

Explaining International Fertility Differences

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Abstract

Why do fertility rates vary so much across countries? Why are European fertility rates so much lower than American fertility rates? To answer these questions we extend the Barro-Becker framework to incorporate the decision to accumulate human capital (that determines earnings) and health capital (that determines life span). We find that cross-country differences in productivity and taxes can go a long way towards explaining the observed differences in fertility and mortality.

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1 Introduction

The Question: Fertility and mortality rates vary considerably across countries. While the average family in the U.S. has 2.1 children and has a life expectancy at age 1 of over 78,

- the average family in Niger has 7.4 children and life expectancy is only 51,
- the average European family has 1.5 children and life expectancy is 78

Our main objective in this paper is to understand the role played by economic forces in the fertility decisions of the typical Niger and European families, and in the allocation of resources that affect life expectancy and schooling. The international evidence is summarized in Figures 1-3.

The Motivation: Differences in fertility and mortality rates can have a very large impact on output per worker. Our previous work (Manuelli and Seshadri (2007a)) suggests that if countries in the bottom decile of the world income distribution were ‘endowed’ with the US demographics, output per worker in these poor countries would more than double. Moreover, demographic changes also affect the dynamics of output and human capital accumulation (Manuelli and Seshadri (2007b)). These results are incomplete since they ignore the endogeneity of demographic variables. In this paper we explore how economic forces affect fertility and mortality.

The Methodology: Our baseline model builds on Barro and Becker (1989), and incorporates some additional features —quantity and quality of schooling and endogenous life spans— that play a major role in the quantitative predictions of the model. Unlike most other papers that endogenize fertility, we

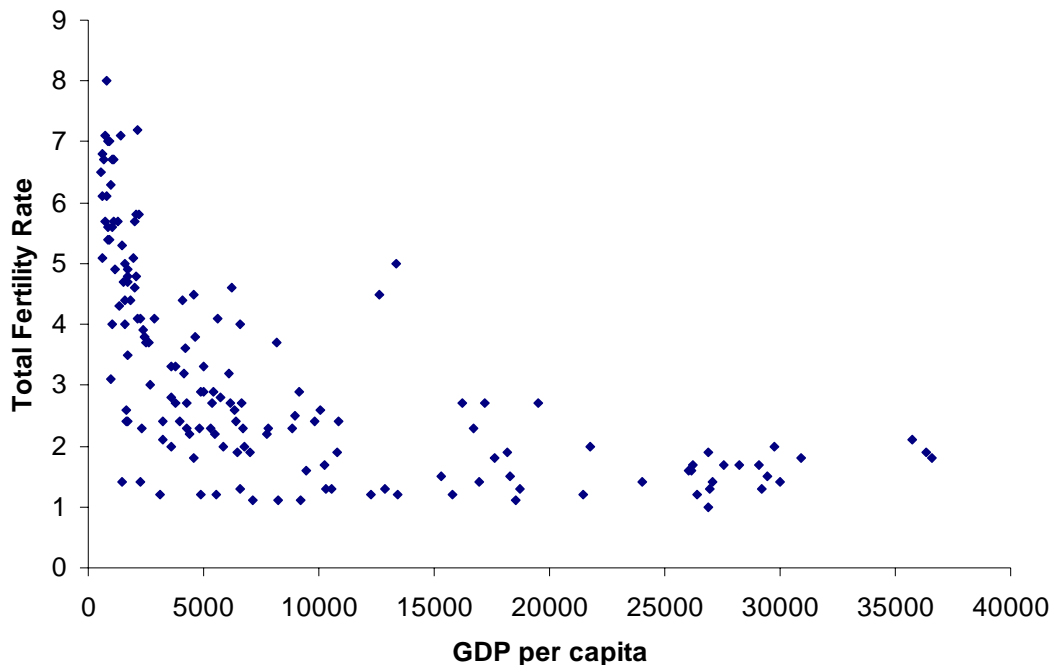


Figure 1: Income and Fertility, 2000

go beyond the more common two or three period overlapping generations set-up and incorporate the full life-cycle of the individual's utility maximization. This permits a more reasonable comparison of the predictions of the model with the data (e.g. years of schooling), and it allows us to use age-earnings profiles to pin down the parameters of the production function.

We also we depart from the literature in the way we model human capital. We follow the tradition in labor economics and use the ideas in Ben-Porath (1967) and Mincer (1974) to model human capital accumulation. A critical distinction with other work in this area is that we do not rely on external effects (see Becker, Murphy and Tamura, 1990) or on shifts in the technology



Figure 2: Life Expectancy and Fertility, 2000

that produces human capital (see Lucas, 2002).

We generalize the original Barro-Becker formulation by letting individuals endogenously determine their life span. Thus, as in Galor (2005) and Acemoglu and Robinson (2006) we consider the interaction between mortality and development.

Our approach relies on differences in productivity (and in tax rates) to explain differences in fertility, education and life span. There is a large literature that emphasizes other factors to explain changes in fertility. For example, Kalemli-Oczan (2002) and Soares (2005) study the role of declining mortality rates; Doepke (2004) stresses the impact of the introduction of skill-

intensive production technologies on fertility choice and Fernandez-Villaverde (2001) emphasizes the role played by the decline in the relative price of capital. We view our work as complementing those explanations.

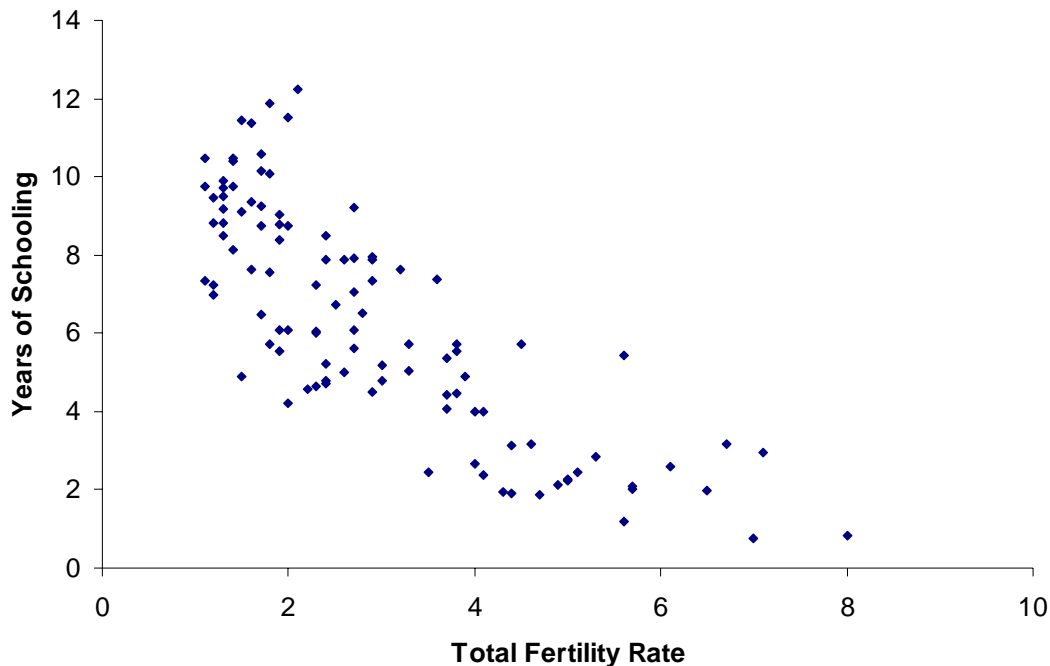


Figure 3: The Quantity-Quality Trade-off, 2000

The Mechanism: As in all versions of the Barro-Becker model, in the steady state — and in the absence of financial market imperfections — there is a positive relationship between the interest rate and fertility. This relationship is one useful tool for understanding the macro channel through which fertility is affected by the exogenous sources of variation that we consider: TFP, retirement age, taxes, and the timing of transfers. It is instructive to analyze the impact of each of these in the context of our model.

