Economics 713, Final Exam
Spring 2007

Answer two of the three equally-weighted questions. Explain your answers. Use a separate blue book for every question you answer.

Question 1: (10 points)

Consider a market which is served by two-quantity setting oligopolists who face inverse demand $p(q) = 1 - q$.

a. (5 points) Suppose the oligopolists have constant marginal costs of production c_i, and firm 1 faces capacity constraint $\bar{q} < 1$ - firm 1 can produce at most $\bar{q}$. Derive best-response functions and characterize the Nash equilibrium of the simultaneous quantity-setting game as $\bar{q}$ varies.

b. (5 points) Now suppose firm 1 can invest in capacity before the Cournot game is played, and that firm 2 observes firm 1's capacity choice before it decides whether or not to enter into the market, so that the Cournot game will only be played if firm 2 decides to enter. In the resulting three-stage game, firm 1 first chooses $\bar{q} \in [0, 1]$ and pays cost $c(\bar{q}) = \alpha\bar{q}$. Firm 2 observes $\bar{q}$ and decides whether or not to enter. If firm 2 does not enter, its profits are zero; if it does enter, it pays entry cost K. In the third stage, quantities are chosen by all firms active in the market. Marginal costs of production are $c_1 = c$, $c_2 = c + \alpha$. Characterize the subgame-perfect Nash equilibrium of this game. Show that when K is not too large, there are values of α such that the resulting market quantities are equal to the equilibrium quantities of the equivalent Stackelberg game.

Question 2: (10 points)

An indivisible good is to be auctioned off to a group of buyers $\mathcal{I} = \{1, ..., n\}$ of unknown valuation $\theta \in [0, \bar{\theta}]$. Buyers' valuations for the good are independently drawn from $F(\theta)$ and are unobservable to the seller of the good. A buyer of type θ has utility $\theta - t$ from winning the good at price t. A *bidding ring* is a contract (i^*, b^*) among buyers such that all buyers $j \neq i^*$ bid 0, and buyer i^* wins the object with bid b^*. Bidding rings are not legally enforceable; a bidding ring succeeds if the contract (i^*, b^*) is an equilibrium outcome of the auction.

a. (3 points) Assume that buyers know each other's valuations and that the object is to be sold in a second-price auction. For any vector θ of type realizations, let $I(\theta) = \{i \in \mathcal{I} |$ there exists a bidding ring (i^*, b^*) that succeeds when buyers' valuations are θ, and $i^* = i\}$. Show that $I(\theta) = \mathcal{I}$ for all θ. What is the designated winners' bid at θ? What is the designated winner's utility from the auction?

b. (2 points) Now assume that the object is sold in a first-price auction. Buyers know each other's valuations. You may assume that no two valuations are identical (a zero-probability event in any case). Show that no bidding ring can succeed.

c. (5 points) Now suppose that the buyers do not know each other's valuations before the auction; they know that the object will be sold in a second-price auction. Hence, if a buyer can come up with a mechanism that will lead all buyers to reveal their valuations to each other, a bidding ring is possible. Consider the two-stage game in which agents first participate (or not) in some mechanism designed to select a designated winner, and then participate in the second-price auction in which the object is sold.

   (i) Set up the mechanism design problem associated with selecting the designated winner i^*. Is there a non-dictatorial mechanism in which all buyers' valuations are revealed without extracting payments from the buyers?

   (ii) Now suppose one buyer's valuation is commonly known among buyers only (all other buyers' valuations are still private). Show that there exists a mechanism for selecting i^* in which all buyer's valuations are revealed to each other before the auction, when all agents participate in this mechanism. (This mechanism will involve payments). Show that this mechanism always selects the agent with the highest valuation as the designated winner, and that there is an equilibrium of the two stage game in which all buyers participate in the first stage and form a successful bidding ring in the second.

Question 3: (10 points)

The owner of a firm wants to hire a manager with unknown cost of effort $\theta \in \{L, H\}$. Effort itself is observable. The firm's profits from a manager's effort of type θ and wages are $\pi(e, t, \theta) = y_\theta(e) - t$ The manager's utility from effort and wages is $u(e, t; \theta) = t - c_\theta(e)$; his reservation utility is 0. The prior probability of $\theta = H$ is $P(H) = \lambda$; the agent's cost is convex, $c'_\theta > 0, c''_\theta \geq 0$ for all θ. Furthermore, $y'_\theta > 0, y''_\theta < 0$, and for $\theta = H, L$, there exists $\tilde{e}_\theta$, $0 < \tilde{e}_\theta < \infty$, with $y'_\theta(\tilde{e}_\theta) = 0$. If the principal cannot observe the agent's type, he will offer a contract $c = ((t_k, e_k))_{k=H,L}$. A *separating* contract has $(t_H, e_H) \neq (t_L, e_L)$.

a. (1 point) Let $y'_H(e) > y'_L(e)$ for all e. What assumptions on c_H, c_L, c'_L, c'_H should you impose to ensure that a separating contract will entail higher effort from the principal's preferred type? Impose these conditions (and $y'_H(e) > y'_L(e)$) for the rest of Question 1.

b. (1 point) Characterize the first-best, that is, the optimal contract offered to the agent when θ is observable.

c. (3 points) Characterize the profit-maximizing contract when θ is not observable. Which constraints bind at the optimum? Prove your assertions.

d. (5 points) Now suppose that the owner of a firm can only commit to the wage t, but not the effort e he requires from an agent: if the owner and manager sign a contract (t, e), the owner will pay the wage t, but he can now force the manager to exert any effort $e' \neq e$. The contract game is otherwise unchanged: the firm's owner proposes a contract c, the manager accepts a wage-effort tuple; then the principal sets some effort level e' and pays the contractual wage t_k.

   (i) What must be true of the effort levels e_H, e_L required of the manager in an equilibrium of the new contract game? Show that the separating contract you derived in c) is no longer implementable.

   (ii) As full separation is not possible, the best the principal can do is a semi-separating contract in which only one type θ of manager accepts (t_θ, e_θ) with certainty. Characterize the optimal semi-separating contract in which only type H accepts the contract with certainty.

   (iii) Can it ever be optimal to offer a semi-separating contract in which only type L accepts with certainty? Give only an intuitive explanation.