Estimation of a Roy/Search/Compensating Differential Model of the Labor Market

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I said earlier that in labor economics the four most important models probably are

- Roy model
- Search Model
- Compensating Differentials
- Human Capital

There is not much of a literature distinguishing between them.

Our goal is to write down a model incorporating all four of these and to use it to think about the relative contributions to earnings inequality

- We take "Roy Model" to mean heterogeneity in pre-market skill levels that varies across individuals and jobs. Workers choose occupation based on comparative advantage
- By “Compensating Differences" we mean that that workers choose jobs that they “like.” With identical skills and job choices may earn different wages because they choose different jobs. Requires:
  - workers care about job characteristics other than wages
  - workers have heterogeneity in these choices
  - We take these as given-we have in mind how much you enjoy the actual job not how much you enjoy your office or health benefits
- By “Search" we mean that workers can not always work at their preferred job but need to wait for an offer
- We incorporate human capital by allowing wages to go up with general experience (learning by doing)

The main goal of this work is to

1. Write down models that incorporate Roy Model skill, search frictions, learning by doing and non-pecuniary tastes for jobs
2. Establish non-parametric identification of (parts of) the model
3. Estimate the model
4. Use it to see how the various features contribute to Earnings Inequality

We are also not trying to write down the most complicated model of the labor market possible, rather we are trying to write down the simplest model that gets at the essence of the goals above

We estimate the model using matched worker/firm data from Denmark

Has two major advantages:

- Panel allows us to follow both workers and firms as matches change
- We observe job-to-job transitions.
  - We use this to make a revealed preference argument.
  - Job ladder will be defined by this revealed preferences rather than by wages (or productivity) -though we allow for some job to job transitions to be involuntary

The Model

Our model has endogenous wage determination in which workers and firms bargain over wages

Determination of entry and exit of firms as well as provision of nonpecuniary aspects is not modeled here and taken as exogenous-thinking more about firm behavior would be an interesting and important extension

We assume a finite number of types of jobs one can take

π_{ij} is the (net) productivity of worker i's human capital at firm type j

Wages will be in efficiency units of human capital so I am paid $w\psi_h$

We denote the flow value that individual i has for job j as

$$U_{ij}(w\psi_h).$$

Human capital takes in a finite number of values $h = 0, ..., H$ and evolves stochastically with a Poisson arrival rate (H=20)

We take time as continuous with

- δ : job destruction rate
- λ_j^e : job arrival rate for a worker employed at another firm
- λ_j^n : job arrival rate for a non-employed worker
- λ_h : arrival rate of human capital accumulation (1.0 in practice)
- P^* Probability of an offer immediately after job destroyed

Let $V_{ijh}(w)$ be the value function for worker i, with human capital ψ_h at firm j at wage w.

V_{i0h} is value function for a nonemployed worker

V_{i0h}^* is immediately after job destruction (incorporates probability of immediate offer)

Clearly we are abstracting from many thing that can be added later

However, we think this gets at the core of what we care about

- The parameter λ_j^e picks up search frictions
- The distinction between $U_{ij}(w\psi_h)$ and $w\psi_h$ picks up compensating differentials
- Variation in wages across individuals but within jobs picks up Roy model heterogeneity (after accounting for bargaining)
- increase in h picks up on the job learning

Given the model there are many ways to decompose earnings inequality.

Wage determination

This is based on Cahuc, Postel-Vinay, and Robin (2006)

Main Components:

- Worker has bargaining power β
- Wage contracts lead to fixed hc rental rate unless the firm wants to offer a higher wage to respond to an outside offer
- Full information about worker tastes and productivity
- When worker gets offer from nonemployment
    1. If there is surplus to the match the worker will work for the firm (i.e. there is a wage that the firm is willing to pay at which the worker prefers the job to nonemployment)
    2. Worker and firms bargains over wage so wage is set to solve
    $$V_{ijh}(w) = \beta V_{ijh}(\pi_{ij}) + (1 - \beta) V_{i0h}$$

- When employed worker gets an outside offer one of three things can happen
  1. If surplus on new job is higher than surplus on older job, worker switches firm. New firm pays wage to solve

$$V_{i\ell h}(w) = \beta V_{i\ell h}(\pi_{i\ell}) + (1 - \beta)V_{ijh}(\pi_{ij})$$