a. *Changing TFP.* When total factor productivity goes up, wages rise. This leads parents to invest more in the human capital and the health capital of their progeny (and in their own human capital). This increases the marginal cost of having children measured in consumption units relative to the marginal benefit — which is proportional to consumption — and results in lower fertility. From a macro perspective, the increase in TFP shifts the demand for capital (due to a wealth effect driven by higher consumption in the retirement period). In equilibrium, the capital-human capital ratio increases. This increase is sufficiently strong to bring the interest rate (and fertility) down.

b. *Changing the Retirement Age.* An increase in the retirement age increases investment in human capital and the present value of net labor income. This, in turn, increases the demand for health capital more than it increases the demand for consumption, provided that the demand for health capital is sufficiently responsive to income (as in this paper). The net result is that the cost of the marginal child increases more than the marginal benefit, and fertility declines.

c. *Changing Taxes.* When the tax rate on labor income goes up, individuals reduce their investments in human capital. The present discounted value of net income associated with human capital investment decreases relative to the decrease in consumption (which is driven by just the negative income effect). This results in a decrease in the marginal benefit of having an additional child that exceeds the decrease in the marginal cost. Consequently, fertility declines. From a macro perspective, the decrease in aggregate human capital corresponds to an increase in the capital-human capital ratio, which,

in turn lowers the interest rate and fertility.

d. *Timing of Transfers.* We consider the effect of using lump sum taxes to finance payments to older individuals. Since, in equilibrium, the population growth rate falls short of the interest rate, any redistribution toward the old results in a decrease in the present discounted value of the net transfers. Consequently, household income falls and this reduces the demand for health capital more than the demand for consumption (again driven by the higher income elasticity of the demand for health capital). Since health capital is an element of the marginal cost of a child, while the benefit is proportional to consumption, additional children become more attractive and fertility increases. From an aggregate perspective, more income in the retirement period reduces the aggregate demand for capital. This lowers the capital-human capital ratio and results in higher interest and fertility rates.

How can these forces explain the cross-country differences in fertility? In our model, rich and poor nations are distinguished only by the level of productivity. Since TFP is higher in the US relative to the poor nations so is life expectancy and fertility. This productivity differential alone accounts for most of the large fertility differences between the United States and poorer nations. In order to make some progress understanding the differences between European countries and the U.S. we study the role of taxes. In the model, higher taxes on labor income (as in Europe) result in lower fertility rates.

Our findings provide renewed support for the use of the Barro-Becker framework. Earlier work had questioned the usefulness of this framework — Fernandez-Villaverde (2005) concludes that the quantitative effects of TFP

differences are very small in the Barro-Becker world, while Boldrin, DeNardi and Jones (2005) argue that the impact of social security on fertility goes in the ‘wrong’ direction in the Barro-Becker world. We show that the addition of human capital á la Ben-Porath and taxes on labor income into the Barro-Becker framework sets these predictions right. Quantitatively, the model does a remarkable job at capturing cross country differences in birth rates. It can also account for the US-Europe birth rate differentials.

2 Economic Environment:

In this section we describe the basic model. We study an economic environment with imperfect altruism and we analyze the choices of quantity and quality of children, as well as lifespan. We show that, with perfect markets, “quality” is determined using the usual investment criteria in models with human capital, while “quantity” is also determined by comparing costs and benefits.

2.1 The Individual Household Problem

The representative household is formed at age I (age of independence). At age B , e^f children are born. The period of ‘early childhood’ (defined by the assumption that children are not productive during this period) corresponds to the (parent) age B to $B+6$. The children remain with the household (and as such make no decisions of their own) until they become independent at (parent) age $B+I$. The parent retires at age R , and dies at age T .

Let a denote an individual’s age. Each parent chooses his own consump-

tion, $c(a)$, as well as consumption of each of his children, $c_k(a)$, during the years that they are part of his household, $a \in [0, I)$, to maximize his utility. We adopt the standard Barro-Becker approach, and we specify that parent's utility depends on his own consumption, as well as the utility of his children. In addition to consumption, the parent chooses the amount of market goods to be used in the production of new human capital, $x(a)$, and the fraction of the time allocated to the formation of human capital, $n(a)$ (and, consequently, what fraction of the available time to allocate to working in the market, $1 - n(a)$) for him and each of his children while they are still attached to his household. The parent also decides to make investments in early childhood, which we denote by x_E (e.g. medical care, nutrition and development of learning skills), that determine the level of each child's human capital at age 6, $h_k(6)$, or h_E for short, and the amount of market goods, g_k , allocated to the production of health capital. While human capital is used to produce income, health capital is used to produce lifespan. Finally, the parent chooses how much to bequeath to each children at the time they leave the household, b_k . We assume that each parent has unrestricted access to capital markets, but that he cannot commit his children to honor his debts. Thus, we restrict b_k to be non-negative.

The utility function of a parent who has h units of human capital, and a bequest equal to b at age I is given by

$$\begin{aligned}
 V^P(h, b, g) = & \int_I^{T(g)} e^{-\rho(a-I)} u(c(a)) da + e^{-\alpha_0 + \alpha_1 f} \\
 & \int_0^I e^{-\rho(a+B-I)} u(c_k(a)) da + e^{-\alpha_0 + \alpha_1 f} e^{-\rho B} V^k(h_k(I), b_k, g_k)
 \end{aligned} \tag{1}$$

Thus, the contribution to the parent's utility of an a year old child still

attached to him is $e^{-\alpha_0 + \alpha_1 f} e^{-\rho(a+B-I)} u(c_k(a))$, since at that time the parent is $a + B$ years old. In this formulation, $e^{-\alpha_0 + \alpha_1 f}$ captures the degree of altruism. If $\alpha_0 = 0$, and $\alpha_1 = 1$, this formulation corresponds to a standard infinitely-lived agent model. Positive values of α_0 , and values of α_1 less than 1 capture the degree of imperfect altruism. The term $V^k(h_k(I), b_k, g_k)$ is the utility of a child at the time he becomes independent.

Each parent maximizes $V^P(h, b, g)$ subject to two types of constraints: the budget constraint, and the production function of human capital. The former is given by

$$\begin{aligned} & \int_I^{T(g)} e^{-r(a-I)} c(a) da + e^f \int_0^I e^{-r(a+B-I)} c_k(a) da + \int_I^R e^{-r(a-I)} x(a) da + \quad (2) \\ & e^f \int_6^I e^{-r(a+B-I)} x_k(a) da + e^f e^{-rB} b_k + e^f e^{-r(B+6-I)} x_E + e^f e^{-r(B-I)} g_k \\ \leq & \int_I^R e^{-r(a-I)} w h(a) (1 - n(a)) da + e^f \int_6^I e^{-r(a+B-I)} [w h_k(a) (1 - n_k(a))] da + b. \end{aligned}$$

We adopt Ben-Porath's (1967) formulation of the human capital production technology, augmented with an early childhood period. We assume that

$$\dot{h}(a) = z_h [n(a)h(a)]^{\gamma_1} x(a)^{\gamma_2} - \delta_h h(a), \quad a \in [6, R] \quad (3)$$

$$h_k(6) = h_B x_E^v, \quad 0 < \gamma_i, v < 1, \quad \gamma = \gamma_1 + \gamma_2 < 1, \quad (4)$$

The technology to produce human capital of each child at the beginning of the potential school years, $h_k(6)$ or h_E is given by (4). Our specification captures the idea that nutrition and health care are important determinants of early levels of human capital, and those inputs are, basically, market goods. Equation (3) corresponds to the standard human capital accumulation model initially developed by Ben-Porath (1967). For a parent who starts indepen-

dent life at age I , the initial human capital (chosen by his parent), $h(I)$, is given.

The function $T(g)$ gives the mapping between expenditures on health—which, for simplicity, we assume take place at the time of birth—and life expectancy.¹ We assume that this function is increasing concave and bounded.

In the steady state, it is possible to separate the optimal consumption decision from the optimal human capital accumulation decision. In particular, the optimal choice of bequests requires that (in the interior case)

$$e^{-\alpha_0 + \alpha_1 f} e^{-\rho B} \frac{\partial V^k(h_k(I), b_k, g_k)}{\partial b} = \Phi e^f e^{-rB},$$

where Φ is the Lagrange multiplier associated with the budget constraint.

