Economics 111  Exam 1  Spring 2008  Prof Montgomery

Answer all questions. Explanations can be brief. 100 points possible.

1) [36 points] Suppose that, within the state of Wisconsin, market demand for cigarettes is given by $Q_D = 570 - 57P$, while market supply is given by $Q_S = 400P - 800$, where $P$ is the price per pack (in dollars) and quantities are given in millions of packs. (Throughout this question, the market for cigarettes is assumed to be perfectly competitive.)

a) Compute the equilibrium price and equilibrium quantity. Then compute consumer surplus, producer surplus, and total surplus. [HINT: You are not required to draw graphs for this problem, but they might help you compute the relevant amounts.]

b) Suppose that the state of Wisconsin now imposes a tax of $2 per pack on cigarettes. Find the new equilibrium price and quantity. Then compute consumer surplus (CS), producer surplus (PS), the state’s revenue from the tax, and total surplus (TS). [HINT: Given the existence of a tax, $TS = CS + PS + \text{tax revenue.}$] Comparing your answers to parts (a) and (b), how has the tax affected total surplus? Briefly discuss the concept of “deadweight loss” and give the size of this loss in the current problem.

c) A state representative argues that, given the state’s projected budget shortfall, the cigarette tax should be set even higher. Compute the equilibrium price and quantity and the state’s tax revenue if the tax is set at $4 per pack.

d) Using your answers to parts (a) and (b), compute the elasticity of demand as the price moves from the initial price (in part a) to the higher price (in part b). Then, using your answers to parts (b) and (c), compute the elasticity of demand if the price rises again. Are these elasticities the same? Briefly explain why or why not.

e) Arguably, cigarette taxes are imposed more for their long-run consequences than their short-run consequences. Qualitatively, how would the demand for cigarettes change in the long run? How would this affect the government’s tax revenue?

2) [24 points] A firm produces output $Q$ from labor input $L$ according to the production function $Q(L) = 20L - L^2$. Derive the firm’s marginal-product-of-labor (MPL) function. [HINT: You can either derive this function analytically using calculus, or you can construct a table that gives the firm’s MPL for various levels of L.] Further assuming that the firm’s output sells at a price of $3 per unit, derive the firm’s value-of-marginal-product-of-labor function. [HINT: Again, you can derive this function analytically, or else add another column to your table.] What is the relationship between the firm’s value-of-marginal-product-of-labor function and its labor-demand function? Draw a graph of the firm’s labor demand curve. If the wage is $12 per unit of labor, what is the optimal level $L^*$ chosen by the firm? Given this (optimal) choice of labor, find the total revenue, total cost, and profit earned by the firm.
3) [30 points] Suppose that workers have 60 hours/week to allocate between work and leisure, are free to choose the number of hours that they work. In the usual way, you should assume that individuals receive positive but diminishing marginal utility from both income and leisure, and that both income and leisure are normal goods.

a) Firm 1 pays $10/hour. At that wage, some of its workers choose to work more than 40 hours/week while others choose to work less than 40 hours/week. Now suppose that firm 1 decides to pay a fixed bonus of $100 (in addition to the hourly wage) to those workers who work at least 40 hours/week. Using a graph (with leisure on the horizontal axis and income on the vertical axis), show how this bonus would affect the constraint facing workers at firm 1. [HINT: Your graph does not need to be exactly to scale, but you should label the level of income and leisure at the horizontal and vertical intercepts and at other important (kink) points on the graph. Messy or improperly labeled graphs will receive less credit.] Given this bonus scheme, would firm-1 workers tend to increase or decrease their number of hours? How would this depend on the number of hours that the worker chose to work before the bonus plan was implemented? What income and/or substitution effects are relevant?

b) Firm 2 also pays $10/hour. Like firm 1, some of its workers choose to work more than 40 hours/week while others choose to work less. Now suppose that firm 2 decides to pay time-and-a-half for overtime. That is, workers are paid $15/hour for each hour beyond 40 hours of work. Using another graph (again with leisure and income on the axes), show how this compensation scheme would affect the constraint facing workers at firm 2. Given this time-and-a-half scheme, would firm-2 workers tend to increase or decrease their number of hours? How would this depend on the number of hours that the worker chose to work before the time-and-a-half plan was implemented? What income and/or substitution effects are relevant?

c) Finally, consider a particular worker who, given the wage of $10/hour, would choose to work more than 40 hours per week. Given that firm 1 and firm 2 have adopted the compensation schemes described above, would this worker choose higher leisure at firm 1 or 2 or is this theoretically ambiguous? Would this worker have higher utility at firm 1 or 2 or is this theoretically ambiguous? Briefly explain, again indicating relevant income and/or substitution effects. [HINT: It may be helpful to draw a graph.]