  2. If surplus on new job is sufficiently low that maximum wage on offered job would be turned down for based on current wage, nothing happens.
  3. If surplus on new job is higher than this, but lower than current job, wages are renegotiated. Current firm offers wage to solve

$$V_{ijh}(w) = \beta V_{ijh}(\pi_{ij}) + (1 - \beta)V_{i\ell h}(\pi_{i\ell})$$

When $h < H$,

$$\left(\rho + \delta + \lambda_h + \Lambda^e_{ik(w)H}\right) V_{ijh}(w)$$
$$= U_{ij}(w\psi_h) + \delta V^*_{i0h} + \lambda_h V_{ijh+1}(w)$$
$$+ \sum_{\{\ell : V_{ijh}(w) < V_{i\ell h}(\pi_{i\ell}) \leq V_{ijh}(\pi_{ij})\}} \lambda^e_\ell \left[\beta V_{ijh}(\pi_{ij}) + (1-\beta) V_{i\ell h}(\pi_{i\ell})\right]$$
$$+ \sum_{\{\ell : V_{i\ell h}(\pi_{i\ell}) > V_{ijh}(\pi_{ij})\}} \lambda^e_\ell \left[\beta V_{i\ell h}(\pi_{i\ell}) + (1-\beta) V_{ijh}(\pi_{ij})\right]$$

(similar but simpler expression for V_{i0h}, V^*_{i0h} and $V_{ijH}(w)$).

Identification

A major goal of this work is to think carefully about identification of the parameters-and importantly what is not identified

For simplicity we assume only two firm types: A and B

Expect everything to generalize to any finite number

We observe individuals employment status and wages from some time 0 to time T

The P^* makes things more difficult so lets assume that this is zero for simplicity

Transition Parameters

δ (job destruction rate) comes directly from job to nonemployment rate

λ_j^n (job arrival rate from non employment) comes from hazard rate from nonemployment to employment

λ_j^e (job arrival rate from employment) identified from hazard rate from job to job:

Revealed Preferences

Define person “types” by their ordering of preference across jobs (at highest wage)

There are a finite set of these.

If ordering does not chance with Human capital, there are 5 types

$$A \succ B \succ 0$$

$$B \succ A \succ 0$$

$$A \succ 0 \succ B$$

$$B \succ 0 \succ A$$

$$0 \succ A, 0 \succ B$$

Conditional on each type and given the λ and δ we can calculate the probability of any particular labor market sequence. This will differ across types. Let $d_{\tau_1}, ..., d_{\tau_K}$ be a sequence of jobs, then

$$Pr\left(d_{\tau_1}, ..., d_{\tau_K}\right) = \sum_{type} Pr(d_{\tau_1}, ..., d_{\tau_K} \mid type) Pr(type).$$

Wages

Given transition parameters and type probabilities by conditioning on timing of when people are hired and using characteristic functions we can identify

- the measurement errors
- the joint distribution of wages for every type in every state of the world that they experience. Includes:
  - wage from nonemployment on each type of job
  - wage on each type of job given each type of outside offer
  - wages on new job after job to job transition
  - how all these vary with human capital

What is not identified

Problem 1: β

revealed preference reveals job ordering and wages in different states of the world, but not utility levels

As a result we can not identify β

However, we do know what happens if $\beta = 1$: that is maximum wage that a worker would ever receive at each job.

When we "normalize" the level of utility in our model as is standard in discrete choice models, β can be identified after this

It is important to keep in mind that β depends on this normaliziation, so it is odd to change model and hold β fixed

In our decompositions we get rid of monopsony power by sending $\beta = 1$, we do this first

Problem 2: The selection problem

We can't observe all workers at all jobs, thus we can not say what their wage would be at all jobs

What you really need for full identification is extreme,

It is not just something that varies occupational choice holding wages constant-you need something like "identification at infinite"

This would allow you to identify for example the wages that Warren Buffett would receive working at Mcdonalds

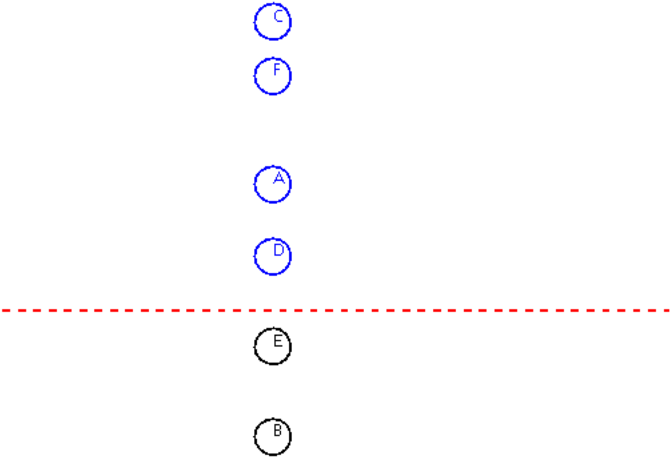
Its hard to imagine you could ever hope to identify this

At the same time, who cares?