Since, in the steady state, it must be the case that

$$\frac{\partial V^k(h_k(I), b_k, g_k)}{\partial b} = \frac{\partial V^P(h, b, g)}{\partial b} = \Phi,$$

it follows that

$$r = \rho + [\alpha_0 + (1 - \alpha_1)f]/B. \quad (5)$$

Thus, as in all the Barro-Becker type of models, there is a one to one mapping between the fertility rate and the interest rate: High fertility countries are also high interest rate countries. However, the elasticity of the market discount factor with respect to the fertility rate is $(1 - \alpha_1)/B$ which can be small.

If the non-negativity constraint on bequests is not binding, the standard separation result obtains: any allocation that maximizes utility should also

¹Given that we do not restrict borrowing and lending, it is possible to view g as the present discounted value of health expenditures.

maximize income. It follows that, to determine equilibrium human capital investment, it is sufficient to focus attention on the problem of maximizing the present discounted value of income. (For details see the Appendix) In our case, the result is a little more delicate as income early in life is appropriated by parents and, even though the model resembles an infinite horizon model, the notion of income that is maximized is just lifetime income.

An intuitive (and heuristic) argument that shows the correspondence between the utility maximization and the income maximization problem is as follows: Suppose that parents (who make human capital accumulation decisions for their children until age I) do not choose investments in human capital to maximize the present value of income of their children (only part of which they keep). In this case, and since $b_k > 0$, the parent could increase the utility of each child by adopting the income maximizing human capital policy and adjusting the transfer to finance this change. It follows that the cost to the parent is the same and the child is made better off.

In the unconstrained case, it is possible to fully characterize the solution to the income maximization problem. The main features of the solution are (see Manuelli and Seshadri (2007a) for details):

1. The optimal allocation of time implies that $n(a) = 1$ (the individual spends all his time producing human capital) for a finite number of years. This period, whose length we denote by s , corresponds to years of schooling.
2. For $a > 6 + s$, the individual is working but he continues to invest — at lower rates — in human capital.

3. Higher wages result in more schooling and in an increase in the amount of human capital per year of schooling.

Given the interest rate, retirement age and wage rate, human capital is independent of fertility decisions (in the unconstrained case). Thus, any effect of fertility upon quality is driven by general equilibrium effects.

In order to characterize the solution to the household problem, we need to describe the optimal choice of consumption (this is standard) and the optimal choice of fertility. The first order condition corresponding to the optimal choice of f is

$$\begin{aligned} & \alpha_1 e^{-\alpha_0 + \alpha_1 f} e^{-\rho B} \left[\int_0^I e^{-\rho(a-I)} u(c_k(a)) da + V^k(h_k(I), b_k, g_k) \right] \quad (6) \\ = & \Phi e^f e^{-rB} \left[\int_0^I e^{-r(a-I)} c_k(a) da + e^{-r(6-I)} x_E + b_k + g_k e^{rI} \right. \\ & \left. - \int_6^I e^{-r(a-I)} (wh_k(a)(1 - n_k(a)) - x_k(a)) da \right]. \end{aligned}$$

The interpretation is simple. The left hand side corresponds to the marginal benefit of a child. It is given by his utility multiplied by the effective discount factor. The right hand side corresponds to the cost — measured in utility units — of an additional child. This cost is the sum of consumption expenditures, investment in early childhood capital and health capital, bequests and net income. Note that both the costs and the potential benefits (i.e. if net income is positive) are considered only during the period that the child spends attached to his parent.

In the steady state, it must be the case that

$$V^k(h_k(I), b_k, g_k) = V(h, b, g), \quad h_k(I) = h, \quad b = b_k, \quad g_k = g.$$

Moreover, in the steady state, the effective discount factor, $-\alpha_0 + \alpha_1 f - \rho B$, equals $f - rB$. Using these steady state restrictions, (6) is

$$\begin{aligned} & \frac{\alpha_1}{1 - e^{f-rB}} \left[\int_0^{T(g)} e^{-\rho(a-I)} u(c(a)) da \right] \frac{1}{u'(c(I))} \\ &= \left[\int_0^I e^{-r(a-I)} [c(a) - (wh(a)(1 - n(a)) - x(a))] da + e^{-r(6-I)} x_E + b + e^{rI} g_k \right]. \end{aligned} \quad (7)$$

The left hand side of (7) — the benefit of an additional child — is the goods equivalent of the an infinite sequence of life cycle utility using the effective discount factor ($f - rB$) to discount future flows multiplied by the semi-elasticity of the value of a child (α_1). The right hand side contains the same cost items that discussed before: consumption and expenditures in human and health capital and transfers net of child labor.

The optimal choice of health capital satisfies

$$e^{-\rho(T(g)-I)} u(c_k(T(g))) T'(g) = u'(c(I)) e^{rI}. \quad (8)$$

The second order condition requires that the left hand side of (8) be a decreasing function of g . Given that this is satisfied (more on this later), the condition implies that decreases in the marginal utility of income — for example driven by increases in productivity — result in increases in health capital and longer life expectancy.

To obtain a more intuitive characterization of the solution, we assume that the utility function is isoelastic and given by,

$$u(c) = \frac{c^{1-\theta}}{1-\theta}.$$

The optimal choice of consumption is given by

$$c(a) = c(I) e^{\frac{r-\rho}{\theta}(a-I)}, \quad a \in [0, T(g)] \quad (9a)$$

To compute the right hand side of (7), we need the equilibrium values of the endogenous variables.²

For $a \geq 0$ let net income be defined as $y(a) = wh(a)(1 - n(a)) - x(a)$. Given our demographic structure, $y(a)$ satisfies

$$y(a) = \begin{cases} 0 & 0 \leq a < 6 \\ -x_E & a = 6 \\ -x(a) & 6 < a \leq 6 + s \\ wh(a)(1 - n(a)) - x(a) & 6 + s < a \leq R \\ 0 & R < a \leq T(g) \end{cases}$$

Similarly, let $h^e(a)$ be the effective supply of human capital by an individual of age a . It follows that,

$$h^e(a) = \begin{cases} 0 & 0 \leq a < 6 + s \\ h(a)(1 - n(a)) & 6 + s < a \leq R \\ 0 & R < a \leq T(g) \end{cases} .$$

Finally, $\tilde{x}(a) = h^e(a) - y(a)$.

Manipulation of the first order conditions, imposing that, in the steady state, $b_k = b$ in the budget constraint (2), and substituting in the first order condition corresponding to the optimal choice of health capital (8) implies that (7) can be written as

$$\bar{B}(g, r) \equiv (\alpha_1 + \theta - 1) \frac{1 - e^{-\lambda(r)T(g)}}{\lambda(r)T'(g)} = g - L(y, r), \quad (10)$$

where

$$\lambda(x) \equiv \frac{r - \rho}{\theta} - x,$$

²For completeness, in the Appendix we present the solution to the income maximization problem.

and, for any function $m(a)$ and discount factor q , the present value operator is defined by

$$L(m, q) \equiv \int_0^{T(g)} e^{-qa} m(a) da$$

The left hand side of (10) gives the marginal benefit of a child (net of his consumption cost) as a function of the amount invested in health capital. The right hand side gives the marginal cost (net of consumption).

As in Barro and Becker's model, a necessary condition for the marginal benefit of a child to be positive (and, hence, for the model to predict non-zero fertility) is that $\theta + \alpha_1 > 1$. This condition simply says that, at the margin, each household wants to have some children. If this condition is violated, each household's utility increases as consumption per child is increased and the number of children decreases. To guarantee that the demand for children is finite, it must be that the cost of an additional child is positive. In our formulation, this requires that the (present discounted value) of investments in life extension must exceed the (present discounted value) of net labor income.

Equation (10) highlights the factors that affect the cost and benefits of an additional child. The net benefit — the left hand side of (10) — increases with the amount of health capital (if $\lambda(r) < 0$, as required by efficiency, and a condition satisfied in all the equilibria we compute). The marginal cost of a child depends positively on expenditures in health and negatively on the present discounted value of net labor income.