4) [10 points] Define “sunk cost.” Given the existence of sunk costs, how does the decision by a firm to enter a market differ from the decision to exit? Give the (mathematical) conditions under which entry and exit should occur.
1a) [8 pts] Setting quantity demanded equal to quantity supplied, we obtain

\[ 570 - 57P = 400P - 800 \]

and thus the (slightly approximated) equilibrium price \( P^* = 3 \) and quantity \( Q^* = 400 \). Consumer surplus (CS) and producer surplus (PS) are given by the areas of the triangles indicted below.

Thus,

\[ CS = (10-3)(400)(1/2) = 1400 \]
\[ PS = (3-2)(400)(1/2) = 200 \]
\[ TS = CS + PS = 1600 \]

b) [14 pts] Given a tax of $2 per unit, consumers pay \( P \) but producers receive \( P-2 \). Thus, setting quantity demanded equal to quantity supplied, we now obtain

\[ 570 - 57P = 400(P-2) - 800 \]

and thus the (again slightly approximated) equilibrium price \( P^* = 4.75 \) and \( Q^* = 300 \).

(Comparing the answers to parts (a) and (b), note that the equilibrium price rises by less than the amount of the tax.) Graphically, the tax can be represented as an upward shift of the supply curve. Regions corresponding to consumer surplus (CS), producer surplus (PS), tax revenue, and deadweight loss (DWL) are indicted on the diagram on the next page. Computing the areas of these regions, we obtain

\[ CS = (10 - 4.75)(300)(1/2) = 787.5 \]
\[ PS = (2.75-2)(300)(1/2) = 112.5 \]
\[ \text{tax revenue} = (2)(300) = 600 \]
\[ TS = CS + PS + \text{tax revenue} = 1500 \]
\[ DWL = (400-300)(2)(1/2) = 100 \]
“Deadweight loss” reflects the value of additional transactions that would have occurred if the tax had not been imposed. These transactions involve consumers whose willingness to pay is below the new price of $4.75 (but above the original equilibrium price of $3) and producers whose willingness to accept is above $2.75 (but below $3).

c) [6 pts] Given a tax of $4 per unit, the equilibrium is determined by the equation

\[ 570 - 57P = 400(P-4) - 800 \]

and thus \( P^* = 6.50 \) and \( Q^* = 200 \). Tax revenue would be \( 4)(200) = 800 \).

d) [5 pts] Using the answers to parts (a) and (b), demand elasticity is

\[ \left| \frac{\% \Delta Q}{\% \Delta P} \right| = \left| \frac{(\Delta Q/Q) / (\Delta P/P)}{(-100/400) / (1.75/3)} \right| = .428 \]

Using the answers to parts (b) and (c), we obtain

\[ \left| \frac{\% \Delta Q}{\% \Delta P} \right| = \left| \frac{(\Delta Q/Q) / (\Delta P/P)}{(-100/300) / (1.75/4.75)} \right| = .905 \]

Elasticity is not constant along a linear demand curve, but rises as we move upward along the curve (to points with higher \( P \) and lower \( Q \)).

e) [3 pts] Demand becomes more elastic in the long run. Intuitively, the increase in cigarette prices would cause people to (eventually) substitute away from cigarettes toward other goods. Thus, while a high tax on cigarettes may generate high tax revenues in the short run, these revenues will be lower in the long run as consumers reduce their consumption of cigarettes.
2) [24 pts] Using calculus, the marginal-product-of-labor function is

\[ MPL(L) = Q'(L) = 20 - 2L \]

and the value-of-marginal-product-of-labor function is

\[ VMPL(L) = P \cdot Q'(L) = (3)(20 - 2L) = 60 - 6L \]

Alternatively, computing MPL as \( \Delta Q / \Delta L \), you could have constructed the table

