Search Theoretic Models of Money

Economics 714

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Essential Models of Money

- Hahn (1965): money is essential if it allows agents to achieve allocations they cannot achieve with other mechanisms that also respect the enforcement and information constraints in the environment.

- Why do we care about essential models of money?

- Three frictions that will make money essential:

  1. Double-coincidence of wants problem.

  2. Long-run commitment cannot be enforced.

  3. Agents are anonymous: histories are not public information.

- Money is a consequence of these frictions in trade: medium of exchange.

Three Generations of Models

1. 1 unit of money, 1 unit of good: Kiyotaki and Wright (1993).
2. 1 unit of money, endogenous units of good: Trejos and Wright (1995).
3. Endogenous units of money, endogenous units of good: Lagos and Wright (2005).

Environment

- $[0, 1]$ continuum of anonymous agents.

- Live forever and discount future at rate r.

- $[0, 1]$ continuum of goods. Good i is produced by agent i.

- Goods are non-storable: no commodity money.

- Unit cost of production $c \geq 0$.

Double-Coincidence of Wants Problem

- I do not produce what I like (non-restrictive: home production, specialization).

- iWj: agent i likes to consume good produced by agent j :.
  1. utility $u > c$ from consuming j.
  2. utility 0 otherwise.

- Probabilities of matching:

$$p\left(iWi\right) = 0$$
$$p\left(jWi\right) = x$$
$$p\left(jWi|iWj\right) = y$$

First Generation: Fixed Money and Fixed Good

- Exogenously given quantity $M \in [0, 1]$ of an indivisible unit of storable good.
- Holding money yields zero utility γ: fiat money.
- Initial endowment: M agents are randomly endowed with one unit of money.
- Agents holding money cannot produce (for example because you need to consume before you can produce again).
- We eliminate (non-trivial) distributions.

Trades

- Pairwise random matching of agents with Poisson arrival time α.

- Bilateral trading is important, randomness is not (Corbae, Temzelides, Wright, 2003).

- Upon meeting, agents decide whether to trade. Then, they part company and re-enter the process.

- History of previous trades is unknown.

- Exchange 1 unit of good for 1 unit of good (barter) or 1 unit of money.

Individual Trading Strategies

- Agents never accept a good in trade if he does not like to consume it since it is not storable.
- They will barter if they like the both agents in the pair like each other goods.
- Would they accept money for goods and viceversa?
- We will look at stationary and symmetric Nash equilibria.

Probabilities

- You meet someone with arrival rate α.
- This person can produce with probability $1 - M$.
- With probability x you like what he produces.
- With probability $\pi = \pi_0 \pi_1$ (endogenous objects to be determined) both of you want to trade.
- If $\pi > 0$, we say that money circulates.

Value Functions

- Value functions with money, V_1:

$$rV_1 = \alpha x (1 - M) \pi (u + V_0 - V_1)$$

- Value functions without money, V_0.

$$rV_0 = \alpha x y (1 - M)(u - c) + \alpha x M \pi (V_1 - V_0 - c)$$

- Renormalize $\alpha x = 1$ by picking time units:

$$rV_1 = (1 - M)\pi (u + V_0 - V_1)$$
$$rV_0 = y (1 - M)(u - c) + M\pi (V_1 - V_0 - c)$$

Individual Trading Strategies

- Net gain from trading goods for money:

$$\Delta_0 = V_1 - V_0 - c = \frac{(1 - M)(\pi - y)(u - c) - rc}{r + \pi}$$

- Net gain from trading money from goods:

$$\Delta_1 = u + V_0 - V_1 = \frac{(M\pi + (1 - M)\,y)(u - c) + ru}{r + \pi}$$

Equilibrium Conditions for π_0 and π_1

- Clearly:

$$\pi_j \begin{cases} = 1 \\ \in [0,1] \\ = 0 \end{cases} \quad \text{as } \Delta_j \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases}$$

- Plug those into the individual trading strategies, and check them.

Characterizing π

- Clearly $\Delta_1 > 0$. Hence $\pi_1 = 1$, i.e., the agent with money always wants to trade.