The relationship between health capital and consumption is given by the appropriate version of (8)

$$c(I) e^{-\frac{r-\rho}{\theta} I} e^{\lambda(r)T(g)} T'(g) = (1 - \theta), \quad (11)$$

The second order condition for a maximum requires

$$\lambda(r)(T'(g))^2 + T''(g) \leq 0.$$

This formulation is intuitively plausible. The second order condition guarantees that the function $e^{\lambda(r)T(g)}T'(g)$ is decreasing in g .

2.2 Equilibrium

Given individual decisions on human capital accumulation and investment as a function of age, all we need is to compute the age structure of the population to determine aggregate human capital. The capital-human capital ratio is pinned down by the condition that the marginal product of capital equal the cost of capital, and this suffices to determine output per worker.

Demographics Since we consider only steady states, we need to derive the stationary age distribution of this economy. Let $N(a, t)$ be the number of people of age a at time t . Thus, our assumptions imply

$$N(a, t) = e^f N(B, t - a)$$

and

$$N(T(g), t) = 0, \quad t > T(g)$$

It is easy to check that, in the steady state,

$$N(a, t) = \phi(a)e^{\eta t}, \tag{12}$$

where

$$\phi(a) = \eta \frac{e^{-\eta a}}{1 - e^{-\eta T(g)}}, \tag{13}$$

and $\eta = f/B$ is the growth rate of population.

Equilibrium From (5) it follows that if the bequest constraint is not binding, the interest rate is given by

$$r = \rho + \frac{\alpha_0}{B} + (1 - \alpha_1)\eta. \quad (14)$$

Optimization on the part of firms implies that

$$p_k(r + \delta_k) = zF_k(\kappa, 1), \quad (15)$$

where κ is the physical capital - human capital ratio. The wage rate per unit of human capital, w , is,

$$w = zF_h(\kappa, 1). \quad (16)$$

Aggregate output and consumption per person satisfy

$$\begin{aligned} & \int_0^{T(g)} c(a)\phi(a)da + \phi(0)g \\ &= [zF(\kappa, 1) - (\delta_k + \eta)\kappa p_k] \int_0^{T(g)} h^e(a)\phi(a)da - \int_0^{T(g)} \tilde{x}(a)\phi(a)da, \end{aligned}$$

which can be expressed as

$$c(I)e^{-\frac{r-\rho}{\theta}I} \frac{e^{\lambda(\eta)T(g)} - 1}{\lambda(\eta)} = -g + L(y, \eta) + (r - \eta)\kappa L(h^e, \eta). \quad (17)$$

This expression shows that aggregate consumption as a function of g and r , denoted \bar{C} , satisfies

$$\bar{C}(g, r) = (1 - \theta) \frac{1 - e^{-\lambda(\eta)T(g)}}{\lambda(\eta)T'(g)} e^{(r-\eta)T(g)} = -g + L(y, \eta) + (r - \eta)\kappa L(h^e, \eta). \quad (18)$$

For this to be an equilibrium, we need to verify that, at the candidate solution, $b > 0$.

The system formed by equations (10) —after substituting in (27a), (27b) and (27c)— (11) and (17) define a solution for the triplet $(c(I), g, f)$ once

the relationship between fertility and prices —as captured in (14), (15), and (16)— is taken into account.

It is possible to use equation (10) to informally discuss the role of TFP. To simplify, let's view the equilibrium level of g as a function of household income, I , the wage rate, w , and the interest rate, r . Consider increasing TFP holding the interest rate constant. If this shock is to decrease fertility, it must be the case that the marginal benefit of a child increases less than the marginal cost. This corresponds to (assuming differentiability)

$$\left[\frac{\partial \bar{B}(g, r)}{\partial g} - 1\right] \frac{dg}{dz} < -\frac{dL(y, r)}{dz}, \quad (19)$$

where

$$\frac{dg}{dz} = \frac{\partial g}{\partial I} \frac{dI}{dz} + \frac{\partial g}{\partial w} \frac{dw}{dz} > 0$$

and

$$\frac{dL(y, r)}{dz} = \frac{\partial L(y, r)}{\partial w} \frac{dw}{dz} > 0.$$

It follows that a necessary condition for (19) to hold is that

$$\frac{\partial \bar{B}(g, r)}{\partial g} < 1,$$

which is equivalent to

$$e^{-\lambda(r)T(g)} + \frac{T''(g)}{\lambda(r)(T'(g))^2} (e^{-\lambda(r)T(g)} - 1) < \frac{1}{\alpha_1 + \theta - 1}. \quad (20)$$

This expression establishes that the impact of TFP changes upon fertility is a quantitative issue. Simple algebra shows that the left hand side of (20) is greater than one.³ Thus, for values of $\alpha_1 + \theta$ close to 2, the inequality will be violated. In order for increases in TFP to induce decreases in the

³To see this, note that the second order condition requires $\lambda(r)(T'(g))^2 + T''(g) < 0$.

number of children, it must be the case that the degree of imperfect altruism is sufficiently low (i.e. α_1 small). Equation (19) also highlights the role played by the response of health capital to increases in income: if the term $\partial g/\partial I$ is sufficiently large, then increases in TFP are more likely to lower fertility.

3 Calibration

We use standard functional forms for the utility function and the final goods production function. As indicated before, the utility function is assumed to be of the CRRA variety

$$u(c) = \frac{c^{1-\theta}}{1-\theta}, \quad 0 < \theta < 1.$$

The production function is assumed to be Cobb-Douglas

$$F(K, H) = zK^\alpha H^{1-\alpha}.$$

We assume that the mapping between health capital and life expectancy is

$$T(g) = \bar{T}(1 - e^{-\mu g}), \quad \mu > 0.$$

Our calibration strategy involves choosing the parameters so that the steady state implications of the model economy presented above match the appropriate moments for the United States (circa 2000). The only exogenous variable that we allow to differ across economies is the level of TFP. It is chosen so that the model's predictions for output per worker match the observed value for each group (decile) of countries. Consequently, while the model is silent about output per worker, it makes predictions about years

of schooling, expenditures in health capital, expenditures on education and lifespan.

We set some parameters consistent with the values commonly accepted in the macro literature. Thus, following Cooley and Prescott (1995), the discount factor is fixed at $\rho = 0.04$ and the depreciation rate is set at $\delta_k = .06$. Capital's share of income is set at 0.33. Less information is available on the fraction of job training expenditures that are not reflected in wages. Following Manuelli and Seshadri (2007a) we assume that half of the investments in human capital in the post-schooling period are not recorded as such in the NIPA. The parameter α_1 determines the degree of curvature in the altruism function of the individual. We proceed by choosing the level of α_1 in the United States so as to match a fertility rate (corresponding to $2 \times e^f$ in the model) of 2.1. Finally, we assume that $B = 25$.

Our theory implies that it is only the ratio $h_B^{1-\gamma}/(z_h^{1-\nu}w^{\gamma_2-\nu(1-\gamma_1)})$ that matters for the moments of interest. Consequently, we can choose z , p_k (which determine w) and h_B arbitrarily and calibrate z_h to match a desired moment. The calibrated value of z_h is common to all countries. Thus, the model does not assume any cross-country differences in an individual's 'ability to learn' or in the ability to produce life span. This leaves us with 10 parameters, $\alpha_0, \delta_h, z_h, \gamma_1, \gamma_2, \nu, \alpha_1, \theta, \bar{T}$ and μ .