<table>
<thead>
<tr>
<th>L</th>
<th>Q</th>
<th>MPL</th>
<th>VMPL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>19</td>
<td>19</td>
<td>57</td>
</tr>
<tr>
<td>2</td>
<td>36</td>
<td>17</td>
<td>51</td>
</tr>
<tr>
<td>3</td>
<td>51</td>
<td>15</td>
<td>45</td>
</tr>
<tr>
<td>4</td>
<td>64</td>
<td>13</td>
<td>39</td>
</tr>
<tr>
<td>5</td>
<td>75</td>
<td>11</td>
<td>33</td>
</tr>
<tr>
<td>6</td>
<td>84</td>
<td>9</td>
<td>27</td>
</tr>
<tr>
<td>7</td>
<td>91</td>
<td>7</td>
<td>21</td>
</tr>
<tr>
<td>8</td>
<td>96</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>9</td>
<td>99</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>11</td>
<td>99</td>
<td>-1</td>
<td>-3</td>
</tr>
</tbody>
</table>

The firm increases L until VMPL(L) equals the wage \( w \). That is, the firm chooses the optimal number of workers \( L^* \) so that \( VMPL(L^*) = w \). Because this equation determines the relationship between the number of workers demanded by the firm \( (L^*) \) and the price per worker \( (w) \), the VMPL function is the firm’s labor demand function. Graphically,

If \( w = 12 \), the firm sets \( L^* \) so that \( VMPL(L^*) = 60 - 6L^* = 12 \), and hence \( L^* = 8 \).
(Alternatively, using the table above, note that the firm would be willing to hire an 8th worker because \( VMPL(8) > 12 \) but wouldn’t hire a 9th worker because \( VMPL(9) < 12 \).)

Given this optimal solution, \( \text{total revenue} = P \cdot Q(L^*) = (3)(96) = 288 \)
\( \text{total costs} = w \cdot L^* = (12)(8) = 96 \)
\( \text{profit} = \text{total revenue} - \text{total costs} = 288 - 96 = 192 \)
3a) [10 pts] Given firm 1’s compensation scheme, the constraint facing their workers is given below. Workers who initially worked less than 40 hours (i.e., choose more than 20 hours of leisure) might be unaffected by the bonus (if their initial utility is greater than their utility at point X) or might increase their work hours to 40 (if their initial utility is less than their utility at point X), but would never decrease their work hours. Workers who initially worked more than 40 hours experience an income effect (reflected by a parallel outward shift of the constraint) and would thus decrease their work hours (note that there is no substitution effect to offset the income effect).

b) [10 pts] Given firm 2’s compensation scheme, the constraint facing their workers is given below. As in firm 1, workers initially choosing less than 40 hours of work might be unaffected by the overtime scheme, or might increase their hours (if their initial utility is less than their utility at some point along the new overtime portion of the constraint). For workers who initially chose to work more than 40 hours, the change in the constraint has an ambiguous effect on their hours choice. On one hand, the constraint is now steeper (reflecting an increase in the “price of leisure”) and so the substitution effect would cause them to increase work hours. On the other hand, there is also an income effect which would cause them to increase their leisure (and hence decrease work hours).
3c) [10 pts] Superimposing the (upper portion of the) preceding graphs, we can see that the constraint for firm 1 lies above the constraint for firm 2 for every choice of choice of leisure between 0 and 20. Thus, any worker who would choose to work more than 40 hours a week (at a wage of $10 per hour) would necessarily have higher utility at firm 1.

To compare the choice of leisure hours across firms, consider a worker who initially faces the firm 2 constraint. This worker is initially at point A. Now suppose that this worker faces the firm 1 constraint. Because the firm 1 constraint is flatter, reflecting the decrease in the “price of leisure” (from $15/hour to $10/hour), the worker will substitute away from income toward leisure (and thus decrease work hours). This is reflected graphically by the movement from point A to point B. Further, given a positive income effect (recall our assumption that leisure is a normal good), the worker will increase his/her hours of leisure further. This is reflected graphically by the movement from point B to point C. Thus, both income and substitution effects go in the same direction, and it is clear that the worker would choose higher leisure (lower working hours) at firm 1.

[A precise specification of the utility function was not necessary to answer this question. However, if you’re interested in solving a numerical example, the graph above was generated under the assumption that the worker’s utility function is

\[
U(Y, L) = \frac{11}{3} \ln(Y) + \ln(L) \quad \text{where } Y \text{ is income and } L \text{ is leisure}
\]

which implies the optimal solution is \(\{L^* = 10, Y^* = 550\}\) at firm 2 (with the worker receiving utility \(U(Y^*, L^*) = 25.439\)), and \(\{L^* = 15, Y^* = 550\}\) at firm 1 (with the worker receiving utility \(U(Y^*, L^*) = 25.844\)).]
4) [10 pts] A sunk cost is a cost that cannot be recovered by the firm if the firm decides to exit.

