Econ 713 posted 4/18/08

Problem Set 4 due 4/25/08

Question 1

A risk-neutral principal who owns a firm employs a manager (the agent), whose effort is unobservable. The agent's effort $e \in \{1, 0\}$ determines the probability distribution of profits $\pi \in \{\pi_H, \pi_L\}$, $\pi_H > \pi_L$, as follows: $P(\pi = \pi_H | e = 1) = p \in (0, 1)$, and $P(\pi = \pi_H | e = 0) = 0$. The agent's utility from effort e and wage w is $v(w, e) = u(w) - e$, where u is continuous and strictly concave with $u(0) = 0$. The agent's reservation utility is 0. Wages are nonnegative.

A *contract* is a tuple $(w(\pi_H), w(\pi_L)) = (w_H, w_L)$ of contingent wages to be paid to the agent after the profit is realized.

a) Consider the standard contracting situation: The principal offers a contract which the agent may accept or reject; upon acceptance, the agent makes an effort choice.

(i) Derive a necessary and sufficient condition that ensures that the principal will always prefer to induce high effort in the agent.

To induce $e = 0, (w_H, w_L)$ *must satisfy*

$$pu(w_H) + (1-p)u(w_L) - 1 \leq u(w_L) \text{ and} \quad \text{(IC)}$$
$$u(w_L) \geq 0. \quad \text{(IR)}$$

Optimality requires $w_L = 0$. w_H *is not pinned down by optimality; one possible contract is* $w_H = w_L = 0$. *The principal's profit from this contract is* π_L.

To induce $e = 1, (w_H, w_L)$ *must satisfy*

$$pu(w_H) + (1-p)u(w_L) - 1 \geq u(w_L) \text{ and} \quad \text{(IC)}$$
$$pu(w_H) + (1-p)u(w_L) - 1 \geq 0. \quad \text{(IR)}$$

Optimality requires both constraints to hold with equality: both constraints slack is clearly not optimal. IC slack and IR tight imply $u(w_L) < 0$, *which* *is not possible as wages are nonnegative. IC tight and IR slack imply* $u(w_L) > 0$, *so that* w'_L *defined by* $u(w'_L) = u(w_L) - \varepsilon$ *is feasible, satisfies both constraints and makes the principal better off. Then we have* $w_L = 0$, $w_H = u^{-1}(\frac{1}{p})$. *The principal's profit is* $p(\pi_H - u^{-1}(\frac{1}{p})) + (1-p)\pi_L$; *inducing* $e = 1$ *is optimal if and only if*

$$p(\pi_H - u^{-1}(\frac{1}{p})) + (1-p)\pi_L \geq \pi_L; \text{ i.e. if and only if}$$

$$\pi_H - \pi_L \geq u^{-1}(\frac{1}{p}).$$

(ii) If your condition in (i) holds, how do the agent's wages depend on p? Interpret.

w_L *does not depend on* p, *whereas* w_H *decreases with* p. *This is a consequence of the fact that the principal is monopolistic and binds the agent to his reservation utility. When* p *increases, an agent expending high effort can expect to receive* w_H *more often; hence the same incentives can be given more cheaply (with a lower* w_H*).*

b) Now consider a contracting situation which is the same as the one in a) until the agent has chosen his effort level; however, *after* e has been chosen and *before* profits realize, the wage contract can be renegotiated. In this renegotiation stage, the principal offers a contract $(\hat{w}_H, \hat{w}_L)$ to the agent (this may be the same as the original contract (w_H, w_L)). If the agent accepts, the new contract $(\hat{w}_H, \hat{w}_L)$ is in force; if the agent rejects, the original contract (w_H, w_L) remains in force.

Let $q := P(e = 1|(w_H, w_L), (\hat{w}_H, \hat{w}_L))$ be the probability that the agent chooses the high effort in this game.

(i) For any $(w_H, w_L), q$, set up the principal's maximization problem when he chooses the optimal renegotiated contract $(\hat{w}_H, \hat{w}_L)$ such that the agent will weakly prefer to accept this contract for any previous action choice. Assume the principal's beliefs are correct.

As the the principal's beliefs are correct, he maximizes

$$\max_{\hat{w}_H, \hat{w}_L} qp(\pi_H - \hat{w}_H) + (1 - qp)(\pi_L - \hat{w}_L) \text{ such that}$$

$$pu(\hat{w}_H) + (1 - p)u(\hat{w}_L) \geq pu(w_H) + (1 - p)u(w_L) \text{ (unless } q = 0), \quad (IR_H)$$

$$\hat{w}_L \geq w_L \text{ (unless } q = 1). \quad (IR_L)$$

(ii) From the optimization problem above, what is the optimal renegotiated contract $(\hat{w}_H, \hat{w}_L)$ when $q = 1$? Give an intuitive explanation.

When $q = 1$, *this is a standard insurance problem with equal probabilities across agents. The optimal allocation is full insurance for the risk averse agent:* $\hat{w}_L = \hat{w}_H$.