The moments we seek in order to pin down these parameters are:

1. Earnings at age R /Earnings at age 55 of 0.8. Source: SSA
2. Earnings at age 50/Earnings at age 25 of 2.17. Source: SSA
3. Years of schooling of 12.08. Source: Barro and Lee, 2000

4. Schooling expenditures as a percentage of GDP of 3.77%. Source: OECD, Education at a Glance, 2003
5. Pre-primary expenditures per pupil relative to GDP per capita of 0.14. Source: OECD, Education at a Glance, 2003
6. Fertility Rate of 2.1. Source: UNDP
7. Lifetime Intergenerational Transfers/GDP of 4.5%. Gale and Scholz, 1994
8. Capital output ratio of 2.52. Source: NIPA
9. Health Expenditures/GDP of 10%
10. Life Span of 78 years

Theory implies that when bequests are in the interior, the human capital allocations that result from the solution to the parent's problem correspond to the allocations that result from the simpler income maximization problem. Consequently, proceed in two steps since the 10 equations in 10 unknowns are 'block-separable'. For a given real interest rate and wage rate, we calibrate the parameters $\delta_h, z_h, \gamma_1, \gamma_2$ and ν so as to match moments 1, 2, 3, 4 and 5. Thus we follow a long-standing tradition in labor economics and use the properties of the age-earnings profile to identify the parameters of the production function of human capital. We then choose the other five parameters so as to match moments 6,7, 8, 9 and 10.

The parameter values that result in a perfect match between model and data are

Parameter	α_0	δ_h	z_h	γ_1	γ_2	ν	α_1	θ	\bar{T}	μ
Value	0.24	0.018	0.361	0.63	0.3	0.55	0.65	0.62	101.2	0.71

Of some interest are our estimates of α_i . Since α_0 is positive and α_1 is less than one, our individuals are imperfectly altruistic. Further, $\alpha_1 + \theta > 1$, which is necessary for the existence of an equilibrium with positive fertility.

4 Results

Before turning to the results, we first describe the data so as to get a feel for the observations of interest. We start with the countries in the PWT 6.1 and put them in deciles according to their output per worker, y . Next, we combine them with observations on years of schooling (s), expenditures on schooling —primary and secondary— relative to GDP (x_s), life expectancy at age 1 (T), and the total fertility rate ($2 \times e^f$) for each of these deciles. The population values are displayed in the following table.

Table 1: World Distribution					
	Relative output	Schooling		Life Span	Fertility
		Years	Expenditures		TFR*(1-inf)
Decile	$\frac{y}{y^{US}}$	s	x_s	T	$2 \times e^f$
90-100	0.921	10.93	3.8	78	1.74
80-90	0.852	9.94	4.0	76	2.1
70-80	0.756	9.72	4.3	73	2.28
60-70	0.660	8.70	3.8	71	2.50
50-60	0.537	8.12	3.1	69	2.82
40-50	0.437	7.54	2.9	64	3.37
30-40	0.354	5.88	3.1	57	3.92
20-30	0.244	5.18	2.7	54	4.76
10-20	0.146	4.64	2.5	51	5.32
0-10	0.052	2.45	2.8	46	5.66

Table 1 illustrates the wide disparities in incomes across countries. The United States possesses an output per worker that is about 20 times as high as countries in the bottom decile. Years of schooling also vary systematically with the level of income — from about 2.5 years at the bottom deciles to about 11 at the top. The quality of education, as proxied by expenditures on primary and secondary schooling as a fraction of GDP, also seems to increase with the level of development. This measure should be viewed with a little caution as it includes only public inputs and not private inputs (including the time and resources that parents invest in their children). Next, notice that demographic variables also vary systematically with the level of development

- higher income countries enjoy greater life expectancies and lower fertility rates. More important, while demographics vary substantially at the lower half of the income distribution, they do not move much in the top half.

4.1 Accounting for International Differences in Fertility

We now examine the ability of the model to simultaneously match the cross country variation in output per capita and years of schooling. To be clear, we choose the level of TFP in a particular country/decile so as to match output per worker.⁴ We then see if the predictions for the fertility rate, life span and schooling are in accordance with the data. In the process we do not assume that the solution is interior; we allow for the constraint on the nonnegativity of bequests to be binding and, in fact, it binds for the poorer countries.

⁴We assume that $R = \min\{64, T\}$.

Table 2: Fertility, Life Expectancy and Schooling - Data and Model										
Decile	$\frac{y}{y^{US}}$	TFP	s		x_s		$2 \times e^f$		T	
			Data	Model	Data	Model	Data	Model	Data	Model
90-100	0.921	0.99	10.93	11.24	3.8	3.73	1.74	2.22	78	76
80-90	0.852	0.98	9.94	10.56	4.0	3.86	2.1	2.31	76	74
70-80	0.756	0.96	9.72	10.11	4.3	3.94	2.28	2.49	73	72
60-70	0.660	0.94	8.70	8.92	3.8	4.12	2.50	2.73	71	68
50-60	0.537	0.92	8.12	7.96	3.1	4.54	2.82	2.99	69	67
40-50	0.437	0.89	7.54	6.44	2.9	4.13	3.37	3.71	64	64
30-40	0.354	0.86	5.88	5.52	3.1	3.83	3.92	3.98	57	59
20-30	0.244	0.83	5.18	4.24	2.7	3.46	4.76	4.58	54	56
10-20	0.146	0.81	4.64	2.94	2.5	2.88	5.32	5.22	51	53
0-10	0.052	0.73	2.45	2.12	2.8	2.29	5.66	5.82	46	48

Table 2 presents the predictions of the model and the data. The model is able to capture reasonably well the variation across countries in the quantity of children, as captured by the fertility rate and the quality of children, as reflected in years of schooling. As we move from the bottom to the top decile of the world income distribution, fertility in the model decreases from 5.82 to 2.22 which compares very favorably with that observed in the data. The change in life span from 48 years to 76 is also in line with the data. Furthermore, the model also captures the variation in schooling quantity and quality across countries.

We conclude that differences in TFP can go a long way toward accounting for the observed cross-country differences in the quality and quantity of

children.

4.1.1 The Role of Human Capital

The above results show that the model can account for a large part of the observed differences in fertility rates across countries. What is the role played by human capital? In order to examine this, imagine shutting down human capital — specifically, let $z_h = 0$ and re-calibrate the model using moments 6 through 10 in order to pin down the 5 parameters (recall that 5 parameters were specific to the human capital sector). With the new parameters at hand, we can now gauge the sensitivity of the economic environment to changes in TFP just as we did in Table 2. The results for the top, middle and the bottom deciles are presented in Table 3.

Decile	Baseline	$z_h = 0$
90-100	2.22	2.11
40-50	3.71	2.23
0-10	5.82	2.56

Shutting down human capital dramatically lowers the sensitivity of fertility rates to changes in TFP. Specifically, without human capital, the model predicts that birth rates in the lowest decile are only about 2.56, which is far from the 5.66 in the data. In the model, human capital amplifies the effect of changes in TFP affecting the incentives to invest in quality.

4.1.2 The Relationship Between Capital Output Ratios and Fertility Rates

Recall from Equations (14) and (15) that one of the key predictions of a Barro-Becker type model is the tight link between interest rates (or capital output ratios) and total fertility rates (under the assumption of a Cobb-Douglas production function). This suggests a testable implication - the correlation between capital output ratios and total fertility rates (after factoring out infant mortality rates) must be rather strong. Before examining the implications of the model, it is instructive to take a brief look at the data. We obtain investment to GDP ratios (in domestic prices) from the Penn World Tables. We then divide this ratio by the sum of the depreciation rate (which we assume to be .06 in every country) and population growth rate so as to get the capital output ratio.

The correlation between fertility rates and capital output ratios is -0.69, a strikingly high correlation. In the model, this correlation is -0.78. We find this close association between the model's predictions and the data quite comforting in that this relationship is central to the mechanism at work.

Perhaps more compelling is that in the data, the correlation is lower amongst the bottom four deciles (where the deciles are constructed as above by GDP per worker). This is precisely what the model predicts - the model predicts a correlation of -0.44 while in the data the correlation is -0.31. In poorer countries, the binding non-negative bequest condition breaks the tight link between capital output ratios and fertility rates. Equation (14) does not hold and a given variation in interest rates translates into a smaller variation in fertility rates. Hence the implied correlation between capital output ratios

and fertility rates is smaller. We view this evidence as providing more support in favor of the mechanism at work.

5 Accounting for US-Europe Fertility Differences

The previous section demonstrated the ability of the model to capture the variation in fertility rates across the different stages of development. Nevertheless, there is one glaring failure — the inability to capture the low fertility rate observed in many European countries. Indeed this feature of the data has been puzzling: Why would the US and European countries, which are at similar stages of economic development, have dramatically different fertility rates? In this section we examine the ability of the model to generate such differential behavior in fertility rates using differences in tax rates on labor income as a way of explaining these differences.

