Econ 713 **Spring 2008**

Lecture 8: Principal-Agent models - Moral Hazard with continuous effort choice

1 Definitions

A single firm (the principal) offers employment to a worker (the agent). The agent's effort choice influences the firm's profit, but is unobservable to the principal. The principal offers contracts specifying the agent's effort levels as well as his compensation as a function of profit. As the principal is a monopolist, the optimal contract maximizes his profit (under informational and acceptance constraints). Note that in all principal-agent models to follow, the set of 'players' is the same.

- the agent's action set $\mathcal{A}_w = E \subseteq \mathbb{R}_0^+$. The choice of e is the agent's private information.
- payoff function for the agent: $V(t,e) = v(t) - g(e)$. The agent's reservation utility is $\bar{u}$.
- the physical environment: output $y \in Y \subseteq \mathbb{R}$ is a random variable, $y \sim F(y|e)$ with associated density $f(y|e)$..
- payoff function for the principal: $u(c)$, $c \in \mathbb{R}$, where c is the principal's consumption.

2 The optimal contract with continuous output

Let $Y = [y_L, y_H]$. We put the following assumptions on utilities and the distribution of y :

Assumption 1: $u' > 0, u'' \leq 0$; $v' > 0, v'' < 0$, $g' > 0, g'' \geq 0$.
Assumption 2: $F_e(y|e) \leq 0$ (strict for some y).
Assumption 3: $f(y|e)$ continuous in y, C^2 in e.

A contract c is a transfer function $t : Y \to \mathbb{R}$ as well as an optimal effort choice by the agent: $c = (t,e)$. Principal's and agent's expected utilities from a contract c are

$$\begin{aligned} U(t,e) &= \int_Y u(y - t(y)) f(y|e) dy, \\ V(t,e) &= \int_Y v(t(y)) f(y|e) dy - g(e). \end{aligned}$$

Let (t^*, e^*) denote the optimal contract under incomplete information. This optimal contract solves

$$\max_{t,e} U(t,e) \text{ s.t. } V(t,e) \geq \bar{u} \text{ and} \tag{P}$$

$$e \in \arg\max V(t,e). \tag{IC}$$

2.1 The first-order approach

The incentive-compatibility constraint as given above is not tractable. The so-called first-order approach consists in replacing (IC) as given above by

$$V_e(t,e) = \int_Y v(t(y)) f_e(y|e)dy - g'(e) = 0. \qquad (IC')$$

This approach is valid if $V(t,e)$ has a unique maximizer e for any t, and this maximizer is always interior. Note that A1-A3 are not sufficient to guarantee this. The two following assumptions together are sufficient to ensure validity of the first-order approach (with appropriate boundary counditions for the case where E is bounded):

Assumption 4: (MLRP) $\frac{f_e(y|e)}{f(y|e)}$ is increasing in y.
Assumption 5: (CDFC) $F_{ee}(y|e) < 0$.

2.2 Optimal contracts under incomplete information

Proposition 1 *Assume A1-A3 and that the first-order approach is valid. Under incomplete information, the first-best will never implement* e^*.

Proof: To show this, we set up the maximization problem for the optimal contract under incomplete information and then show that μ, the multiplier on (IC), is always strictly positive. (t^*, e^*) solve

$$\max_{t,e} U(t,e) \text{ s.t. } (P) \text{ and } (IC)'.$$

Therefore, for all y, $t^*(y), e^*$ must satisfy $(P), (IC)'$, and

$$\frac{u'(y - t^*(y))}{v'(t^*(y))} = \lambda + \mu \frac{f_e(y|e^*)}{f(y|e^*)}, \qquad (FOC_{t(y)})$$

$$\int_Y u(y - t^*(y)) f_e(y|e^*)dy + \lambda \left[\int_Y v(t^*(y)) f_e(y|e^*)dy - g'(e^*)\right] + \qquad (FOC_e)$$

$$\mu \left[\int_Y v(t^*(y)) f_{ee}(y|e^*)dy - g''(e^*)\right] = 0$$

Assuming $\mu = 0$, we then derive a contradiction with first-order stochastic dominance.■

Properties of t^* :

- A2 implies that P binds;
- if A4 holds, t^* is increasing in y.

3 The optimal contract with discrete output

Let $Y = \{y_L, y_H\}$, $E = \mathbb{R}_0^+$ and $P(y = y_H|e) = p(e)$. We put the following assumptions on utilities and the distribution of y :

Assumption 1: $u' > 0, u'' \leq 0$; $v' > 0, v'' < 0$, $g(e) = e$.

Assumption 2: $p' > 0, p'' < 0$
Assumption 3: $p'(0)$ large, $\lim_{e\to\infty} p'(e) = 0$.

A contract c is a tuple of transfers (t_H, t_L) as well as an optimal effort choice by the agent: $c = (t_H, t_L, e)$. Principal's and agent's expected utilities from a contract c are

$$\begin{aligned} U(c) &= p(e)u(y_H - t_H) + (1 - p(e))u(y_L - t_L), \\ V(c) &= p(e)v(t_H) + (1 - p(e))v(t_L) - e. \end{aligned}$$

3.1 The first-best contract

Let $(\tilde{t}_H, \tilde{t}_L, \tilde{e})$ denote the first-best contract that solves

$$\max_{t_H, t_L, e} U(c) \text{ such that } V(c) \geq \bar{u}.$$

The first-order conditions with respect to $\tilde{t}_H, \tilde{t}_L$ are

$$\frac{u'(y_H - \tilde{t}_H)}{v'(\tilde{t}_H)} = \lambda = \frac{u'(y_L - \tilde{t}_L)}{v'(\tilde{t}_L)},$$

implying $\tilde{t}_H > \tilde{t}_L$.

3.2 The optimal contract with incomplete information

Let (t_H^*, t_L^*, e^*) denote the optimal contract under incomplete information.

Claim 1 *The first-order approach is valid if A1-A3 hold and we add the monotonicity constraint*

$$t_H > t_L \tag{M}$$

Proposition 2 *The optimal contract (t_H^*, t_L^*, e^*) is the solution to the maximization problem*

$$\begin{aligned} \max U(c) \text{ s.t. } V(c) &\geq \bar{u}, && (P) \\ p'(e^c)(v(t_H) - v(t_L)) &= 1. && (IC') \end{aligned}$$

At the optimum, both P and IC' are binding.