It is hard to imagine any interesting counterfactual that would actually involve this

Our strategy here is to focus on identifying what can be identified

That is we will recognize this problem in the work-and as a result we will not simulate counter-factuals that involve these unidentified features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Econometric Specification

We assume that log productivity for worker i at firm j is

$$\log(\pi_{ij}) = \theta_i + \mu_j^w + v_{ij}^w$$

Flow value to firm is $\pi_{ij}\psi_h$

We observe log wages with i.i.d measurement error with variance σ_ξ^2

Workers utility is determined by

$$U_{ij}(W\psi_h) = \alpha log(W\psi_h) + \mu_j^n + v_{ij}^n.$$

with θ_i independent of (μ_j^w, μ_j^u) independent of (v_{ij}^w, v_{ij}^n).

Flow utility for non-employment is

$$U_{i0} = \alpha E_\theta + \gamma_\theta \left(\theta_i - E_\theta\right) + \nu_{i0}^n$$

We assume that the arrival and destruction rates are not heterogeneous so we estimate the three parameters $(\delta, \lambda^n, \lambda^e)$.

Human capital evolves as

$$\log(\psi_h) = b_1 h + b_2 h^2 + b_3 h^3$$

We use a cubic spline so there are two free parameters and we choose the third to impose that

$$\frac{\partial \log(\psi_H)}{\partial h} = 0$$

We fix λ_h=1

We parameterize the model with parametric functional forms

$$
\begin{aligned}
\theta_i &\sim N\left(E_\theta, \sigma_\theta^2\right) \\
\log \delta_i &\sim N\left(d_0, \sigma_\zeta^2\right) \\
\log \nu_{i0}^n &\sim N\left(0, \sigma_\nu^2\right) \\
\xi_{it} &\sim N\left(0, \sigma_\xi^2\right) \\
(v_{ij}^n, v_{ij}^w) &\sim N(0, \Sigma_v) \\
\mu_j^n &= f_1 \left[U_1(j) + f_3 U_2(j)\right] \\
\mu_j^w &= f_2 \left[f_3 U_1(j) + U_3(j)\right]
\end{aligned}
$$

We normalize var$(v_{ij}^n) = 1$ and cov$(v_{ij}^n, v_{ij}^w) = 0$

This gives us 18 parameters

- Transitions: $d_0, \sigma_\zeta, \lambda^n, \lambda^e, P^*$
- General Ability: E_θ, σ_θ
- Measurement Error: σ_ξ
- Reservation Utility: $\gamma_\theta, \sigma_\nu$
- Idiosyncratic Tastes and Productivity: σ_{ν^w}, α
- Firm Tastes and Productivity: f_1, f_2, f_3
- Human Capital: b_1, b_2
- Bargaining Process: β

Doing this requires pretty cool data, but luckily we have pretty cool data

Data

Really two parts:

1: Essentially every November for every worker in the country we observe

- Demographic stuff
- Whether they are working
- If working we observe
  - firm identifier
  - wage

2: We observe spell data

Spells are of 5 types, employment at a firm, unemployment, OLF, retired, self-employed

For each spell we observe the firm, start, and stop date

Sample Criterion

To be in our sample you need:

- 1985-2003
- done with schooling
- younger than 56

For each parameter we will choose a statistic in the data to help identify it

In doing this we will use the identification discussion to guide us

In the identification model we first discussed how to identify turnover and then wages. We will use this strategy as well in thinking of an auxiliary model.