- For π_0, you have

$$\Delta_0 = \frac{(1-M)(u-c)\pi_0}{r+\pi_0} - \frac{(1-M)y(u-c)+rc}{r+\pi_0}$$

- Then, Δ_0 has the same sign as

$$\pi_0 - \frac{rc+(1-M)y(u-c)}{(1-M)(u-c)} = \pi_0 - \widehat{\pi}$$

Multiple Equilibria

- Nonmonetary equilibrium: we have an equilibrium where $\pi_0 = 0$.

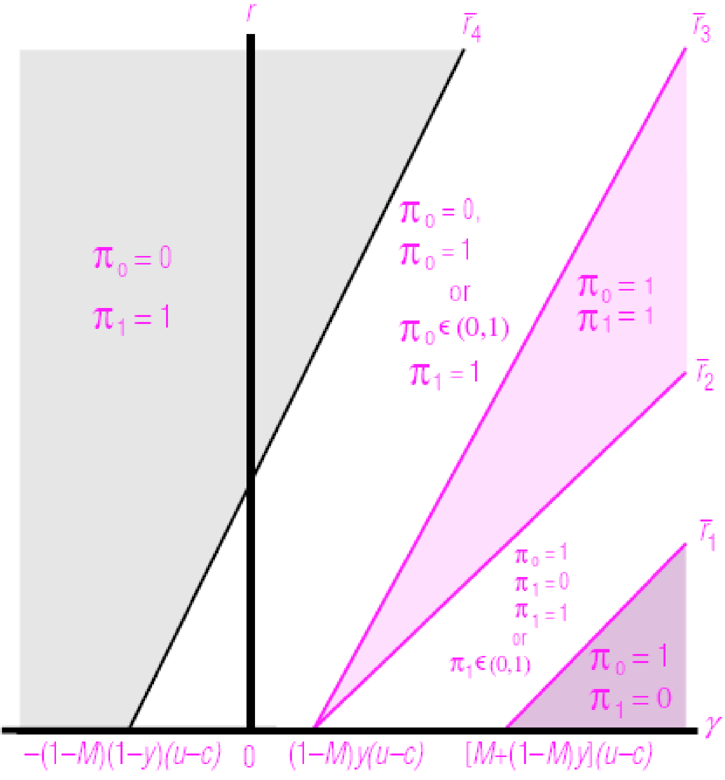
- Monetary equilibrium: if

$$c < \frac{(1-M)(1-y)}{r+(1-M)(1-y)}u$$

then $\widehat{\pi} < 1$ and $\pi_0 = 1$ is an equilibrium as well.

- Mixed-monetary equilibrium: $\pi_0 = \pi^\frown$. However, not robust (Schevchenko and Wright, 2004).

Equilibria in (γ, r)-Space When Money Holders Cannot Produce



Welfare

- Define welfare as the average utility:

$$W = MV_1 + (1 - M)\, V_0$$

- Then:

$$rW = (1 - M)\, [(1 - M)\, y + M\pi]\, (u - c)$$

- Note that welfare is increasing in π.

Welfare $\pi = 1$

- Note:

$$rW = (1 - M)[(1 \underset{2}{\,} - M)y + M](u - c)$$

- Maximize W with respect to M :

$$\begin{aligned} M^* &= \frac{1 - 2y}{2 - 2y} \text{ if } y < \frac{1}{2} \\ M^* &= 0 \text{ if } y \geq \frac{1}{2} \end{aligned}$$

- Intuition: facilitate trade versus crowding out barter.

Welfare $\pi = 0$

- Note:

$$rW = (1 - M)\,[(1 - M)\,y]\,(u - c)$$

- Monotonically decreasing in $M \Rightarrow M^* = 0$.

- Result is a little bit silly: it depends on the absence of free disposal of money. Otherwise, welfare is independent of M.