Let $Q^*$ denote the firm’s optimal production level.

A firm *enters* the market if $\pi(Q^*) > 0$

given that $\pi(Q^*) = TR(Q^*) - TC(Q^*)$, this entry condition is equivalent to

$$TR(Q^*) > TC(Q^*)$$

dividing through by $Q^*$, the entry condition can also be written as

$$P > AC(Q^*)$$

in words, the firm enters when price is greater than average cost (evaluated at the optimal level of production)

If a firm *exits* the market, it receives $-SC$ (because sunk costs cannot be recovered)

thus, exit occurs if $\pi(Q^*) < -SC$

given $TC(Q^*) = VC(Q^*) + OC + SC$, this exit condition is equivalent to

$$TR(Q^*) < VC(Q^*) + OC$$

dividing through by $Q^*$, the exit condition can also be written as

$$P < AVC(Q^*)$$

where $AVC(Q^*) = [VC(Q^*) + OC] / Q^*$

in words, the firm exits when price is below average variable cost (evaluated at the optimal level of production)

[In scoring this question, I deducted one point if you didn’t indicate that the relevant cost functions in these conditions are evaluated at the optimal level of production $Q^*$]
Answer all questions. 100 points possible.

1) [40 points] A monopolist faces an industry demand curve that can be written

\[ p = 60 - Q \]

where \( p \) is price and \( Q \) is the quantity chosen by the monopolist.

a) Assuming that the monopolist has zero costs, write the monopolist’s profit \( \pi \) as a function of \( Q \). [HINT: Given zero costs, profit is equal to revenue.] Find the profit-maximizing quantity and the level of profit that the monopolist earns by choosing this quantity.

b) Now suppose there is a duopoly in this industry, so that industry demand can be written

\[ p = 60 - Q_1 - Q_2 \]

where \( p \) is price, \( Q_1 \) is the quantity chosen by firm 1, and \( Q_2 \) is the quantity chosen by firm 2. Assume that both firms have zero costs. Write firm 1’s profit \( \pi_1 \) as a function of \( Q_1 \) and \( Q_2 \), and then write firm 2’s profit \( \pi_2 \) as a function of \( Q_1 \) and \( Q_2 \).

c) Suppose that each of the duopolists in part (b) has three possible choices: set quantity at 15, 20, or 30 units. Substituting the firms’ choices into the profit functions from part (b), construct the 3×3 payoff matrix that gives the pair of profit levels \( (\pi_1, \pi_2) \) for each possible pair of quantity choices \( (Q_1, Q_2) \). Which pair of quantity choices would maximize joint profit (i.e., \( \pi_1 + \pi_2 \))? Is this joint profit higher than, lower than, or the same as the monopoly profit derived in part (a)? Briefly explain.

d) Given the payoff matrix in part (c), and assuming that firms choose quantities simultaneously, what is the Nash equilibrium? Is the joint profit of the firms higher than, lower than, or the same as the monopoly profit derived in part (a)? Briefly explain.

e) Now suppose that firm 1 chooses its quantity before firm 2 chooses its quantity. What is the equilibrium of this sequential game? [HINT: Draw the game tree.] Compared to the equilibrium in part (d), does firm 1 gain or lose by moving first? Briefly explain.

2) [15 points] What is a natural monopoly? Why are natural monopolies usually regulated by the government rather than broken up into smaller firms? What problem faces the government regulator in setting the price charged by the natural monopoly? What solution have regulators usually adopted? Draw a graph to illustrate your answer.
3) [20 points] Consider a used-car market with asymmetric information. Each seller knows the quality of her car. Buyers cannot observe quality directly, but merely know the number of cars of each type that could (potentially) be sold on the market, along with the seller’s valuation of each type of car.

Suppose there are three types of cars: low quality (L), medium quality (M), and high quality (H). For each of these types of cars, the following table lists the buyer’s value, seller’s value, and the number of sellers.

