Efficient Risk Sharing with Unobservable Income and Enforcement Constraints
SUPPLEMENT A: Policy functions

Johanna Hertel*
University of Wisconsin-Madison

October 2005

In this supplement, we present some further computational results concerning policy functions under incomplete information for the interested reader. As in Hertel (2007), we set $\pi = 0.5$, $h = 1.9$, $l = 0.1$. Agent 2's sure income is again set to 1. We assume identical utility functions for both agents, and derive our numerical results for varying degrees of impatience and risk aversion: $\sigma \in \{0.5, 2, 4\}$, $\delta \in \{0.9, 0.99\}$.

Figure 1 presents policy functions for the problem W, selected to represent the greatest range of variation. As risk aversion and impatience become large, the policy functions become (visually) almost indistinguishable, and are represented in Figure 1 by $\sigma = 4$, $\delta = .99$.

As is expected, agent 1's high state-consumption is a strictly increasing function of his expected utility with maximal value h (see Lemmas 10 and 12). Agent 1's consumption in the low state, however, is shown in Figure 1 to be nonmonotonic in the interval $[\kappa^P, \kappa^{**}]$. This is in fact a general feature of b (see Lemmas 5,7 and 8) and is a consequence of the intertemporal distortion of agent 1's low-state consumption profile. Recall that agent 1's continuation utility after a low income shock is usually distorted downwards, his present consumption distorted upwards to provide for cheaper incentives. While this is optimal overall, such a distortion lowers both agents' utility in the low state and has to be undone when it becomes no longer sustainable (i.e. when agent 2's participation constraint binds). Hence, b is strictly decreasing on the interval $[\kappa^P, \kappa^R]$.

Agent 1's continuation utility is also shown to be strictly increasing in today's utility. Though this can be generally shown to be true for G when participation constraints are not too tight, we have not been able to prove the same result for B generically. Again, this is a consequence of the intertemporal distortion of low-state consumption profiles, with agent 1's continuation utility after a low income shock distorted downwards. Loosely speaking, B will only be monotone if this distortion does not become too large as low-state consumption increases; but note that both agents' risk aversion

*Department of Economics, Social Science Building, 1180 Observatory Drive, Madison, WI 53706-1393. Phone: 608-263-3861, email: jhertel@ssc.wisc.edu

(and hence, their consumption level) in large part determines the size of the intertemporal distortion. It is therefore not too surprising that one can show B to be monotonically increasing for CARA utility functions[1].