Employment Dynamics Parameters (before wages):

- d_0 : Mean length of employment spells
- σ_ζ: Var. length of employment spells
- λ^n : Length of non-employment spells
- λ^e : Length of Job spells
- σ_ν : Var. nonemployment Duration
- P^*: Fraction job-to-job that are involuntary
- f_1 : $E(\widetilde{S}_{i\ell j}\widetilde{h}_{-i\ell j})$

For auxiliary parameters involving Wages we have 11 parameters left: $E_\theta, \sigma_\theta, \sigma_\xi, var\left(v_{ij}^w\right), \alpha, f_2, f_3, b_1, b_2, \beta, \gamma_\theta$

For $\sigma_\theta^2, \sigma_\xi^2, var\left(v_{ij}^w\right)$: We use the decomposition

$$\frac{\sum\left(w_{it}^m - \overline{w}\right)^2}{\sum T_{i\ell j}} = \frac{\sum\left(w_{i\ell jt}^m - \overline{w_{i\ell j}}\right)^2}{\sum T_{i\ell j}} + \frac{\sum\left(\overline{w_{i\ell j}} - \overline{w_i}\right)^2}{\sum T_{i\ell j}} + \frac{\sum\left(\overline{w_i} - \overline{w}\right)^2}{\sum T_{i\ell j}}$$

That is we use each of the three expressions on the right hand side

For the next 5 we use the moments:

- E_θ :Sample mean of w_{it}
- f_2:E$(\widetilde{w}_{it}\widetilde{w}_{-it})$
- f_3:E$(\widetilde{w}_{it}\tilde{h}_{-it})$
- α: Fraction wage drops
- γ_θ: Cov($\overline{w}$, non-empl dur)

That leaves three parameters: $b_1, b_2,$ and β

We use a worker$\times$ match fixed effect regression:

$$w_{i\ell jt} = \beta_{i\ell j} + \beta_1 E_{i\ell jt} + \beta_2 E^2_{i\ell jt} + \beta_3 TE^2_{i\ell jt} + \epsilon_{i\ell jt}$$

(the level of tenure and experience are perfectly collinear within a job spell)

Idea is that β determines the rate at which wages grow on the job.

From λ_e we know the rate at which new jobs come

The idea is that β_3 picks up the magnitude

Results

Moment	Data	Model
Avg. Length Emp. Spell	377	382
	(0.202)	
Avg. Length Nonnemp. Spell	91.4	91.5
	(0.086)	
Avg. Length Job	108	107
	(0.101)	
$E(\widetilde{S}_{i\ell j}\widetilde{h}_{-i\ell j}) \times 100$	1.53	1.51
	(0.003)	
Between Persons$\times 100$	8.03	8.02
	(0.012)	
Between Jobs$\times 100$	2.87	2.89
	(0.006)	
Within Job$\times 100$	1.49	1.49
	(0.003)	
Sample mean w_{it}	4.50	4.50
	(0.000)	

E$(\widetilde{w}_{it}\widetilde{w}_{-it}) \times 100$	0.284	0.284
	(0.002)	
$E(\widetilde{w}_{it}\widetilde{h}_{-it}) \times 100$	0.108	0.108
	(0.001)	
Fraction Wage Drops	0.400	0.408
	(0.000)	
Coeff Exper$\times 100$	2.48	2.44
	(0.008)	
Coeff Exper$^2 \times 1000$	-0.291	-0.295
	(0.002)	
Coef Tenure$^2 \times 1000$	-0.460	-0.462
	(0.005)	
Var(Nonemployment)	16000	15992
	(47.472)	
Cov($\overline{w}_i$, Non-employment)	-3.42	-3.43
	(0.013)	
Var(Employment Dur)	102000	99688
	(71.434)	
Invol Job to Job	0.205	0.205
	(0.011)	

Parameter Estimates

Parameter	
d_0	-2.89
	(0.035)
λ^n	0.98
	(0.007)
λ^e	2.07
	(0.061)
E_θ	4.34
	(0.013)
σ_θ	0.231
	(0.002)
σ_ξ	0.134
	(0.001)
f_1	2.207
	(0.139)
f_2	0.122
	(0.001)
f_3	-0.000
	(0.009)

σ_{ν^w}	0.197
	(0.003)
α	2.82
	(0.123)
b_1	0.019
	(0.001)
b_2	-0.001
	(0.000)
β	0.812
	(0.008)
P^*	0.435
	(0.019)
σ_{ν_0}	0.434
	(0.003)
γ_θ	-0.387
	(0.025)
σ_d	1.99
	(0.026)

Simulations

The goal is to understand how the various components lead to wage variation.

First note that measurement error is outside the model. The total variance in wages is 0.120.