Figure 5 shows the marked divergence in the fertility rates of the United States and the European nations starting around 1976.⁵ While American fertility rates increased by more than 17% over the next two decades, European fertility rates fell by a little more than 11%. At the same time, while taxes on labor income in the United States virtually stayed constant, tax rates on labor income in most European countries as well as Japan and Canada went up. Prescott (2003) argues that these higher taxes explain the lower hours

⁵The European nations included in Figure 4 are Finland, France, Germany, Hungary, Italy, Netherlands, Norway, Portugal, Sweden, Switzerland and the UK.

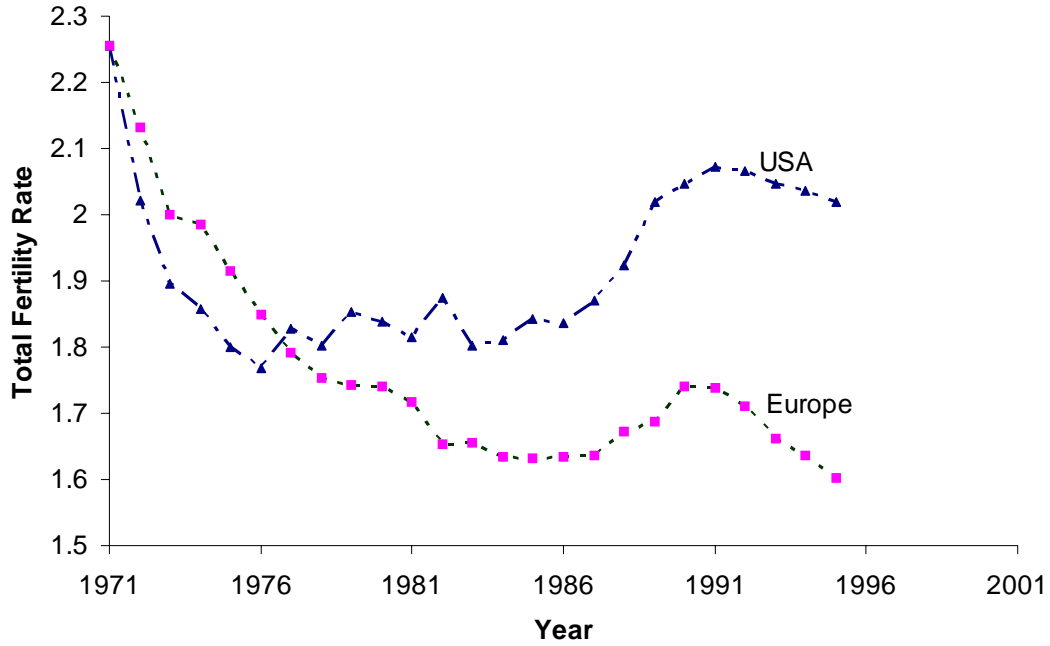


Figure 4: US-Europe Fertility Differences, 1970-1995

worked in Europe relative to the US. Davis and Henrekson (2004) present evidence in support of the negative effect of taxes on labor supply. Table 4 presents data on tax rates, total fertility rates and GDP and the percentage change in these variables between 1975 and 1995.

Table 4: Taxes and Fertility, G7 - Data						
Country	Tax rate on labor income		Total Fertility Rate		GDP per worker	
	1975	% change	1975	% change	1975	% change
Germany	0.54	19	1.77	-28	0.76	41
France	0.43	35	2.36	-29	0.81	39
Italy	0.38	100	2.37	-49	0.81	57
Canada	0.43	34	2.06	-20	0.93	22
U.K.	0.47	-5	2.2	-21	0.71	42
Japan	0.23	67	2.02	-22	0.53	75
U.S.A.	0.45	2	2.1	-3	1.00	41

5.1 The Effects of Taxes on Fertility

To study the effects of taxes on fertility, imagine adding a tax on labor income (τ^h) and capital income (τ^k) into the baseline model. The effective prices that the consumer faces are $\tilde{r} = r(1 - \tau^k)$ and $\tilde{w} = w(1 - \tau^h)$. Further, assume that the revenues from these taxes are rebated back to individuals in a lump-sum fashion.

In order to describe the equilibrium in a model with taxes and transfers, let

$$y^\tau(a) = \begin{cases} 0 & 0 \leq a < 6 \\ -x_E & a = 6 \\ -x(a) & 6 < a \leq 6 + s \\ w(1 - \tau^h)h(a)(1 - n(a)) - x(a) & 6 + s < a \leq R \\ 0 & R < a \leq T(g) \end{cases}$$

and let the transfer received by an individual of age a be denoted $q(a)$. Let

the youngest age that makes a person eligible for the transfer be denoted p . The appropriate version of the condition that equates the net marginal benefit of an additional child with the marginal cost (equation (10)) is

$$\bar{B}(g, r) = g - L(y^\tau, r) - \tilde{L}(q, r, p), \quad (21)$$

where

$$\tilde{L}(m, r, p) \equiv \int_p^{T(g)} e^{-ra} m(a) da.$$

In this formulation, an increase in p corresponds to a redistribution toward the old.

Once taxes are introduced, the remaining equilibrium conditions are

$$\bar{C}(g, r) = -g + L(y, \eta) + \left(\frac{r - (1 - \tau^k)\eta}{1 - \tau^k} \right) \kappa L(h^e, \eta),$$

and the government budget constraint satisfies.

$$(\tau^k r \kappa + \tau^h w) L(h^e, \eta) = \tilde{L}(q, \eta, p).$$

At a heuristic level, it is possible to describe how changes in tax rates, in particular τ^h , affect the benefits and costs of an additional child. As before, assume that all expressions are differentiable, then the condition that implies that taxes and fertility move in opposite directions is

$$\left(\frac{\partial \bar{B}(g, r)}{\partial g} - 1 \right) \frac{dg}{d\tau^h} < -\frac{dL(y^\tau, r)}{d\tau^h} - \frac{d\tilde{L}(q, r, p)}{d\tau^h}.$$

If tax increases result in lower expenditures on health capital ($dg/d\tau^h < 0$), and since a necessary condition for TFP increases to decrease fertility is that $\partial \bar{B}(g, r)/\partial g - 1 < 0$, then it must be the case that increases in τ^h have a sufficiently large (negative) impact on $dL(y^\tau, r)/d\tau^h$ so that the right hand

side of the previous expression is positive.⁶ This is the case in our formulation since human capital accumulation decisions are very responsive to changes in the after tax wage rate.

To evaluate the quantitative effects of changes in the tax rates on human capital, we hold τ_k fixed in what ensues. In order to re-calibrate the model to match the targets for the United States, we set $\tau_k = 0.3$ and $\tau_h = 0.46$.⁷ The parameter values change slightly. Now, imagine changing the tax rate on labor income and solving for the new steady state. The results are in Table 5.

Table 5: Effect of Taxes of Fertility - Model	
τ_h	$2 \times e^f$
0.40	2.10
0.45	1.77
0.50	1.61
0.55	1.42
0.60	1.22

What happens when τ_h rises? An increase in τ_h reduces the effective wage rate thereby leading to a reduction in human capital investment. Hence, the marginal cost of children declines. However, the reduced wage rate, also implies lower consumption for the parent. A rise in the tax rate, increases marginal cost relative to consumption and consequently fertility declines. An

⁶Here we assume the “standard” case in which increases in the tax rate result in increases in revenue and, through the government budget constraint, this increases transfers. Thus, $\partial L(q, r)/\partial \tau^h > 0$.

⁷Our quantitative results do not hinge on $\tau_k = 0.3$.

alternative way to see this is to think about the impact of taxes on aggregate physical and human capital. When τ_h rises, the stock of human capital falls. Furthermore, since the proceeds are rebated back to the consumer in a lump-sum fashion, the individual does not have much of a need to access the capital market in order to smooth the receipts across his life-cycle. Consequently, the stock of physical capital falls by less than that of human capital. This implies that the capital output rises and the real interest rate falls. Correspondingly, the fertility rate must also fall.

