Econ 713 posted 5/02/08

Problem Set 6 due 5/09/08

Question 1:

Take a dynamic monopoly problem with $T = 2$ and $\delta = 1$ where the monopolist faces a continuum of buyers whose identity he cannot keep track of - that is, he does not know any particular buyer's purchasing history at any time. The buyers' per-period valuations θ are distributed uniformly on $[0, 1]$, and the monopolist has valuation 0 for the good as in class. The monopolist cannot commit.

a) What is the optimal sequence of spot prices $\{p_1^*, p_2^*\}$? *(Hint: first find the optimal* $p_2(p_1)$ or $p_2(q_1)$).

b) If the monopolists sets rental prices $\{r_1, r_2\}$, buyers need to purchase every period (if at all). Is the monopolist acquiring useful information about buyers' types in $t = 1$ with a rental scheme? What is the optimal sequence of rental prices?

c) Show that a sequence of rental prices $\{r_1^*, r_2^*\}$ yields strictly higher profits for the monopolist than a sequence of spot prices $\{p_1^*, p_2^*\}$. Interpret.

a) In $t = 2$, given quantity q_1 already sold in $t = 1$, inverse demand is $p_2 = 1 - q_1 - q_2$. The monopolist sets quantity $p_2 = \frac{1-q_1}{2}$ for profits $\left[\frac{1-q_1}{2}\right]^2$. To derive first-period demand, one needs to find $\hat{\theta}$ such that $\hat{\theta}$ is exactly indifferent between buying in period 1 or 2: $2\hat{\theta} - p_1 = \hat{\theta} - p_2$. Note that all $\theta < \hat{\theta}$ will buy in period 2, and hence $\hat{\theta} = 1 - q_1$. Therefore, first-period demand is $p_1 = \frac{3}{2}(1 - q_1)$. The monopolist now maximizes $\max_{q_1} \frac{3}{2}(1 - q_1)q_1 + \left[\frac{1-q_1}{2}\right]^2$, leading to an overall profit from spot prices of $\pi^S = \frac{45}{100}$.

b) With spot prices, the monopolist clears the market of high valuations in the first period, so that he can update his beliefs about θ in the second period (as the high-valuation buyers won't come back). With rental prices, as the seller cannot keep track of buyers' identities, he learns nothing at all from the sales in the first period - all types must come back for a sale in the second period. Hence, he can set monopolistic prices $r_1 = r_2 = \frac{1}{2}$. He therefore has a profit from renting of $\pi^R = \frac{1}{2}$.

c) Note that when the monopolist uses spot prices, he is in competition with himself as he cannot commit himself to not lowering the price in the second period (i.e. he cannot commit not to use what he has learnt in period 1, and this decreases his profits in t=1 as buyers know this). However, the rental scheme destroys the information acquisition - therefore, the principal can commit again as he is no longer in competition with himself.

Question 2

Prove Proposition 2 on the handout for Lecture 12, for $T = 2$ (where the first period is $t = 0$). You may want to use the following steps:

a) Show that if θ_L accepts the equilibrium rental price r_t^*, so does θ_H. To show this, pay particular attention to equilibrium beliefs $\mu_{t+1}^*(a_t) = P(\theta = \theta_H|$ acceptance decision in t) and what these beliefs imply for $r_{t+k}^*(a_t)$.

b) What condition does a price r_t have to satisfy that induces only θ_H to accept in t? Use this condition to show that both types would accept such a price in $t = 0$ whenever $\delta + \delta^2 > 1$, and that separating is therefore infeasible at $t = 0$.

c) Find the optimal pricing schedule $\{r_1^*, r_2^*, r_3^*\}$.

d) Why is it so hard for the monopolist to separate types in the renting model, as opposed to the buying model? Give an intuitive explanation.

a) This is obvious in $t = T$. For $t < T$, show this by contradiction: suppose $a_t^*(\theta_H) = 0$, $a_t^*(\theta_L) = 1$. Note that this implies equilibrium beliefs $\mu_{t+1}^*(a_t = 1) = 0$, $\mu_{t+1}^*(a_t = 0) = 1$, and hence rental prices from then on must be $r_{t+k}^*(a_t = 1) = \theta_L$, $r_{t+k}^*(a_t = 0) = \theta_H$, $k \geq 1$. Hence, θ_L will have utility 0 from $t+1$ onwards whether or not he accepts, so that $a_t^*(\theta_L) = 1$ implies $r_t^* \leq \theta_L$. Type θ_H has 0 utility from not accepting in t (0 today and 0 from then on). His utility from accepting, however, is $\theta_H - r_t^* + \delta(\theta_H - \theta_L)\sum_0^{T-t-1}\delta^i > 0$.

b) Again use beliefs: We have $a_t^*(\theta_H) = 1$, $a_t^*(\theta_L) = 0$. Note that this implies equilibrium beliefs $\mu_{t+1}^*(a_t = 1) = 1$, $\mu_{t+1}^*(a_t = 0) = 0$, and hence rental prices from then on must be $r_{t+k}^*(a_t = 1) = \theta_H$, $r_{t+k}^*(a_t = 0) = \theta_L$, $k \geq 1$. Then θ_H only accepts if $\theta_H - r_t + 0 \leq \delta(\theta_H - \theta_L)\sum_0^{T-t-1}\delta^i = (\theta_H - \theta_L)\sum_1^{T-t}\delta^i$, or $r_t \leq \theta_H - (\theta_H - \theta_L)\sum_1^{T-t}\delta^i$. However, if $\sum_1^{T-t}\delta^i > 1$, this implies $r_t < \theta_L$, which will cause θ_L to accept r_t as well (as his continuation utility is 0 whether or not he accepts).

c) a) and b) imply that $\mu_1^* = \lambda$ - there can be no information acquisition in $t = 0$. A1 then implies that it is better to set a separating rental schedule at $t = 1, 2$, with $r_1^* = \theta_H - (\theta_H - \theta_L)\delta$, $r_2^*(0) = \theta_L$, $r_2^*(1) = \theta_H$. In t=0, it is obviously better to acquire no information by selling to both ($r_0^* = \theta_L$) than neither. The result then follows.

d) Note that in both models, the monopolist really would like to know whether he is facing the high type. However, in the selling model he can only acquire that information by selling to the high type, which means that he is no longer able to use this information against the high type (he can only extract rents once). In the renting model, once the high type has made himself known, he will be held down to his reservation utility from then on, which means that he requires bigger informational rents to reveal himself than he needs in the buying model. Another way to view this is that the renting model is the situation with the least commitment possibilities for the seller - one can view the spot price model as one in which the principal offers fixed rental contracts which will no longer be changed once they are accepted (though he can change the rental contract he's offering if it's not accepted). The less the principal can commit, the more bargaining power the buyer has, leading to very low rental prices and the impossibility of separating *types.*