<table>
<thead>
<tr>
<th>type of car</th>
<th>buyer’s value</th>
<th>seller’s value</th>
<th>number of sellers</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>30</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>M</td>
<td>72</td>
<td>50</td>
<td>200</td>
</tr>
<tr>
<td>H</td>
<td>98</td>
<td>80</td>
<td>100</td>
</tr>
</tbody>
</table>

Finally, suppose that there are 200 buyers.

a) Compute the buyer’s expected value for each price range.

b) What is the market equilibrium (price and quantity of cars sold)? [HINT: There may be more than one equilibrium. You might want to draw a supply-and-demand graph.]

4) [25 points] Consider a small country which imports sugar and also produces some sugar domestically. Because this country is small relative to the world sugar market, it can import as much sugar as desired at the international price of \( p^\ast = 40 \) per unit. Further suppose that domestic supply and demand are given by

\[
Q_S = \frac{5}{2}p - 80 \\
Q_D = 100 - \frac{4}{5}p
\]

a) What is a tariff?

b) In the absence of a tariff, what quantity is purchased by domestic consumers? What quantity is produced by domestic producers? Compute the domestic consumer surplus (CS) and domestic producer surplus (PS). [HINT: While you don’t need to draw a supply-and-demand graph, it might help you compute CS and PS.]

c) Now suppose that the government of this small country imposes a tariff of \( t = 5 \) on each unit of sugar. What quantity is now purchased by domestic consumers? What quantity is now produced by domestic producers? What are the new levels of domestic consumer surplus (CS) and domestic producer surplus (PS)? How much revenue does the government collect? What is the societal loss induced by the tariff? [HINT: Again, you might want to draw a supply-and-demand graph. It will look like the one pictured in the discussion of tariffs in Chapter 19 (International Trade) of your textbook.]
1a) [9 pts]

\[ \pi(Q) = (60-Q)Q \]
\[ MR(Q) = (60-2Q)Q \]
\[ MC = 0 \]
\[ MR(Q^*) = MC \]
\[ 60 - 2Q^* = 0 \]
\[ Q^* = 30 \]
\[ \pi(Q^*) = 900 \]

b) [4 pts]

\[ \pi_1(Q_1, Q_2) = (60-Q_1-Q_2)Q_1 \]
\[ \pi_2(Q_1, Q_2) = (60-Q_1-Q_2)Q_2 \]

c) [9 pts]

<table>
<thead>
<tr>
<th></th>
<th>firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>450, 450</td>
</tr>
<tr>
<td>15</td>
<td>450, 450</td>
</tr>
<tr>
<td>20</td>
<td>500, 375</td>
</tr>
<tr>
<td>30</td>
<td>450, 225</td>
</tr>
</tbody>
</table>

Joint profit is maximized at (15,15), which implies \( \pi_1 + \pi_2 = 900 \). This is the same as the monopoly profit in part (a). Essentially, the duopolists act together as a monopolist, splitting the monopoly production and monopoly profit in half.

d) [9 pts] In Nash equilibrium, each firm chooses its best response to the quantity chosen by the other firm. The (payoffs corresponding to) best responses are underlined in the payoff matrix in part (c). For this game, the (unique) Nash equilibrium is (20, 20), which generates a joint profit of 400 + 400 = 800, which is below the monopoly profit of 900. If one firm agreed to collude (setting \( Q = 15 \)), the best response of the other firm is to set \( Q = 20 \). Thus, the collusive outcome (15, 15) is not a Nash equilibrium. Note that, if firms could only produce 15 or 20 units, the payoff structure is a Prisoners’ Dilemma.

e) [9 pts] Given the game tree for the sequential game (see next page), we solve this game using backward induction (i.e., we solve the game “backwards”). After firm 1 has chosen its quantity, firm 2 is at one of three decision nodes. Firm 2’s best response at each node is indicated by an arrow. We then move backward in the game tree to firm 1’s decision. Because firm 1 knows that firm 2 will make a best response at the second stage, firm 1’s best response is to choose \( Q_1 = 30 \) (yielding profit 450). Thus, in the second stage, firm 2 chooses \( Q_2 = 15 \) (yielding profit 225). In this game, there is a “first mover” advantage because firm 1 is able to commit to a higher quantity than it would choose in the simultaneous-move game. Essentially, firm 1 can choose its preferred point on firm 2’s reaction function.
1e continued) game tree for sequential game:

2) [15 pts] A natural monopoly has high fixed costs, and thus its average cost $AC(Q)$ falls as its output $Q$ rises. Because of the high fixed costs, it would be inefficient to have more than one firm in the industry. Thus, natural monopolies are usually regulated by the government, not broken up. To eliminate deadweight loss, government regulators would ideally like to set $P$ at the level where the demand curve intersects the marginal cost curve. However, because $AC(Q) > MC(Q)$, this would cause the firm to lose money. As an alternative, regulators often set $P$ at the level where the demand curve intersects the average cost curve, which allows the firm to break even (i.e., receive zero profit).