5.2 The Effect of Social Security on Fertility

To model the impact of an increase in the generosity of the social security regime financed by taxes on labor income, we proceed in two steps. First, increase taxes and redistribute the proceeds in a lump sum fashion to everyone. Second, change the redistribution scheme to give the proceeds to the retirees. The first stage was completed in the previous section. We now address the second stage.

Consider a marginal version of that exercise. To be precise, let's explore (heuristically) the effect of an increase in p , the minimum age that makes an individual eligible for a transfer. Given revenue \bar{R} , the flow transfer, q , must satisfy

$$\bar{R} \equiv [\tau^h z F_h(\kappa, 1) + \tau^k z F_k(\kappa, 1) \kappa] L(h^e, \eta) = q \int_p^{T(g)} \phi(a) da.$$

The present discounted value of such a transfer is

$$\tilde{L}(q, r, p) = \bar{R} e^{rI} \frac{\int_p^{T(g)} e^{-ra} da}{\int_p^{T(g)} \phi(a) da}.$$

Since $r > \eta$, $d\tilde{L}/dp < 0$. An increase in p increases fertility if it increases the marginal benefit of an additional child more than the marginal cost. From condition (21), it follows that such a policy increases fertility if

$$\left(\frac{\partial \bar{B}(g, r)}{\partial g} - 1\right) \frac{dg}{dp} > -\frac{d\tilde{L}(q, r, p)}{dp}.$$

However, as a first approximation $dL(q, r, p)/dp \approx dI/dp$, and $dg/dp \approx (\partial g/\partial I)(dI/dp)$. Thus, such a redistribution increases fertility if

$$\left[\left(\frac{\partial \bar{B}(g, r)}{\partial g} - 1\right) \frac{\partial g}{\partial I} + 1\right] \frac{dI}{dp} > 0.$$

Since $dI/dp < 0$, it must be the case that the term in square brackets is negative. As before, a large response of health capital to changes in income guarantees that this inequality holds.

To summarize, increases in tax rates decrease fertility, while a redistribution of tax proceeds increases fertility. In order to evaluate the quantitative impact of an increase in a social security like regime, we consider an increase in tax rates with the proceeds allocated to individuals aged 65 or higher. Table 6 displays the results.

Table 6: Effect of Social Security on Fertility - Model	
τ_h	$2 \times e^f$
0.40	2.10
0.45	1.95
0.50	1.84
0.55	1.76
0.60	1.66

Notice that social security has a negative effect on fertility rates. This is driven by the way it is financed — using labor income taxes — and not by the timing of payments. Social security leads to a decrease in fertility rates only because the presence of human capital makes the supply of effective labor elastic. Absent human capital — or if the social security program was financed using lump-sum taxes — there is only one effect at play: a rise in social security receipts would lead to a fall in the stock of capital, which would lead to a fall in the capital output ratio and hence raise the fertility rate. Indeed, this is the argument in Boldrin et. al. (2005). Thus, the addition of human capital, and the major role played by taxation in this version of the Barro-Becker model, implies that more generous social security regimes financed by higher taxes on labor income have a negative net effect on fertility.

5.3 Taking Stock: Fertility change in the G7 countries

The above discussion suggests that distortionary taxes on human capital acquisition can have large effects on fertility choice. We now see whether the model is capable of accounting for the changes in fertility rates between the 70s and the 90s for the US and the G7 countries. We take as a starting point the parameterization (for the U.S.) from the previous section. Then, as we move across countries we vary three things:

- TFP (to match output per worker),
- Taxes on labor income (using the estimates in Prescott (2003)),

- Ratio of Social Security payments to GDP (OECD, Quarterly Labour Force Statistics).

Each of these countries saw a rise in the share of GDP spent on social security payments with some, such as Japan, experiencing almost a threefold rise, while others such as Germany seeing only a modest increase. Table 7 contains the relevant data from the G7 countries, and the predictions of the model.

Country	Tax rate on labor		Total Fertility Rate		Change in Fertility Rate	
	1975	1995	Model - 1975	Data - 1975	Model	Data
Germany	0.54	0.64	1.86	1.77	-0.21	-0.28
France	0.43	0.58	1.92	2.30	-0.32	-0.29
Italy	0.38	0.76	2.23	2.33	-0.61	-0.49
Canada	0.43	0.58	2.11	2.10	-0.21	-0.20
U.K.	0.47	0.45	2.30	2.20	-0.13	-0.21
Japan	0.23	0.38	2.45	2.02	-0.09	-0.22
U.S.A.	0.45	0.46	2.20	2.10	-0.09	-0.03

The match between model and data though not perfect is pretty reasonable, especially given that it abstracts from differences in the prices of many services (e.g. child care subsidies) and transfers (e.g. maternity leave) that may well affect fertility choices. The predictions of the model for 1975 agree with the evidence with two exceptions: France —where the model underpredicts fertility — and Japan —where the model overpredicts fertility.

In all cases the model predicts that the combination of observed tax increases and changes in the social security regime result in fewer children per person. The magnitude of the decrease is in line with the data with two exceptions: Japan and Italy. As before, the errors have different signs, which suggests that other factors that differentiate these countries probably play a major role.

The model implies lower fertility rates are associated with higher capital output ratios. With the exception of the United Kingdom, this is exactly what we see. The correlation between capital output ratios and total fertility rates is -0.72 in the model and -0.59 in the data.

Despite having abstracted away from such differences, we find it remarkable that changes in tax rates and TFP can go a long way towards understanding the puzzling behavior of fertility rates in the richer nations over the last few decades.

6 The U.S.: 1900 vs. 2000

In this section we use the calibrated model to predict life expectancy, fertility and schooling for the U.S. in 1900. To be precise, we take our base calibrated model (with taxes) as a good description of the (steady state) of the U.S. economy circa 2000. We take the (extreme) view that the U.S. economy circa 1900 was in a steady state as well. Clearly, this is not realistic, but accounting for transition effects is beyond the limits of this paper and, in any case, the purpose of this exercise is to evaluate how large changes in taxes and TFP can affect fertility. For that purpose, the steady state assumption is not a

bad approximation.

The only differences between 2000 and 1900 are the tax rates (which are assumed to be 0 in 1900) and TFP (which is chosen to match output per worker). The results of the experiment are in Table 8.

Period	$\frac{y}{y^{US}}$	s		g/y		$2 \times e^f$		T	
		Data	Model	Data	Model	Data	Model	Data	Model
1900	0.19	5.4	4.83	na	0.04	3.8	4.32	52	48
2000	1	12.08	12.08	.10	0.10	2.1	2.1	78	78

The model does a reasonable job of accounting for the changes over the last century. The predictions of the model for fertility, schooling and life expectancy are not perfect but they are reasonably close to the data. If anything, it predicts — relative to the data — larger responses in the endogenous variables as a result of changes in TFP.

7 Conclusions

This paper integrates a life-cycle model of human and physical capital accumulation where life expectancy is endogenous with the Barro-Becker framework. This permits an interesting trade-off between the quantity and quality of children and the quantity and quality of life. The model is able to capture the wide variation in fertility rates seen across the income distribution. Further, the model suggests that a substantial part of the lower fertility rates in the G7 countries are due to their higher labor income tax rates.

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8 Appendix

In this appendix we show that, in the interior case, utility maximization and lifetime income maximization coincide. To be precise, we assume that $r = \rho + [\alpha_0 + (1 - \alpha_1)f]/B$. In this case, the solution to the optimal human capital accumulation corresponding to the maximization of (1) subject to (2)-(4) is identical to the solution of the following income maximization problem

$$\max \int_6^R e^{-r(a-6)} [wh(a)(1 - n(a)) - x(a)] da - x_E \quad (22)$$

subject to

$$\dot{h}(a) = z_h [n(a)h(a)]^{\gamma_1} x(a)^{\gamma_2} - \delta_h h(a), \quad a \in [6, R), \quad (23)$$

and

$$h(6) = h_E = h_B x_E^v \quad (24)$$

with h_B given.

