Human Capital

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Human Capital

Topics

- Returns to Schooling
- On-the-job Training
- Tenure
- Education Production Function

Lets think of skill as endogenous

What makes human capital special?

- Non-tradable
- Not observable; easily measured

Examples:

- Schooling
- OJT (Experience)
- Health
- Migration
- Manners
- Sports

Simplest Possible Model

2 periods

Can buy human capital at cost $\psi(I)$

I represents investment

Rent out human capital at rate R_t

other assets pay $1 + r$

Max

$$u(C_0) + \delta u(C_1)$$

subject to budget constraint

$$\begin{aligned} A_t &= (1+r)A_{t-1} - C_t + R_t H_t - \psi(I_t) \\ A_2 &\geq 0 \\ H_t &= h(I_{t-1}, H_{t-1}) \end{aligned}$$

where

$$\begin{aligned} \frac{\partial h(I,H)}{\partial I} &> 0 \\ \frac{\partial^2 h(I,H)}{\partial I^2} &< 0 \end{aligned}$$

First notice that

- $A_2 = 0$ you can't take it with you
- $I_1 = 0$ for exactly the same reason, no point in investing today with no benefit tomorrow

Taking this into account we can rewrite the budget constraint as:

$$C_0 + \frac{1}{1+r} C_1 \leq A_0 + R_0 H_0 - \psi(I_0) + \frac{1}{1+r} R_1 h(I_0, H_0)$$

Solving for first order conditions we get

$$
\begin{aligned}
u'(C_0) &= \frac{\delta}{1+r} u'(C_1) \\
\psi'(I_0) &= \frac{R_1}{1+r} \frac{\partial h(I_0, H_0)}{\partial I_0}
\end{aligned}
$$

we can rewrite this as

$$
1 + r = \frac{R_1}{\psi'(I_0)} \frac{\partial h(I_0, H_0)}{\partial I_0}
$$

What do we learn from this?

- Rate of return on assets is equal to rate of return on I_0
- Units of I are irrelevant
- Only interest rate and human capital production function matter

General Human Capital and On-the-job Training

Firm has two roles:

- Productive Activity
- Training Workers

Frictionless Markets

Workers can always find another firm

Firm must pay cost of training $\psi(I_0)$

Assume that the value of worker to all firms is $R_t H_t-$lots of firms lots of workers

Let w_t represent wage

Workers and firm can contract on current allocation (w_t, I_t) but not on future

This is because in practice a worker can walk away pretty easily (i.e. slavery or indentured servitude is illegal)

What will contracts look like?

In second period it is pretty clear that firms will only offer a contract $I_1 = 0$.

(worker would never accept a lower wage for higher I_1 so no point)

Thus in the second period the firms will all offer $(R_1 H_1, 0)$

In Period 0:

Firm gets "profit" from worker

$$R_0 H_0 - w_0 - \psi(I_0)$$

Since there is free entry for the contracts we see:

$$R_0 H_0 - w_0 - \psi(I_0) = 0$$

So what contract will the worker want?

They know that profits have to be zero so think of them as choosing (w_0, I_0) to maximize

$$w_0 + \frac{1}{1+r} R_1 h_1 \left(I_0, H_0 \right)$$

subject to

$$R_0 H_0 - w_0 - \psi \left(I_0 \right) = 0.$$

This will be the unique contract you will see in equilibrium (straight forward to show)

Solving for the first order condition we find:

$$\frac{1}{1+r} R_1 \frac{\partial h_1 \left(I_0, H_0\right)}{\partial I_0} = \psi' \left(I_0\right)$$

This is exactly the conditions from before so:

- Investment is optimal
- Workers implicitly pay for the human capital investment through lower wages
- Typically will see higher wages in second period than first period in part because

$$w_0 = R_0 H_0 - \psi \left(I_0\right).$$

In fact we might think that most investment on the job is time

Assume that workers spend I of their time investing

and the rest $(1 - I)$ producing the good

I can write the problem now as choosing I_0 to maximize

$$\begin{aligned} & R_0 H_0 (1 - I_0) + \frac{1}{1+r} R_1 h(I_0, H_0) \\ = \quad & R_0 H_0 - R_0 H_0 I_0 + \frac{1}{1+r} R_1 h(I_0, H_0) \end{aligned}$$

This is exactly the same as before with

$$\psi(I_0) = R_0 H_0 I_0$$

Under the conditions before we get

$$\begin{aligned} w_0 &= R_0 H_0 - \psi(I_0) \\ &= R_0 H_0 (1 - I_0) \end{aligned}$$

The firm pays you for the hours that you actually spend producing the final good

In this case the first order condition is

$$1 + r = \frac{R_1}{R_0} \frac{\partial h(I_0, H_0)}{H_0 \partial I_0}$$

Notice that investment rises with $\frac{R_1}{R_0}$

Specific Human Capital

Skills learned at one firm are not valuable at other firms

The model is the same as above except that in the second period the worker is worth:

- $R_1 H_1$ if he/she stays at the same firm
- $R_1 H_0$ if he/she switches to a different firm

First consider the case in which the firm makes a take it or leave it offers (no bargaining)

The second period wage is thus $R_1 H_0 \ (+\varepsilon)$

Worker will take it

Since the worker knows this they get no benefit from training it is irrelevant

All that matters is w_0

The firm chooses I_0 to maximize

$$\pi = R_1 H_0 - w_0 - \psi(I_0) + \frac{1}{1+r}\left[R_1 h(I_0, H_0) - R_1 H_0\right]$$

