Econ 713 **Spring 2008**

Lecture 12: Indivisible good problems - Dynamic monopoly pricing

A monopolist has a durable good that he may sell (or rent) to consumers with unknown (flow) valuations θ. Opportunities for sale occur every period $t = 1, ..., T$. Once a consumer has bought the good, he keeps it until T, deriving utility θ from it every period he has the good in his possession.

A sequence of prices is said to exhibit *Coasian dynamics* if it is (weakly) decreasing. It has the *skimming property* if high-valuation buyers buy first.

1 Definitions

- buyer's valuations $\theta \in \Theta = \{\theta_L, \theta_H\}$, $0 < \theta_L < \theta_H < \infty$. θ is the buyer's private information. $P(\theta_H) = \lambda$.
- buyer's action space $\mathcal{A}_b = \{1, 0\}$ (1 being interpreted as buying, 0 as not buying)
- seller's action space (prices p_t) $\mathcal{A}_s = \mathbb{R}_0^+$
- buyer's per-period utility $u(p_t, a_t; \theta) = a_t(\theta - p_t)$
- seller's per-period utility $\pi(p_t) = p_t$.
- both buyer and seller discount the future with discount factor δ.
- game tree $\mathcal{T}$: in period 0, nature draws a type for the buyer. In every period $t = 1, ..., T$, buyer and seller repeat the stage game: first the seller proposes p_t, then the buyer makes an acceptance decision a_t. Once the buyer has accepted an offer, no further transactions occur in the sales model - in the rental model with rental prices r_t, the game is repeated every period whether or not the buyer has rented in the previous period.

Assumption 1: $\lambda > \frac{\theta_L}{\theta_H}$

2 Spot prices: The full commitment solution

Claim 1 *If A1 holds and the principal can commit to a sequence of prices, the optimal price sequence* $\{p_1^C, ..., p_t^C\}$ *has* $p_t^C = \theta_H \sum_{i=0}^{T-t} \delta^i$.

Define π^C as the seller's profit with full commitment. We have $\pi^C = \theta_H \sum_{i=0}^{T-t} \delta^i$.

3 Spot prices without commitment

Define $\mu_t = P(\theta_H|\text{no sale before } t)$. We will analyze pure strategy Perfect Bayesian equilibria, specifically the sequence of prices $\{p_1^*, ..., p_t^*\}$ on the equilibrium path.

Proposition 1 *If A1 holds,* $p_1^* = \theta_H + \theta_L \sum_{i=1}^{T-1} \delta^i$, $p_t^* = \theta_L$, $t = 2, ..., T$.

Step 1: If type L accepts p_t^*, so does H.

Step 2: $p_t^* \in [\theta_L \sum_{i=0}^{T-t} \delta^i, \theta_H \sum_{i=0}^{T-t} \delta^i]$ for all t.

Step 3: Assume A1. If $\tau < T$ is the first period in which the seller expects acceptance, $p_\tau^* = \theta_H + \theta_L \sum_{i=1}^{T-\tau} \delta^i$, $p_{\tau+1}^* = \theta_L$.

Step 4: $\tau = 1$.

Define π^S as the seller's profit from spot prices without commitment. We have $\pi^S = \lambda\theta_H + \theta_L \sum_{i=1}^{T-1} \delta^i$. It is easy to see that the seller would prefer to have the power of commitment.

4 Renting without commitment

When the identity of buyers is unknown, the seller can rent instead of sell to escape the time-inconsistency problem we see above. However, in our model, the buyer is identifiable, and one can show that renting is worse for the seller than spot prices without commitment, when agents are patient and $T > 2$. We will show this for $T = 3$; it is easy to use the argument below to show that for every $T > 2$, there exists δ_T such that $\pi^R < \pi^S$ whenever $\delta > \delta_T$.

If the seller sets rental prices $\{r_1, ..., r_t\}$, the buyer will have to buy every period. Define π^R to be the profit from renting without commitment.

Proposition 2 *If A1 holds, δ is large enough ($\delta + \delta^2 > 1$) and $T \geq 3$, $\pi^R < \pi^S$.*

Define $\mu_t = P(\theta_H |$ history of transactions before $t)$.

Step 1: If type L accepts r_t^*, so does H.

Step 2: $r_t^* \in [\theta_L, \theta_H]$

Step 3: Under the assumptions of the proposition, separating prices are not feasible for $t \leq T - 2$.

Step 4: $\pi^R = \theta_L \sum_{i=0}^{T-2} \delta^i + \delta^{T-1}(\lambda\theta_H + \delta\theta_L)$.