**Econ 713** posted 3/28/08

**Problem Set 1** due 4/4/08

**Question 1**

Consider a 2-player Cournot game with linear inverse demand $p(q) = a - bq$, where the oligopolists have different constant marginal costs $c_i$.

a) Derive the Nash equilibrium. Under what condition does it involve only one firm producing? Which firm will this be?

b) When both firms produce in equilibrium, show how equilibrium outputs and profits for both firms vary with firm 1's marginal cost.

*The solution is straightforward.*

**Question 2**

Take a general $n-$player Cournot game with asymmetric marginal costs $c_i$ and inverse demand $p(q)$ that satisfies $A1 - A3$.

a) Show that the Cournot equilibrium price and total output do not vary with marginal costs as long as their sum remains unchanged. Go on to show that when marginal costs change so that their sum remains unchanged, a firm's equilibrium output is only affected by its own changes in marginal cost. *(Hint: Use firms' first-order conditions for this part.)*

*Sum up the individual first-order conditions to get* $p'(q^*)q^* + np(q^*) = \sum_i c_i$. *Hence,* $p(q)$ *remains unchanged. Then the second claim follows from the individual firm's foc, rearranged to give*

$$q_i^*(c_i) = -\frac{p(q^*) - c_i}{p'(q^*)}.$$

b) Suppose the government considers imposing excise taxes $t_i \in (0, 1)$, $i = 1, ..., n$ on the oligopolists. At present, there are no taxes in this market. (An excise tax requires the producer to pay $t_i$ for every unit of the good sold). The government wants to maximize tax revenue, but it does not want to upset consumers by causing changes of price or quantity in the market. Set up the government's maximization problem. What conditions do tax rates $t_i$, $t_j$ have to satisfy for any $i, j$? Interpret.

*The imposition of* $t_i$ *is equivalent to an increase in the marginal cost of size* $t_i$*, that is, given* $t_i$*, the firm produces NE quantity* $q_i^*(c_i + t_i)$ *The government's maximization problem is*

$$\max_{t_1,..,t_n} \sum_i t_i q_i^*(c_i + t_i) \text{ s.t. } \sum_i t_i = 0.$$

*From the first-order conditions for* $t_i, t_j$*, we get the following condition:*

$$t_i - t_j = -\frac{1}{2}(c_i - c_j).$$

*The government subsidizes high-cost firms with low output in order to collect high tax revenues from low-cost firms with high output.*

**Question 3**

Take a modified two-firm Bertrand model in which firms are allowed to commit to match prices. That is, firms simultaneously announce prices $p_i^a$ and policies $r_i \in \{M, \varnothing\}$, where policy $M$ means a public commitment to match the competitor's price if it is lower (for example, by advertising the fact). Firms can also choose not to commit to anything (policy $\varnothing$). After these announcements are made, firms' prices $p_i$ are transmitted to the market as

$$p_i = \left\{ \begin{array}{c} p_i^a, r_i = \varnothing \\ \min\{p_i^a, p_j^a\}, r_i = M \end{array} \right. .$$

Firms are symmetric with marginal cost $c$ and have payoffs from realized prices $p_i$ as in the classic Bertrand model. You may also impose Assumptions 1 and 2 from Lecture 1. What is the highest possible market price that can result in a Nash equilibrium of this game? Is this equilibrium unique?

*The equilibrium involving the highest market price has* $p_1^{a*} = p_2^{a*} = p^m$ *(the monopoly price), and* $r_1 = r_2 = M$*. Here, both firms share monopoly profits equally. Deviations to higher prices (with* $r_i' = \varnothing$*) are not profitable as they lead to zero profit. Deviations to lower prices will result in the competitor matching the price, so that both firms equally share a profit that is smaller than the monopoly profit. Clearly, this is not profitable either.*

*This equilibrium is not unique - for example, the old equilibrium* $p_1^{a*} = p_2^{a*} = c$ *and* $r_1 = r_2 = \varnothing$ *still survives in this setup. Both firms make 0 profit and have no profitable deviations.*