy from the Canadian side only (the way McCallum did in his 1995 paper; includes CAN-US observations and excludes US-CAN observations):

$$\ln \frac{X_{ni}}{X_i X_n} = k + A * \text{border_dummy} + \varepsilon,$$

where $n \in CAN, i \in CAN$ or US . What is the value of the regression coefficient on the border dummy?

d. Suppose you run the same regression as in point *c* but from the US side. What is the value of the regression coefficient on the border dummy?

e. How do your answers to *c* and *d* compare to the coefficients that Anderson and Wincoop found in the data by running McCallum’s regression separately from the US side and the Canadian side? Explain briefly the implications of your findings.

f. Redo points *c* and *d* with $\sigma = 8$.

g. Comparing the answers in *e* and *f*, what fraction of the border puzzle is accounted for by this simple model?

h. What is the average share of trade with the US for a representative Canadian province in the model (measure it by the ratio $(90X_{CAN,US})/(10X_{CAN,CAN} + 90X_{CAN,US})$)? Consider two levels of trade cost: $d_{US,CAN} = 1.2$ (same as before), and $d_{US,CAN} = 1.175$. Given that

the median and average value of this object in the data is about¹ .45, which level of the border cost accounts better for this number?

i. Would the answers to *b-g* depend on the choice of N (presuming that 10%, 90% of regions is an integer)? (Verify numerically by using $N = 50$. You can also show it analytically, but it is not required – just give me the answer, yes/no.)

¹Pulled out from the data available from Anderson's website.

Problem 4 (Monopolistic Competition Model)

Consider our benchmark monopolistic competition model with $\gamma = 1$ (see course website). Assume the world is symmetric and the transportation cost is $d_{ni} = d$, all $n \neq i$, and 1 otherwise.

- a. Define equilibrium in this environment.
- b. Derive the welfare of a representative household in terms of model primitives (labor endowment, d , f and other parameters).
- c. How does the welfare vary with the size of the world (total labor endowment), transportation cost and the fixed cost f ? (In this exercise, I would suggest to interpret L as the measure of identical atomless households with 1 unit of labor endowment – and measure welfare accordingly.)

HINT: Use the note at the end of problem 1, and the fact that under symmetry the numeraire normalization gives us $w = 1$ for all countries.

Problem 5 (Ricardian Model with a Continuum)

Consider a world with two countries and a continuum of goods indexed on a unit interval. Preferences in each country are identical and given by

$$U_i = \left(\int_0^1 c_i(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}, \quad i = 1, 2,$$

and all markets are perfectly competitive.

Assume each country has access to a linear technology to produce each good using labor,

$$y_i(\omega) = z_i(\omega)l_i(\omega),$$

where $z_i(\omega)$ is the efficiency level in producing good ω in country i , and $l_i(\omega)$ is the labor input.

Assume that the labor endowment of the stand-in household in each country is *one*, and the production efficiency schedules are given by the following functions:

$$\begin{aligned} z_1(\omega) &= 5 - 5\omega, \\ z_2(\omega) &= 5\omega. \end{aligned}$$

There is also a positive tariff rate between the two countries that amounts to 10% of the value of the transported goods across the border. The revenue from the tariff is lump-sum rebated to the domestic households.

- a.** Define competitive equilibrium in this economy.
- b.** Compute the competitive equilibrium. (Exploit the fact that equilibrium allocation is symmetric.)
- c.** Applying the US NIPA rules compute GDP of each economy. What happens to GDP in equilibrium when the tariffs are increased? (This is a tricky question. I would suggest first checking the online guide how GDP is computed in the US data. Income side of Table A in http://www.bea.gov/national/pdf/nipa_primer.pdf and the document <http://www.bls.gov/ore/pdf/ec010020.pdf> on page 6 provide enough information...)