Welfare π

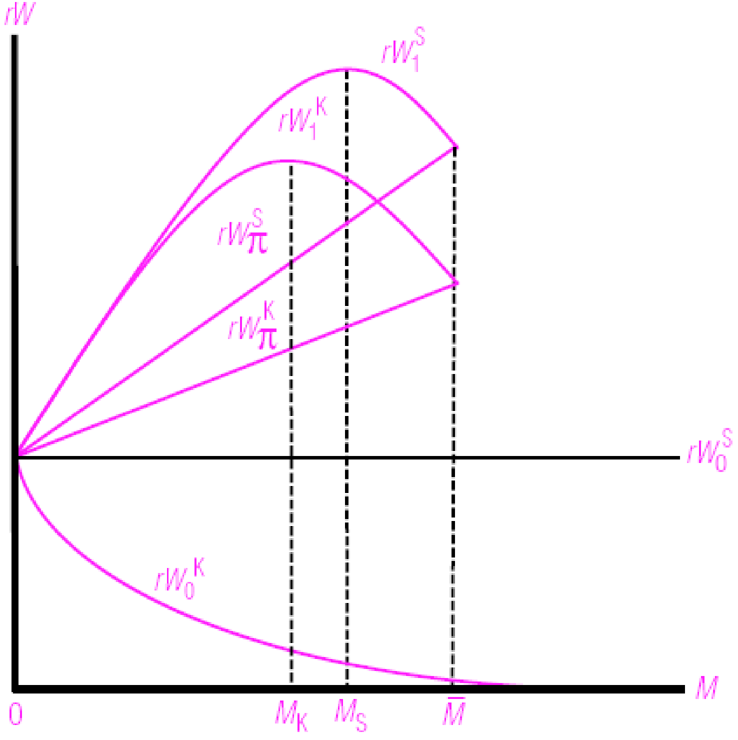
Define $\underline{\mathbf{M}}$ **such that** $\boldsymbol{\pi}$**=1,**

- Note:

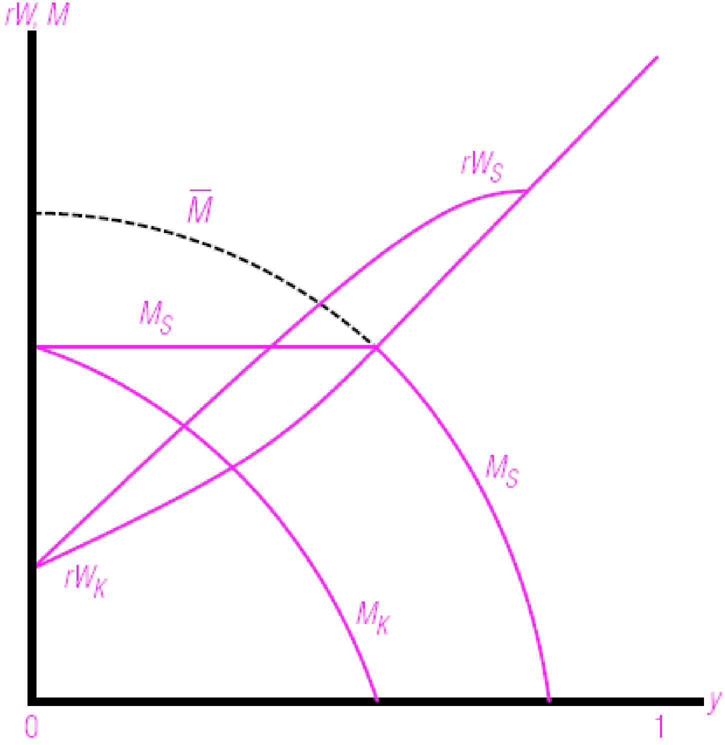
$$rW = (1 - M)\left[(1 - M)\,y + M\pi\right](u - c)$$

- Monotonically increasing in M in the $[0, \underline{\mathrm{M}}]$ interval.

Welfare as a Function of M



Welfare as a Function of y (optimal M)



Comparison with Alternative Arrangements

- Imagine that we have the credit arrangement: "produce for anyone you meet that wants your good."

- Value function

$$rV_c = u - c$$

- Clearly

$$rV_c > rW$$

- However, this arrangement is not self-enforceable: histories are not observed.

Second Generation: Endogenous Prices

- We make the very strong assumption that we exchanged one good for one unit of money.

- What if we let prices be endogenous? Shi (1995) and Trejos and Wright (1995).

- We set $y = 0$ and we let goods be divisible.

- When agents meet, they bargain about how much q will be exchanged, or equivalently, about price $1/q$.

Utility and Cost Functions

- Utility is $u\,(q)$ and cost of production is $c\,(q)$.