This give the familiar first order condition

$$\frac{1}{1+r} R_1 \frac{\partial h_1(I_0, H_0)}{\partial I_0} = \psi'(I_0)$$

We get optimal investment

Since there is free entry

$$w_0 = R_1 H_0 - \psi(I_0) + \frac{1}{1+r}\left[R_1 h(I_0, H_0) - R_1 H_0\right]$$

It is all financed by the firm-they pay the full cost and get the full benefit

Nash Bargaining

Suppose that the worker gets δ of the surplus in the second period

Then the period 1 wage is

$$w_1 = R_1 H_0 + \delta \left(R_1 h \left(I_0, H_0 \right) - R_1 H_0 \right)$$

Suppose further that everyone knows this ahead of time

Once again we can set up the problem as if the worker chooses the contract to maximize his own present value of income

$$w_0 + \frac{1}{1+r}\left[R_1 H_0 + \delta\left(R_1 h\left(I_0, H_0\right) - R_1 H_0\right)\right]$$

subject to the free entry condition:

$$\begin{aligned}
\pi &= R_1 H_0 - \psi\left(I_0\right) - w_0 + \frac{1}{1+r}\left[R_1 h\left(I_0, H_0\right) - w_1\right] \\
&= R_1 H_0 - \psi\left(I_0\right) - w_0 + \frac{(1-\delta)}{1+r}\left[R_1 h\left(I_0, H_0\right) - R_1 H_0\right] \\
&= 0
\end{aligned}$$

The first order condition for I_0 is

$$\frac{\delta R_1}{1+r}\frac{\partial h\left(I_0, H_0\right)}{\partial I_0} = \psi'\left(I_0\right) - \frac{(1-\delta)}{1+r}R_1\frac{\partial h\left(I_0, H_0\right)}{\partial I_0}$$

but this solves to the optimal investment

$$\frac{1}{1+r}R_1\frac{\partial h_1\left(I_0, H_0\right)}{\partial I_0} = \psi'\left(I_0\right)$$

With

$$w_0 = R_1 H_0 - \psi\left(I_0\right) + \frac{(1-\delta)}{1+r}\left[R_1 h\left(I_0, H_0\right) - R_1 H_0\right]$$

Once again we will see optimal investment

Can set up bargaining problems with inefficient investment

For example suppose workers are risk averse and uncertain about δ

Other things will change model a bit as well:

- costs of switching
- exogenous separations
- borrowing constraints on workers

Acemoglu and Pischke go through some of these

Is there really specific and general human capital?

Probably not: very few skills are either purely general or purely specific.

We will return to this.

Schooling

Think about school in this framework

Often schooling is not a continuous decision, you are either in school or you are not

With two periods, the question is simply whether you spend the first period in school or not

Receive

$$R_0 H_{0,0} + \frac{1}{1+r} R_1 H_{0,1}$$

if no school

$$0 - T + \frac{1}{1+r} R_1 H_{1,1}$$

if attend school

All we do is compare the two profiles

This is really just a Roy model

This is the same as model above with

$$\begin{aligned} I &\in \{0, 1\} \\ \psi(I) &= R_0 H_{0,0} + T \end{aligned}$$

How do we measure "return to schooling?"

For a 2 period model this is straight forward

Cost of asset is $R_0 H_{0,0} + T$

Future payment is $R_1 H_{1,1} - R_1 H_{0,1}$

Thus we can write the return as:

$$\frac{R_1 H_{1,1} - R_1 H_{0,1}}{R_0 H_{0,0} + T}$$

Notice that people are indifferent between investing or not if

$$\begin{aligned} R_0 H_{0,0} + \frac{1}{1+r} R_1 H_{0,1} &= -T + \frac{1}{1+r} R_1 H_{1,1} \\ (1+r) &= \frac{R_1 H_{1,1} - R_1 H_{0,1}}{R_0 H_{0,0} + T} \end{aligned}$$

This all seems nice and clean-but it isn't really

Now suppose there are three periods,

if no school

$$R_0 H_{0,0} + \frac{1}{1+r_1} R_1 H_{0,1} + \frac{1}{(1+r_1)(1+r_2)} R_2 H_{0,2}$$

if school

$$-T + \frac{1}{1+r_1} R_1 H_{1,1} + \frac{1}{(1+r_1)(1+r_2)} R_2 H_{1,2}$$

Cost of asset in period 0: $R_0 H_{0,0} + T$

Payoff in period 2: $R_2 H_{1,2} - R_2 H_{0,2}$

What about period 1?

Is

$$R_1 H_{1,1} - R_1 H_{0,1}$$

a payoff from the investment or a cost?

If people invest in human capital on the job so that $H_{0,1} > H_{0,0}$ then it is hard to call it just a benefit

Since asset is not tradable, one can not use standard asset pricing formulas

need to compare the lifecycle profiles to each other

Internal Return to Schooling

Define the internal rate of return r_I as

$$\begin{aligned} & R_0 H_{0,0} + \frac{1}{1+r_I} R_1 H_{0,1} + \frac{1}{(1+r_I)^2} R_2 H_{0,2} \\ = \quad & -T + \frac{1}{1+r_I} R_1 H_{1,1} + \frac{1}{(1+r_I)^2} R_2 H_{1,2} \end{aligned}$$

More generally:

$$\sum_{t=0}^{T} \frac{1}{(1+r_I)^t} R_t H_{0,t} = \sum_{t=0}^{S} -\frac{1}{(1+r_I)^t} T_t + \sum_{t=S+1}^{T} \frac{1}{(1+r_I)^t} R_t H_{S,t}$$

Choose to invest in schooling S if $r_I > r$

Don't invest otherwise (assuming only two choices)