To see this we show that the first order conditions corresponding to both problems coincide. Since the problems are convex, this suffices to establish the result. Consider first the first order conditions of the income maximization problem given the stock of human capital at age 6, $h(6) = h_E$. Let $q(a)$ be the costate variable. A solution satisfies

$$whn \leq q\gamma_1 z_h (nh)^{\gamma_1} x^{\gamma_2}, \quad \text{with equality if } n < 1, \quad (25a)$$

$$x = q\gamma_2 z_h (nh)^{\gamma_1} x^{\gamma_2}, \quad (25b)$$

$$\dot{q} = rq - [q\gamma_1 z_h (nh)^{\gamma_1} x^{\gamma_2} h^{-1} - \delta_h] - w(1 - n), \quad (25c)$$

$$\dot{h} = z_h (nh)^{\gamma_1} x^{\gamma_2} - \delta_h h, \quad (25d)$$

where $a \in [6, R]$. The transversality condition is $q(R) = 0$.

Let Φ be the Lagrange multiplier associated with the budget constraint (2). Then, the relevant (for the decision to accumulate human capital) problem solved by a parent is

$$\begin{aligned} & \max \Phi \left\{ \int_I^R e^{-r(a-I)} [wh(a)(1 - n(a)) - x(a)] da \right. \\ & + e^f \int_B^{B+I} e^{-r(a-I)} [wh_k(a)(1 - n_k(a)) - x_k(a)] da \\ & \left. - e^f e^{-rB} b_k - e^f e^{-r(B+6)} x_E \right\} + e^{-\alpha_0 + \alpha_1 f} e^{-\rho B} V^k(h_k(B+I), b_k), \end{aligned}$$

where, in this notation, a stands for the parent's age. It follows that the first order conditions corresponding to the choice of $[h(a), n(a), x(a), q_p(a)]$ are identical to those corresponding to the income maximization problem (25), including the transversality condition $q_p(R) = 0$ for $a \in [I, R]$. It follows that $q_p(a) = q(a)$. Simple algebra shows that the first order conditions corresponding to the optimal choices of $[h_k(a), n_k(a), x_k(a), q_k(a)]$ also satisfy (25) for $a \in [6, I)$. However, the appropriate transversality condition for this problem is

$$q_k(B+I) = e^{-[\alpha_0 + (1-\alpha_1)f]} e^{-(\rho-r)B} \frac{1}{\Phi} \frac{\partial V^k(h_k(B+I), b_k)}{\partial h_k(B+I)}.$$

However, given (5), and the envelope condition

$$\frac{\partial V^k(h_k(B+I), b_k)}{\partial h_k(B+I)} = \Phi_k q_p(I),$$

evaluated at the steady state $\Phi = \Phi_k$, it follows that

$$q_k(B+I) = q_p(I).$$

Thus, the program solved by the parent (for $a \in [I, R]$) is just the continuation of the problem he solves for his children for $a \in [6, I)$. It is clear that

if (5) does not hold, then there is a ‘wedge’ between how the child values his human capital after he becomes independent, $q_p(I)$, and the valuation that his parent puts on the same unit of human capital, $q_k(B + I)$.

Solution to the Income Maximization Problem In Manuelli and Seshadri (2007a), it is proved that the solution to the income maximization problem is given by the following conditions:

a. Time allocated to human capital accumulation:

$$n(a) = \frac{m(a)^{\frac{1}{1-\gamma}}}{e^{-\delta_h(a-s-6)}m(6+s)^{\frac{1}{1-\gamma}} + \frac{(r+\delta_h)e^{-\delta_h(a-R)}}{\gamma_1\delta_h} \int_{e^{\delta_h(6+s-R)}}^{e^{\delta_h(a-R)}} (1-x^{\frac{r+\delta_h}{\delta_h}})^{\frac{\gamma}{1-\gamma}} dx}, \quad (26)$$

for $a \in [6+s, R]$.

b. Market goods allocated to human capital accumulation:

$$x(a) = \left(\frac{\gamma_2 w}{r + \delta_h} \right) C_h(z_h, w, r) m(6+s)^{\frac{1}{1-\gamma}} e^{\frac{r+\delta_h(1-\gamma_1)}{(1-\gamma_2)}(a-s-6)}, \quad a \in [6, 6+s], \quad (27a)$$

$$x(a) = \left(\frac{\gamma_2 w}{r + \delta_h} \right) C_h(z_h, w, r) m(a)^{\frac{1}{1-\gamma}}, \quad a \in [6+s, R]. \quad (27b)$$

$$x_E = v \left[\frac{\gamma_1^{\gamma_1(1-\gamma_2)} \gamma_2^{\gamma_1\gamma_2} z_h^{\gamma_1} w^{(1-\gamma_1)(1-\gamma_2)}}{(r + \delta_h)^{(1-\gamma_2)}} \right]^{\frac{1}{1-\gamma}} \frac{m(6+s)^{\frac{(1-\gamma_2)}{1-\gamma}}}{e^{(r+\delta_h(1-\gamma_1))s}} \quad (27c)$$

c. Level of human capital of an individual of age a in the post-schooling period (i.e. $a \geq 6+s$):

$$h(a) = C_h(z_h, w, r) \left\{ e^{-\delta_h(a-s-6)} \frac{\gamma_1}{r + \delta_h} m(6+s)^{\frac{1}{1-\gamma}} + \frac{e^{-\delta_h(a-R)}}{\delta_h} \int_{e^{\delta_h(6+s-R)}}^{e^{\delta_h(a-R)}} (1-x^{\frac{r+\delta_h}{\delta_h}})^{\frac{\gamma}{1-\gamma}} dx \right\}, \quad a \in [6+s, R]. \quad (28)$$

d. Stock of human capital at age 6, h_E :

$$h_E = v^v h_B \left[\frac{\gamma_1^{\gamma_1(1-\gamma_2)} \gamma_2^{\gamma_1\gamma_2} z_h^{\gamma_1} w^{(1-\gamma_1)(1-\gamma_2)}}{(r + \delta_h)^{(1-\gamma_2)}} \right]^{\frac{v}{1-\gamma}} \quad (29)$$

$$e^{-v(r+\delta_h(1-\gamma_1))s} m(6+s)^{\frac{v(1-\gamma_2)}{1-\gamma}}$$

e. Supply of human capital to the market by an individual of age a (for $a \geq 6 + s$):

$$h(a)(1 - n(a)) = C_h(z_h, w, r) w \left\{ \gamma_1 e^{-\delta_h(a-6-s)} \frac{m(6+s)^{\frac{1}{1-\gamma}}}{r + \delta_h} \right. \quad (30)$$

$$\left. - \gamma_1 \frac{m(a)^{\frac{1}{1-\gamma}}}{r + \delta_h} + \frac{e^{-\delta_h(a-R)}}{\delta_h} \int_{e^{\delta_h(6+s-R)}}^{e^{\delta_h(a-R)}} [(1 - x^{\frac{r+\delta_h}{\delta_h}})^{\frac{\gamma}{1-\gamma}} dx] \right\}$$

where

$$C_h(z_h, w, r) = \left[\frac{\gamma_2^{\gamma_2} \gamma_1^{\gamma_1} z_h w^{\gamma_2}}{(r + \delta_h)^\gamma} \right]^{\frac{1}{1-\gamma}}.$$

f. Income during the working years satisfies

$$y(a) = C_h(z_h, w, r) w \left\{ \gamma_1 e^{-\delta_h(a-6-s)} \frac{m(6+s)^{\frac{1}{1-\gamma}}}{r + \delta_h} - (\gamma_1 + \gamma_2) \frac{m(a)^{\frac{1}{1-\gamma}}}{r + \delta_h} + \right.$$

$$\left. \frac{e^{-\delta_h(a-R)}}{\delta_h} \int_{e^{\delta_h(6+s-R)}}^{e^{\delta_h(a-R)}} [(1 - x^{\frac{r+\delta_h}{\delta_h}})^{\frac{\gamma}{1-\gamma}} dx] \right\}$$

during the working life (i.e. $6 + s < a < R$).