After getting rid of measurement error we are down to 0.107

In terms of the experiments:

- Eliminating Roy means setting wages to the mean log wage by firm type (holding preferences for jobs constant)
- Eliminating Search means setting

$$\lambda^e \to \infty$$

- Eliminating Compensating Differentials means preference over wage only (conditional on preferring job to nonemployment)

Decompositions

(A)		(B)		(C)		(D)	
Total	0.106	Total	0.106	Total	0.106	Total	0.106
No HC	0.101	No HC	0.101	No HC	0.101	No HC	0.101
No Monop	0.093	No Monop	0.093	No Monop	0.093	No Monop	0.093
No Roy	0.006	No Search	0.091	No Comp	0.089	No Comp	0.089
No Search	0.005	No Roy	0.005	No Search	0.068	No Roy	0.002

Understanding importance of Search

Workers are searching for four different things:

1. firm specific wage
2. firm specific nenpecuniary aspects
3. individual$\times$firm type productivity match
4. individual$\times$firm type utility match

In our different simulations above we changed which of these are being searched for and it matters a lot

Simulation	Searching For	Importance
(A)	1,2,4	1%
(B)	1,2,3,4	2%
(C)	1,3	21%
(D)	1	2%

Compensating Differentials and Search Frictions

Just because these are not the major contributors to earnings inequality does not mean they aren't important

Search Frictions

- Obviously important for turnover and unemployment-in our model they drive both
- Wages would be about 0.17 log points higher without search frictions (about 0.07 is the negotiation)

Compensating Differentials

- On turnover, roughly 1/3 of competing offers would change if people only cared about wages
- people earn on average 0.20 log points less to take jobs they like

Utility Decomposition

Another way to show the importance of search and compensating differentials is to do a “variance in utility” decomposition rather than just wages

Recall that

$$U_{ij}(W\psi_h) = \alpha log(W\psi_h) + \mu_j^n + v_{ij}^n.$$

To get this in the same units as log wages we can rescale simply by dividing by α

$$U_{ij}(W\psi_h) = log(W\psi_h) + \left(\frac{\mu_j^n + v_{ij}^n}{\alpha}\right).$$

We then do things exactly as the decomposition above (for example only looking at workers) for comparison

Utility Decomposition

(A)		(B)		(C)		(D)	
Total	0.315	Total	0.315	Total	0.315	Total	0.315
No HC	0.303	No HC	0.303	No HC	0.303	No HC	0.303
No Monop	0.284	No Monop	0.284	No Monop	0.284	No Monop	0.284
No Roy	0.218	No Search	0.099	No Comp	0.089	No Comp	0.089
No Search	0.060	No Roy	0.018	No Search	0.068	No Roy	0.002

Next we divide into four groups

- College Men
- College Women
- High School Men
- High School Women

Moment Fit: All Groups

	Col Men		HS Men		Col Women		HS Women	
Moment	Data	Model	Data	Model	Data	Model	Data	Model
Avg. Length Emp. Spell	430	435	379	382	392	391	346	348
	(0.173)		(0.200)		(0.165)		(0.175)	
Avg. Length Nonnemp. Spell	60.1	60.3	80	80	63	63	119	119
	(0.073)		(0.086)		(0.072)		(0.104)	
Avg. Length Job	119	120	104	103	109	109	109	109
	(0.087)		(0.091)		(0.081)		(0.098)	
$E(\widetilde{S}_{i\ell j}\widetilde{h}_{-i\ell j}) \times 100$	1.59	1.61	1.45	1.44	1.08	1.08	1.68	1.67
	(0.004)		(0.003)		(0.003)		(0.003)	
Between Persons$\times 100$	9.88	9.92	6.20	6.23	5.05	5.04	4.49	4.50
	(0.013)		(0.010)		(0.008)		(0.008)	
Between Jobs$\times 100$	3.21	3.21	3.13	3.11	2.55	2.55	2.57	2.55
	(0.007)		(0.006)		(0.006)		(0.006)	
Within Job$\times 100$	1.83	1.83	1.39	1.39	1.44	1.44	1.50	1.50
	(0.004)		(0.003)		(0.014)		(0.003)	
Sample mean w_{it}	4.78	4.78	4.56	4.56	4.48	4.48	4.33	4.33
	(0.000)		(0.000)		(0.000)		(0.000)	
E$(\widetilde{w}_{it}\widetilde{w}_{-it}) \times 100$	4.60	4.61	0.407	0.407	0.203	0.203	1.81	1.81
	(0.004)		(0.002)		(0.002)		(0.002)	
$E(\widetilde{w}_{it}\widetilde{h}_{-it}) \times 100$	1.74	1.74	0.113	0.113	0.083	0.083	0.726	0.726
	(0.001)		(0.001)		(0.001)		(0.001)	
Fraction Wage Drops	0.335	0.342	0.408	0.410	0.409	0.404	0.414	0.494
	(0.000)		(0.000)		(0.000)		(0.000)	

