Econ 713 posted 4/18/08

Problem Set 4 due 4/25/08

Question 1

A risk-neutral principal who owns a firm employs a manager (the agent), whose effort is unobservable. The agent's effort $e \in \{1, 0\}$ determines the probability distribution of profits $\pi \in \{\pi_H, \pi_L\}$, $\pi_H > \pi_L$, as follows: $P(\pi = \pi_H | e = 1) = p \in (0, 1)$, and $P(\pi = \pi_H | e = 0) = 0$. The agent's utility from effort e and wage w is $v(w, e) = u(w) - e$, where u is continuous and strictly concave with $u(0) = 0$. The agent's reservation utility is 0. Wages are nonnegative.

A *contract* is a tuple $(w(\pi_H), w(\pi_L)) = (w_H, w_L)$ of contingent wages to be paid to the agent after the profit is realized.

a) Consider the standard contracting situation: The principal offers a contract which the agent may accept or reject; upon acceptance, the agent makes an effort choice.

i. Derive a necessary and sufficient condition that ensures that the principal will always prefer to induce high effort in the agent.

ii. If your condition in (i) holds, how do the agent's wages depend on p? Interpret.

b) Now consider a contracting situation which is the same as the one in a) until the agent has chosen his effort level; however, *after* e has been chosen and *before* profits realize, the wage contract can be renegotiated. In this renegotiation stage, the principal offers a contract $(\hat{w}_H, \hat{w}_L)$ to the agent (this may be the same as the original contract (w_H, w_L)). If the agent accepts, the new contract $(\hat{w}_H, \hat{w}_L)$ is in force; if the agent rejects, the original contract (w_H, w_L) remains in force.

Let $q := P(e = 1 | (w_H, w_L), (\hat{w}_H, \hat{w}_L))$ be the probability that the agent chooses the high effort in this game.

i. For any $(w_H, w_L), q$, set up the principal's maximization problem when he chooses the optimal renegotiated contract $(\hat{w}_H, \hat{w}_L)$ such that the agent will weakly prefer to accept this contract for any previous action choice. Assume the principal's beliefs are correct.

ii. From the optimization problem above, what is the optimal renegotiated contract $(\hat{w}_H, \hat{w}_L)$ when $q = 1$? Give an intuitive explanation.

iii. *Now* consider the agent's effort choice when he correctly anticipates the renegotiated wages the principal will offer (which you have derived in (ii)). Can you find conditions under which $q = 1$ could be a Perfect Bayesian equilibrium?

iv. Assuming the condition you derived in a (i), compare your results from (iii) with the results you derived in a(i). Give an intuitive explanation.

Question 2

Take a moral hazard model with a risk neutral principal, a risk averse agent, continuous output $y \in [0, 1]$ and continuous effort $e \in [0, 1]$. Make the usual assumptions on the agent's utility. The agent's reservation utility is 0. The distribution of output is as follows: for any effort e, $P(y \in \{0\}|e) = 1 - e$, and $P(y \in (0, 1]|e) = e$.

a) Show that the distribution of y given e satisfies first-order stochastic dominance. Give an interpretation of the random process that determines y.

b) Assume the first-order approach is valid. Write down the principal's maximization problem. Argue that all constraints must bind. Then characterize the optimal contract. (Hint: pay special attention to the first order conditions with respect to the transfer $t(y)$.) Why do optimal transfers have this shape despite the fact that output is continuous? Give an intuitive explanation.