- Assumptions:

$$
\begin{gathered}
u\,(0) = c\,(0) = 0 \\
u'\,(0) > c\,'\,(0) \\
u'\,(0) > 0, u''\,(0) \leq 0 \\
c'\,(0) > 0, c''\,(0) \geq 0
\end{gathered}
$$

- Also, $\widehat{q}$ and q^* are such that

$$
\begin{aligned}
u(\hat{q}) &= c(\hat{q}) \\
u'(q^*) &= c'(q^*)
\end{aligned}
$$

Value Functions and Bargaining

- Take $q = Q$ as given. Then:

$$rV_1 = (1 - M)\left[u(Q) + V_0 - V_1\right]$$
$$rV_0 = M\left[V_1 - V_0 - c(Q)\right]$$

- Bargaining is the generalized Nash bargaining solution:

$$q = \text{a r g m a x}\,[\,u(q) + \;V_0\,(Q) - T_1]^{\theta} \times [V_1\,(Q) - c(\text{q}) - T_0]^{\theta}$$
$$u(q) + V_0 \geq V_1$$
$$V_1 - c(q) \geq V_0$$

where T_j is the threat point of the agent with j units of money.

- We will set $T_j = 0$ and $\theta = 1/2$.

Equilibria

- Necessary condition taking $V_0(Q)$ and $V_1(Q)$ as given:

$$[V_1(Q) - c(q)]\, u'(q) = [u(q) + V_0(Q)]\, c'(q)$$

- The bargaining solution defines a function

$$q = e(Q)$$

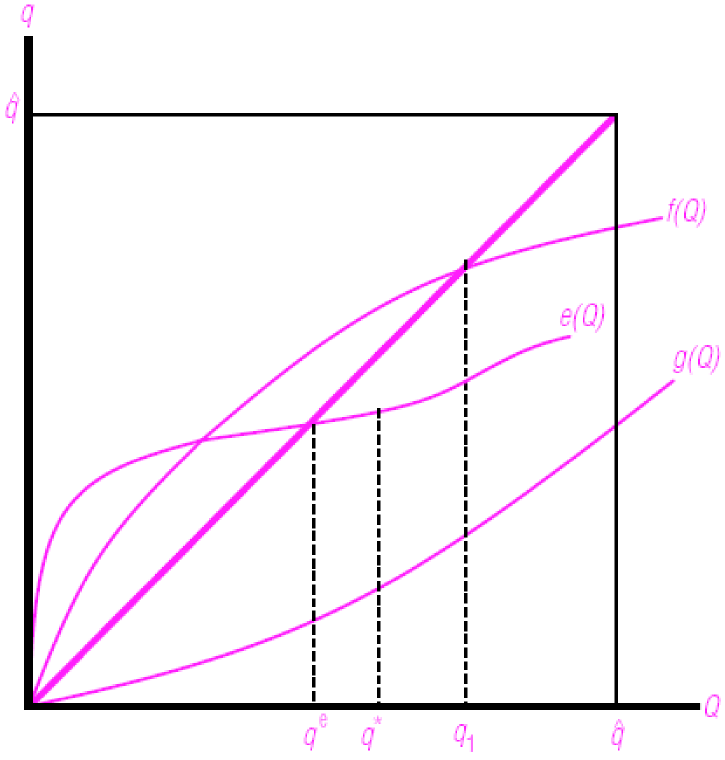
and we look at its fixed points.

- Two fixed points:

1. $q = 0$: nonmonetary equilibrium.

2. $q = q^e > 0$: monetary equilibrium.

Monetary Equilibrium in the Divisible-Goods Model



Efficiency

- Note that the efficient outcome is q^*, i.e. $u'(q^*) = c'(q^*)$.

- In the monetary equilibrium:

$$u'(q^e) = \frac{u(q^e) + V_0(q^e)}{V_1(q^e) - c(q^e)} c'(q^e) > u'(q^*)$$

  since $u(q^e) + V_0(q^e) > V_1(q^e) - c(q^e)$.

- Hence $q^* > q^e$, or equivalenty, the price is too high.

Third Generation: Endogenous Prices and Goods

- Relax the assumption that agents hold 0 or 1 units of money.

- Problem: endogenous distribution of money that we (and the agents!) need to keep track of.

- Computational: Molico (2006).

- Theoretical:

  1. Families: Shi (1997).

  2. Two markets: Lagos and Wright (2005).