In this example, if regulators set $P = 5$ to eliminate deadweight loss, the firm would sell $Q = 17.5$ at an average cost $AC(17.5) = 11$ and thus lose $(11-5)(17.5) = 105$.

To allow the firm to break even, regulators would set $P = 12$, so that the firm would sell $Q = 14$. At this quantity, $P = AC(Q)$, so the firm’s profit is zero.
3a) [10 pts] Cars are placed on the market (i.e., put up for sale) when the market price exceeds the sellers’ valuations. Thus, for P < 20, no cars are placed on the market.

For P between 20 and 50, only low-quality cars are placed on the market. In this price range, buyers are certain to purchase a low-quality car, and so their expected value is 30.

For P between 50 and 80, both L and M cars are placed on the market. In this price range, buyers’ expected value is \((\text{probability of an L}) \times (\text{buyer’s value of an L}) + (\text{probability of an M}) \times (\text{buyer’s value of an M}) = (100/300)(30) + (200/300)(72) = 58\).

For P higher than 80, all cars are placed on the market. In this price range, buyers’ expected value is \((\text{prob L})*(\text{b’s value of L}) + (\text{prob M})*(\text{b’s value of M}) + (\text{prob H})*(\text{b’s value of H}) = (100/400) * 30 + (200/400) * 72 + (100/400) * 98 = 68\).

b) [10 pts] Buyers are willing to purchase cars when their expected value is greater than or equal to the market price. Thus, given the answers to part (a), all 200 buyers are willing to buy when the market price is between 20 and 30, or when the market price is between 50 and 58. Otherwise (for all other prices), no buyer is willing to buy. This leads to the (dotted) demand curve shown below. Superimposing the (solid) supply curve, we obtain the graph below. For any price between 20 and 30, there is excess demand, and so the market price would rise to 30. For any price between 50 and 55, there is excess supply, and so the market price would fall to 50. Thus, there are two possible equilibria: \((P^* = 30, Q^* = 100)\) and \((P^* = 50, Q^* = 200)\).
4a) [3 pts] A tariff is a tax on imports.

b) [8 pts] Given no tariff, the domestic price is equal to the international price $p^* = 40$. Thus, domestic consumers will purchase 68 units, and domestic producers will produce 20 units. (And thus 48 units are imported.) Using the graph below to determine the area of the relevant triangles, $CS = (1/2)(125-40)(68) = 2890$ and $PS = (1/2)(40-32)(20) = 80$. (And thus total surplus $= TS = 2890 + 80 = 2970$.)

c) [14 pts] Given the tariff, the domestic price is now $p^* + t = 45$. Thus, domestic consumers will purchase 64 units, and domestic producers will produce 32.5 units. Now $CS = (1/2)(125-45)(64) = 2560$, and $PS = (1/2)(45-32)(32.5) = 211.25$. Given that 31.5 units are imported, the government collects $(31.5)(5) = 157.5$ in revenue from the tariff. Now, total surplus (which includes government revenue) $= TS = 2560 + 211.25 + 157.5 = 2928.75$. Thus, the societal (deadweight) loss from the tariff is $2970 - 2928.75 = 41.25$. Graphically, this is the sum of the areas of the two small triangles indicated on the diagram below. (See also the discussion in the textbook, in Chapter 19 on trade.)
Answer all questions. 100 points possible. Explanations can be brief.

1) [10 pts] Consider a simple economy with 3 firms (A, B, C). Each firm produces a different type of good. The goods may be used either by firms (as intermediate goods) or by consumers (as final goods). The following input-output table reports the quantity produced by each firm and used by firms or consumers within the previous year.