Coeff Exper$\times 100$	4.26	4.38	2.34	2.40	2.62	2.57	1.83	1.79
	(0.008)		(0.009)		(0.007)		(0.010)	
Coeff Exper$^2 \times 1000$	-0.640	-0.648	-0.259	-0.259	-0.352	-0.356	-0.207	-0.211
	(0.002)		(0.002)		(0.002)		(0.002)	
Coef Tenure$^2 \times 1000$	-0.793	-0.759	-0.470	-0.468	-0.493	-0.498	-0.286	-0.287
	(0.005)		(0.006)		(0.005)		(0.006)	
Var(Nonemployment)	7830	7775	12057	12000	8571	8550	22800	22231
	(33.52)		(43.23)		(35.92)		(54.97)	
Cov($\overline{w}_i$, Non-employment)	-1.96	-1.96	-3.02	-3.02	-1.38	-1.38	-1.43	-1.43
	(0.005)		(0.008)		(0.003)		(0.007)	
Var(Employment Dur)	102000	98663	106032	107000	96283	98100	94700	90903
	(54.24)		(71.39)		(65.26)		(69.65)	
Invol Job to Job	0.182	0.182	0.243	0.243	0.086	0.086	0.218	0.218
	(0.010)		(0.011)		(0.007)		(0.012)	

Parameters: All Groups

Parameter	Col Men	HS Men	Col Women	HS Women
d_0	-4.38	-2.76	-3.57	-2.41
	(0.131)	(0.026)	(0.018)	(0.022)
λ^n	1.92	1.14	1.78	0.737
	(0.011)	(0.016)	(0.036)	(0.006)
λ^e	2.59	2.35	2.34	1.57
	(0.039)	(0.015)	(0.037)	(0.093)
E_θ	4.64	4.41	4.35	4.169
	(0.009)	(0.009)	(0.006)	(0.012)
σ_θ	0.264	0.181	0.150	0.134
	(0.005)	(0.003)	(0.001)	(0.001)
σ_ξ	0.095	0.128	0.124	0.143
	(0.011)	(0.001)	(0.000)	(0.001)
f_1	5.067	2.85	0.91	2.92
	(0.199)	(0.198)	(0.007)	(0.394)
f_2	0.202	0.145	0.121	0.103
	(0.006)	(0.002)	(0.003)	(0.001)
f_3	-0.043	-0.015	0.095	0.044
	(0.088)	(0.010)	(0.006)	(0.008)

σ_{v^w}	0.177	0.202	0.201	0.198
	(0.014)	(0.002)	(0.002)	(0.003)
α	15.85	3.07	2.64	4.61
	(1.00)	(0.067)	(0.123)	(0.312)
b_1	0.015	0.012	0.041	-0.001
	(0.033)	(0.001)	(0.007)	(0.003)
$b_2 \times 10$	-0.021	-0.005	-0.006	0.008
	(0.053)	(0.001)	(0.002)	(0.002)
β	0.310	0.791	0.665	0.845
	(0.047)	(0.009)	(0.009)	(0.018)
P^*	0.691	0.451	0.258	0.409
	(0.057)	(0.020)	(0.018)	(0.018)
σ_{ν_0}	0.188	0.358	0.331	0.366
	(0.015)	(0.053)	(0.024)	(0.013)
γ_θ	0.583	-1.221	-0.357	-0.595
	(0.021)	(0.182)	(0.050)	(0.041)
σ_d	2.42	2.25	2.05	1.65
	(0.053)	(0.014)	(0.013)	(0.024)

Decompositions: All Groups

	Col Men	HS Men	Col Women	HS Women
Total	0.141	0.091	0.075	0.065
No HC	0.140	0.087	0.073	0.061
No Monop	0.108	0.076	0.063	0.057
No Roy	0.012	0.008	0.006	0.004
No Search	0.085	0.071	0.060	0.047
No Comp	0.095	0.071	0.056	0.053
No Roy/Search	0.006	0.007	0.005	0.003
No Roy/Comp	0.004	0.003	0.0002	0.002
No Search/Comp	0.080	0.047	0.036	0.029