(iii) *Now* consider the agent's effort choice when he correctly anticipates the renegotiated wages the principal will offer (which you have derived in (ii)). Can you find conditions under which $q = 1$ could be a Perfect Bayesian equilibrium?

$q = 1$ *can never be an equilibrium: If* $q = 1$, *the agent will be fully insured by the principal* ($\hat{w}_L = \hat{w}_H$). *But then the agent's optimal effort choice is* $e = 0$.

(iv) Assuming the condition you derived in a (i), compare your results from (iii) with the results you derived in a(i). Give an intuitive explanation.

In the game with renegotiation, high effort can never be induced with certainty, while high effort is certainly induced in the game without renegotiation for a range of parameter values. Renegotiation makes high effort harder to sustain because it introduces a time-inconsistency problem: ex ante, high effort can only be induced by making the (risk-averse) agent bear some risk. However, once the agent has exerted the effort, the principal would like to perfectly insure him as this will lower the wage bill. If renegotiation is not *possible, it means the principal can commit not to offer the agent insurance after effort is exerted; hence high effort can be induced. With renegotiation, this power for commitment is lost.*

Question 2

Take a moral hazard model with a risk neutral principal, a risk averse agent, continuous output $y \in [0,1]$ and continuous effort $e \in [0,1]$. Make the usual assumptions on the agent's utility. The agent's reservation utility is 0. The distribution of output is as follows: for any effort e, $P(y \in \{0\}|e) = 1-e$, and $P(y \in (0,1]|e) = e$.

a) Show that the distribution of y given e satisfies first-order stochastic dominance. Give an interpretation of the random process that determines y.

Write $F(y|e) = P(y \in \{0\}|e) + \int_0^y e dy = 1 - e(1-y)$*. For any* y*, this is clearly decreasing in* e*, hence satisfies first-order stochastic dominance.*

Note that the agent's effort only determines the probability of $y = 0$*; the distribution is uniform for all* $y > 0$*. This is like a two-output model, where the high output* $y > 0$ *is 'fuzzy' - subject to some random process that is not influenced by the agent.*

b) Assume the first-order approach is valid. Write down the principal's maximization problem. Argue that all constraints must bind. Then characterize the optimal contract. (Hint: pay special attention to the first order conditions with respect to the transfer $t(y)$.) Why do optimal transfers have this shape despite the fact that output is continuous? Give an intuitive explanation.

The principal maximizes

$$\max_{t,e}(1-e)(-t(0)) + e\int_0^1 (y - t(y))dy \text{ s.t.}$$

$$(1-e)v(t(0)) + e\int_0^1 v(t(y))dy - g(e) \geq 0 \qquad (P)$$

$$\int_0^1 v(t(y))dy - v(t(0)) - g'(e) = 0 \qquad (IC')$$

leading to first-order conditions P, IC'*, and*

$$\frac{1}{v'(t(0))} = \lambda - \mu\frac{1}{1-e} \quad (FOC_{t(0)})$$

$$\frac{1}{v'(t(y))} = \lambda + \mu\frac{1}{e},\ \forall y > 0 \quad (FOC_{t(y)})$$

$$t(0) + \int (y - t(0)dy + \lambda[\int_0^1 v(t(y))dy - v(t(0))] - \qquad (FOC_e)$$

$$g'(e) + \mu[\int_0^1 v(t(y))dy - v(t(0)) - g''(e)] = 0.$$

$FOC_{t(0)}$ *implies* $\lambda > 0$*. If* $\mu = 0$*, both* $FOC_{t(0)}$ *and* $FOC_{t(y)}$ *imply* $t(y) = \bar{t}$ *for all* y*. Plugging that and* $\mu = 0$ *into* FOC_e*, we find that it requires* $E[y] = 0$*, a contradiction.*

The optimal contract is a two-part payment scheme:

$$t(y) = \begin{cases} t_L, y = 0 \\ t_H, y > 0. \end{cases},$$

where $t_L < t_H$*. This is implies by both* $FOC_{t(0)}$ *and* $FOC_{t(y)}$*.*

You see that the optimal contract is independent of y *for* $y > 0$*. The intuition for this result comes out of the shape of the density (as most results in moral hazard come straight out of the density, i.e. how the probabilities of high output vary with effort). There are two motives for a transfer scheme that varies with output:a risk-sharing motive, which here is absent as the principal is risk neutral, and an incentives motive - give higher payments* *for output levels that are relatively more likely under high effort. Here, however, the relative likelihood of any* $y, y' > 0$ *is independent of the agents' effort, so that the incentives motive is also absent (for* $y > 0$*). To put it differently, the density of y is equivalent to the density that would result if the agent could only produce two distinct output levels. He then puts them in the bank, who sends them to the principal; but the bank is not very accurate with the higher output, which may arrive as any* $y \in (0, 1]$. *Clearly, the agent should not bear an incentive cost for the bank's behavior; the optimal incentive scheme is one resulting from a model where the agent can produce two output levels.*