<table>
<thead>
<tr>
<th>quantity used by</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>consumers</th>
</tr>
</thead>
<tbody>
<tr>
<td>quantity</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>produced by</td>
<td>2</td>
<td>10</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>15</td>
</tr>
</tbody>
</table>

Thus, of the output produced by firm A, 4 units were used by firm A, 5 units were used by firm B, 6 units were used by firm C, and 0 units were used by final consumers. Note that firm A thus produced a total of 4+5+6+0 = 15 units of output. The other rows of the table can be interpreted similarly. Further assume the price of good A equals 2, the price of good B equals 6, and the price of good C equals 3.

Compute GDP using the final-goods approach, and then compute GDP using the value-added approach. [NOTE: To receive full credit, you need to show your computations for both approaches.]

2) [20 pts] Consider an open economy (so that the circular-flow model includes the government as well as capital, product, and foreign-exchange markets).

a) For each of the following nodes in the circular-flow diagram, give the accounting identity corresponding to the condition that inflows equal outflows. [HINT: You don’t need to draw the circular-flow diagram, but it might help you keep track of these flows.]

(i) households
(ii) product market
(iii) capital market
(iv) government
(v) foreign exchange market

b) What is a trade surplus? Is it possible for the US to have a trade surplus at the same time that we have positive net capital flows into the US? Explain using the relevant accounting identity from part (a).
3) [12 pts] Consider an economy in which, over the last decade, the number of hours worked grew by 1.5%, the population grew by 1.2%, and productivity grew by 2.3%.

Compute the growth rate in
(a) number of hours worked per capita
(b) total output
(c) output per capita

4) [36 pts] Consider a closed economy (with no foreign sector). Consumption (C), private savings (Sp), taxes (T), and investment (I) are given by the following functions:

\[
C = 10 + 0.7(1-\tau)Y \\
Sp = -10 + 0.3(1-\tau)Y \\
T = \tau Y \\
I = 30 - 3r
\]

where \( \tau \) is the tax rate, \( Y \) is income, and \( r \) is the interest rate. Further suppose that the government holds its spending fixed at \( G = 20 \) while decreasing the tax rate from \( \tau = 0.3 \) to \( \tau = 0.2 \).

a) Using the full-employment model with the assumption that full-employment output is \( Y = 100 \), compute the equilibrium levels of \( C, Sp, T, r, \) and \( I \) both before and after the change in the tax rate. Indicate the direction of change for each of these variables.

b) Using the unemployment model with the assumption that the interest rate is fixed at \( r = 3 \), compute the equilibrium levels of \( Y, C, Sp \) and \( T \) both before and after the change in the tax rate. Indicate the direction of change for each of these variables.

5) [10 pts] Using the full-employment model, question (4a) considered the effect of a decrease in the tax rate in a closed economy. If we now assume that there is a foreign sector, would the change to \( r \) and \( I \) have been larger or smaller than in part (4a)? How would the exchange rate (\( e = \text{yen}/\$ \)) and foreign-exchange-market flows be affected? Briefly explain. [HINT: You don’t have enough information to find numerical solutions. I am merely looking for qualitative answers.]

6) [12 pts] Using the unemployment model, question (4b) considered the short-run effect of a decrease in the tax rate while ignoring potential long-run consequences. Assuming that the economy was initially (before the decrease in the tax rate) at its long-run equilibrium, discuss these longer-run consequences using the ADI-AS diagram. Be sure to discuss the role of the Fed in this process. [HINT: Again, you don’t have enough information to find numerical solutions. I am merely looking for qualitative answers.]
Econ 111 Exam 3 Spring 2008 Solutions

1) [10 pts] Using the final-goods approach,

\[ \text{GDP} = \sum_i \text{(value of final goods produced by firm i)} \]
\[ = \sum_i \text{(price of good produced by firm i \times quantity of final goods produced by firm i)} \]
\[ = 2 \times 0 + 6 \times 5 + 3 \times 15 = 0 + 30 + 45 = 75 \]

Using the value-added approach,

\[ \text{GDP} = \sum_i \text{(value added by firm i)} \]
\[ = \sum_i \text{(revenues for firm i – cost of intermediate goods for firm i)} \]
\[ = [2 \times 15 – (2 \times 4 + 6 \times 2)] + [6 \times 20 – (2 \times 5 + 6 \times 10 + 3 \times 2)] \]
\[ + [3 \times 20 – (2 \times 6 + 6 \times 3 + 3 \times 3)] \]
\[ = 10 + 44 + 21 = 75 \]

2a) [15 pts]

i) \( Y = C_d + S_p + T + IM \) (or, equivalently, \( Y = C + S_p + T \))
ii) \( Y = C_d + I + G + EX \) (or, equivalently, \( Y = C + I + G + EX – IM \))
iii) \( S_p + S_g +NCF = I \)
iv) \( T = S_g + G \)
v) \( IM = NCF + EX \)

b) [5 pts] A “trade surplus” occurs when exports are higher than imports. The trade surplus equals \( EX – IM \). Using (v), we obtain \( EX – IM = –NCF \). Thus, the trade surplus is positive if and only if net capital flows are negative.