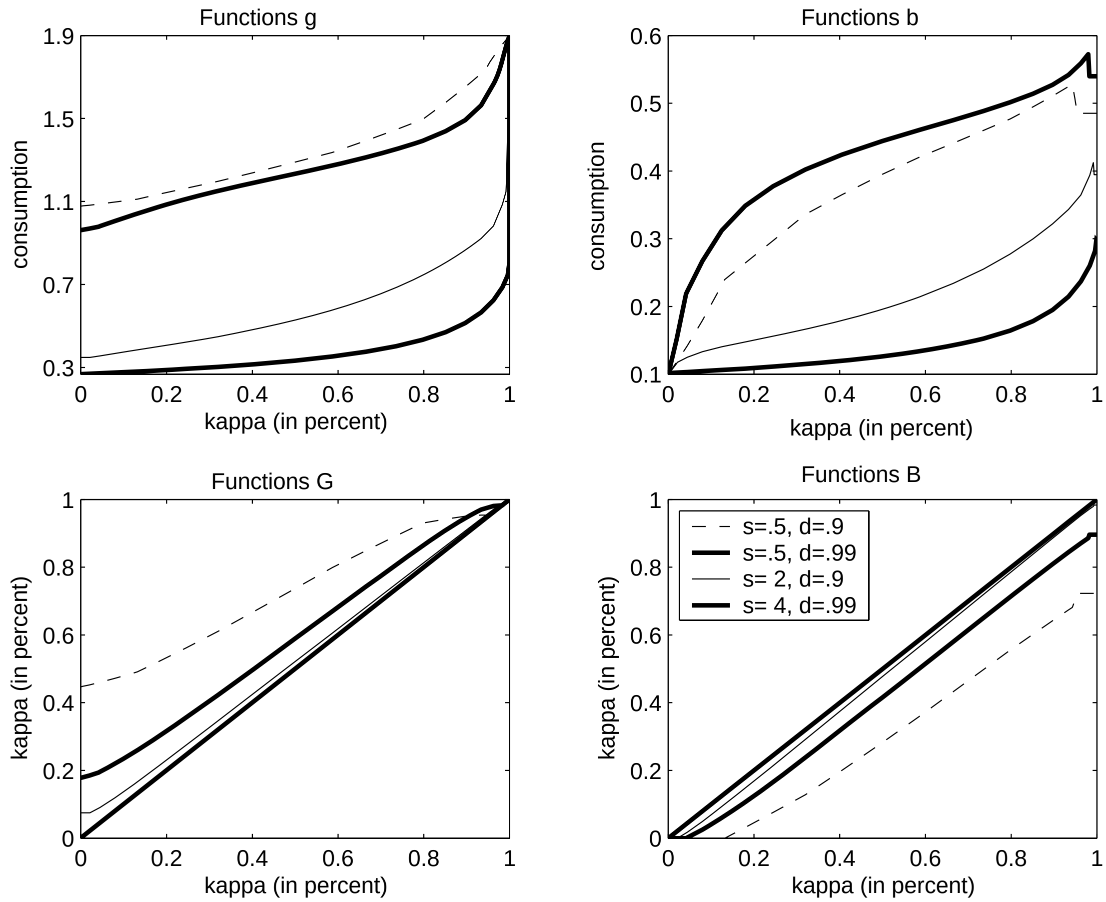

Figure 1: Selected policy functions (κ in percent)

The importance of monotonicity of the continuation utility lies in its consequences for what we call *cross-sectional history dependence*, or lasting effects of income on consumption. We define consumption to be cross-sectionally history dependent if $c(h_t) > c(h'_t)$, for two histories h_t, h'_t, that only differ at $t-k$, such that $s_{t-k} = H$, $s'_{t-k} = L$, for $k \geq 1$. It is this form of history dependence that results from optimal transfers in Atkeson and Lucas (1994), and hence came to known as one of the distinguishing features of optimal transfers with incomplete information. It is easy to see that one needs all policy functions to be strictly monotone in order for cross-sectional history-dependence to obtain at all histories. Atkeson and Lucas assume exponential utility as well as full commitment and can therefore show all their policy functions to be monotonic. However, the

[1]One can also show that the result is true for logarithmic utility.

discussion above suggests that this might not always be a feature of optimal transfers under incomplete information. In fact, if $B(\kappa^{**}) > \kappa^*$ so that $\kappa^R > \kappa^P$, b monotonically decreasing on $[\kappa^P, \kappa^R]$ and G strictly increasing everywhere, it is easy to show that we can always find a finite history that occurs with positive probability in the long run, such that the result is overturned: $c(h_t) < c(h_t')$. To generate these histories, note that there exists a finite number n such that a point in $[\kappa^P, \kappa^R]$ can be reached from κ^* with n successive high income shocks. κ^* is always reached with positive probability in the long run. Take a history h_k such that $U^1(c_{h_k}) = \kappa^*$. Then take $h_{k+n+1} = (h_k, H, H, ..., H)$ and $h'_{k+n+1} = (h_k, L, H, ..., H)$, where H is repeated n times from time $k+2$ onward, and the claim is proved.

We also note that as the discount factor increases, the functions G, B approach the diagonal, so that continuation utilities become closer to today's utilities, and consumption varies less and less over time. This is in line with results from Fudenberg, Levine and Maskin (1994) who show that optimal sharing rules under incomplete information converge to the first best as the discount factor approaches 1.

References

Atkeson, A. and R.E. Lucas, jr. (1992), "On Efficient Distribution with Private Information", *Review of Economic Studies 59,* 427-453.

Fudenberg, D., D. Levine and E. Maskin (1994), "The Folk Theorem with Imperfect Public Information", *Econometrica 62,* 997-1039.

Hertel, J. (2007), "Efficient Risk Sharing with Unobservable Income and Enforcement Constraints".