Problem 6 (Heckscher-Ohlin Model)

Consider the world with 2 countries and 2 tradable goods. Preferences of the stand-in household in each country are

$$U_i = \sum_{j=1,2} \ln C_i^j, \quad i = 1, 2$$

where C_i^j denotes consumption in country i of good j . The stand-in household in country 1 has 2 units of labor (L) and 3 units of capital (K), and the stand-in household in country 2 has 3 units of labor and 2 units of capital. Firms in each country have access to the same CRS technology to produce both goods. The technology to produce good 1 is $Y = K^{\frac{1}{3}}L^{\frac{2}{3}}$ (sector 1), and good 2 is $Y = K^{\frac{2}{3}}L^{\frac{1}{3}}$ (sector 2). There is no transportation cost.

a. Assume factors are perfectly mobile across countries. Define the competitive equilibrium.

b. Refers to a. It can be shown, using the First and the Second Welfare Theorems, that the competitive equilibrium is unique up to the underdetermined allocation of capital and labor across countries and sectors, and it solves the following planning problem for $\mu = \frac{1}{2}$:

$$\max_{(C_i^j, K_i^j, L_i^j)_{i,j=1,2}} \mu \sum_{j=1,2} \ln C_1^j + (1 - \mu) \sum_{j=1,2} \ln C_2^j$$

subject to

$$\begin{aligned} \sum_{i,j=1,2} K_i^j &= 5, & \sum_{i,j=1,2} L_i^j &= 5, \\ \sum_{i=1,2} C_i^1 &= \sum_i (K_i^1)^{\frac{1}{3}} (L_i^1)^{\frac{2}{3}}, \\ \sum_{i=1,2} C_i^2 &= \sum_i (K_i^2)^{\frac{2}{3}} (L_i^2)^{\frac{1}{3}}. \end{aligned}$$

Compute the competitive equilibrium you defined in *a*.

HINT: Step 1. Show that $C_i^1 = C_i^2$ all i from the symmetry of the planning problem above. Step 2: Note that the relative price of good 1 versus good 2 is *one* from MRS. Step 3: The ratio of unit production costs you found in problem 2 is MRT, and since MRS=MRT in equilibrium, find w/r . Step 5: Find C_i^j 's by exploiting symmetry.

c. Assume factors are immobile across countries. Define the competitive equilibrium.

d. Refers to c. It can be shown that there is a unique competitive equilibrium, and it solves the following planning problem for $\mu = \frac{1}{2}$:

$$\max_{(C_i^j, K_i^j, L_i^j)_{i,j=1,2}} \mu \sum_{j=1,2} \ln C_1^j + (1 - \mu) \sum_{j=1,2} \ln C_2^j$$

subject to

$$\begin{aligned}
 (*) \quad \sum_{j=1,2} K_1^j &= 3, \quad \sum_{i,j=1,2} L_1^j = 2, \quad \sum_{j=1,2} K_2^j = 2, \quad \sum_{j=1,2} L_2^j = 3, \\
 \sum_{i,j=1,2} K_i^j &= 5, \quad \sum_{i,j=1,2} L_i^j = 5, \\
 \sum_{i=1,2} C_i^1 &= \sum_i (K_i^1)^{\frac{1}{3}} (L_i^1)^{\frac{2}{3}}, \\
 \sum_{i=1,2} C_i^2 &= \sum_i (K_i^2)^{\frac{2}{3}} (L_i^2)^{\frac{1}{3}}.
 \end{aligned}$$

Compute the competitive equilibrium you defined in d .

HINT: Guess that the solution to the planning problem above solves a relaxed problem with (*) constraints omitted (like in the planning problem in point b). Verify the guess by showing that factor markets clear – use (*) in combination with the factor demand functions you derived in problem 2 to find market clearing production pattern (write it in matrix form, will be easier...).

e. What is the pattern of trade in the competitive equilibrium you found in d ? More precisely, in which good the labor abundant country is a net exporter?

f. Note that ‘The trick’ you used in d to solve for the equilibrium would not work in general, i.e. for an arbitrary distribution of factor endowment levels. Show which step of your solution in e would break down, and explain why. **HINT:** Recall that there are non-negativity constraints on all the variables...

Comment: Since we did not cover H-O model in this course, I should mention that you have just went through two standard results: H-O theorem (the country will export the good which uses its abundant factor intensively), Factor Price Equalization Theorem (showing that within the cone of diversification, i.e. the domain of initial endowment levels for which ‘the trick’ works, factor prices w/r are equalized across countries).