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Problem Set 5 due 5/2/08

Question 1

A monopolist produces quantities $q \in \mathbb{R}_0^+$ of a good with costs $c(q) = q^2$. Consumers' valuation of this good depends on their type $\theta \in \{\theta_L, \theta_H\}$, where $0 < \theta_L < \theta_H < \infty$ and $2\theta_L > \theta_H$. Both types are equally likely ex ante; the type is not observable for the monopolist. Given a monetary transfer $t \in \mathbb{R}_0^+$ and quantity q, consumers' utility from a purchase is $\theta q - t$, and the monopolist's profit is $t - c(q)$. The monopolist uses the usual direct revelation mechanism.

a) Set up the monopolist's optimization problem if he wants both types of customers to buy. Which constraints will bind at the optimum? Why?

b) Derive the optimal menu of contracts $(t_H^*, q_H^*), (t_L^*, q_L^*)$.

c) Suppose the monopolist had tried using a two-part tariff instead of a direct revelation mechanism; that is, he offers a price schedule $T(q) = a + bq$ for some a, b, and leaves the choice of q to the consumers. Could he implement $(t_H^*, q_H^*), (t_L^*, q_L^*)$ with such a tariff? If not, are there other nonlinear pricing schemes $t = T(q)$ that might work?

Question 2

A social planner is regulating a monopolist whose type $\theta \in \Theta = [0, 1]$ he cannot observe. The monopolist produces two different goods, 1 and 2. A monopolist of type θ has marginal costs θ of producing good 1; his marginal cost of producing good 2 is $(1-\theta)$. A monopolist who produces quantity $q = (q_1, q_2)$ and receives transfer t from the planner has profits $\pi(t, q; \theta) = t - \theta q_1 - (1-\theta)q_2$. The monopolist's reservation utility is zero. The social planner maximizes consumer surplus $S(q_i)$ in both markets, net of the transfer; his objective function is $V(t, q) = S(q_1) + S(q_2) - t$, where $S(q_i) = 1 - (1 - q_i)^2$. θ is distributed uniformly on Θ.

A *direct revelation contract* is a tuple of functions (t, q_1, q_2), $t : \Theta \to \mathbb{R}$, $q_i : \Theta \to \mathbb{R}$, $i = 1, 2$. Rewrite the monopolist's profit from such a contract as $\pi(\hat{\theta}; \theta) = \pi(t(\hat{\theta}), q(\hat{\theta}); \theta)$, where $\hat{\theta}$ is the monopolist's announcement of his type, θ is his true type. The optimal direct revelation contract maximizes the social planner's expected utility, subject to the monopolist's participation and revelation constraints:

$$\max_{q,t} \int_\Theta V(t(\theta), q(\theta)) f(\theta) d\theta \text{ s.t. } \pi(\theta; \theta) \geq 0, \forall \theta, \quad (P_\theta)$$

$$\theta \in \arg\max_{\hat{\theta}} \pi(\hat{\theta}; \theta), \forall \theta, \quad (R_\theta)$$

You may use the facts that for any $\bar{\theta} \in \Theta$, $g : \Theta \to \mathbb{R}$, $\int_0^{\bar{\theta}} [\int_\theta^{\bar{\theta}} g(y) dy] f(\theta) d\theta =$

$\int_0^{\bar{\theta}} g(\theta)F(\theta)d\theta$ and $\int_{\bar{\theta}}^1 [\int_{\bar{\theta}}^{\theta} g(y)dy] f(\theta)d\theta = \int_{\bar{\theta}}^1 g(\theta)(1 - F(\theta))d\theta$.

a) First suppose that production of good 2 is prohibited for some reason. Derive the function q_1^* of quantities set by the optimal direct revelation contract (t^*, q_1^*). Point out the monopolist's information rent along the way. Is the contract for the best type distorted away from the first-best? What about all other types?

b) Now suppose the monopolist is producing both goods, 1 and 2. The optimal direct revelation contract is a tuple of functions $(t^* q_1^*, q_2^*)$.

i. In the optimal contract, there exists (at least one) $\tilde{\theta} \in (0, 1)$ such that $q_1^*(\tilde{\theta}) = q_2^*(\tilde{\theta})$. Show that the revelation constraint of any θ, together with the participation constraint of type $\tilde{\theta}$, implies the participation constraint for type θ. *(Hint: A change of variables to make this problem look like the problem in a) may be useful).*

ii. Rewrite the revelation constraint in the usual manner, taking care to note whether you have to impose additional conditions on the optimal contract. Then use your result from (i) to derive expressions for $t(\theta)$ for all θ in an incentive compatible contract.

iii. (optional - do this part only if you want practice) Derive the optimal quantities q_1^*, q_2^*. Which types receive informational rents in the optimal contract? Why?