3) Following the notation in Chapter 27, let \( H \) denote aggregate hours, \( N \) denote population size, and \( Y \) denote output. Using this notation, you are told that

\[ \% \Delta H = + 1.5 \]
\[ \% \Delta N = + 1.2 \]
\[ \% \Delta (Y/H) = + 2.3 \]

a) [4 pts] \( \% \Delta (H/N) = \% \Delta H – \% \Delta N = 1.5% – 1.2% = +0.3% \)
b) [4 pts] \( \% \Delta Y = \% \Delta (Y/H) + \% \Delta H = 2.3% + 1.5% = +3.8% \)
c) [4 pts] \( \% \Delta (Y/N) = \% \Delta Y – \% \Delta N = 3.8% – 1.2% = +2.6% \)

(Note that this problem appears in the textbook as Problem 5 in Chapter 27.)
4a) [18 pts] Given $Y = 100$, the equations become

\[
\begin{align*}
C &= 10 + 70(1-\tau) \\
S_p &= -10 + 30(1-\tau) \\
T &= 100\tau \\
I &= 30 - 3r \\
G &= 20
\end{align*}
\]

In the full-employment model, $r$ adjusts to maintain equilibrium, which is given by

\[
Y = C + I + G
\]

\[
100 = 10 + 70(1-\tau) + 30 - 3r + 20
\]

\[
r = (1/3)[30 - 70\tau]
\]

Thus, given $\tau = .3$, we obtain $r = 3$, $I = 21$, $C = 59$, $S_p = 11$, and $T = 30$.

Given $\tau = .2$, we obtain $r = 5.333$, $I = 14$, $C = 66$, $S_p = 14$, and $T = 20$.

Given the full-employment model, a decrease in the tax rate ($\tau$) thus causes the interest rate ($r$) to rise and investment ($I$) to fall (i.e., the government has “crowded out” investment). Tax revenues ($T$) fall, while consumption ($C$) and private savings ($S_p$) rise due to the increase in disposable income.

b) [18 pts] Given $r = 3$, the equations become

\[
\begin{align*}
C &= 10 + (.7)(1-\tau)Y \\
S_p &= -10 + (.3)(1-\tau)Y \\
T &= \tau Y \\
I &= 21 \\
G &= 20
\end{align*}
\]

In the unemployment model, $Y$ adjusts to maintain equilibrium, which is given by

\[
Y = C + I + G
\]

\[
Y = 10 + 70(1-\tau)Y + 21 + 20
\]

\[
Y = 51 / [1 - (.7)(1-\tau)]
\]

Thus, given $\tau = .3$, we obtain $Y = 100$, $C = 59$, $S_p = 11$, and $T = 30$.

Given $\tau = .2$, we obtain $Y = 115.9$, $C = 74.9$, $S_p = 17.8$, and $T = 23.2$.

Given the unemployment model, a decrease in the tax rate ($\tau$) thus causes output ($Y$) and consumption ($C$) and private savings ($S_p$) to rise, while tax revenues ($T$) fall.
5) [10 pts] Problem 4a illustrated the “crowding out” effect: a decrease in the tax rate reduces government savings, thus increasing the interest rate and “crowding out” investment. Given an open economy, this “crowding out” effect is less severe – the changes in r and I will be smaller. As r begins to rise, net capital flows increase. This causes the exchange rate to rise, which reduces exports and increases imports.

6) [12 pts] The decrease in the tax rate causes the ADI curve to shift rightwards. In the short run, the economy moves from point A to point B. However, because output is now above potential output (point B is to the right of the AS curve), inflation begins to rise. Assuming that the Fed maintains the same monetary policy rule, the increase in inflation will cause the Fed to increase interest rates, thus decreasing output. As inflation continues to rise and the Fed continues to increase the interest rate, the economy gradually moves along the ADI curve from point B to point C.