

# Lecture 9: Optimal Fiscal Policy

Economics 714, Spring 2018

## 1 Ramsey Optimal Taxation

### 1.1 Setup

Look for linear taxes that fund given  $\{G_t\}$  and maximize household welfare

First, find implementability constraint summarizing equilibria. Rewrite HH BC:

$$\sum_{t=0}^{\infty} q_t [C_t - (1 - \tau_t^N) w_t N_t] = \sum_{t=0}^{\infty} q_t [R_t^K K_t - K_{t+1}] = q_0 R_0^K K_0$$

Then use HH first order conditions to substitute out for prices

$$\sum_{t=0}^{\infty} \beta^t [U_C(C_t, 1 - N_t) C_t - U_L(C_t, 1 - N_t) N_t] = U_C(C_0, 1 - N_0) K_0 [1 + (1 - \tau_0^K)(F_K(K_0, N_0) - \delta)]$$

Primal approach: solve for allocation first, back out supporting taxes from equilibrium conditions:

$$\left( \frac{u_C(C_t, 1 - N_t)}{\beta u_C(C_{t+1}, 1 - N_{t+1})} - 1 \right) \frac{1}{F_K(K_{t+1}, N_{t+1}) - \delta} = 1 - \tau_{t+1}^K$$
$$\frac{u_L(C_t, 1 - N_t)}{u_C(C_t, 1 - N_t) F_N(K_t, N_t)} = 1 - \tau_t^N$$

$\tau_0^K$  only affects period zero: initial capital is inelastic, tax it as much as possible. To make problem interesting, restrict  $\tau_0^K \leq \bar{\tau}^K$ .

Same problem re-occurs, so if the government could re-optimize in any period it would have an incentive to extract the accumulated capital: time consistency problem.

Define  $U(C, N) = U(C, 1 - N)$  so  $U_N = -U_L$ . Also define:

$$W(C_t, N_t; \lambda) = U(C_t, N_t) + \lambda[U_C(C, N)C + U_N(C, 1 - N)N]$$

Then Ramsey problem can be written:

$$\max_{\{C_t, K_{t+1}, N_t\}} \sum_{t=0}^{\infty} \beta^t W(C_t, N_t; \lambda) - \lambda U_C(C_0, N_0) K_0 [1 + (1 - \tau_0^K)(F_K(K_0, N_0) - \delta)]$$

subject to (multiplier  $\mu_t$ ):

$$C_t + G_t + K_{t+1} = F(K_t, N_t) + 1 - \delta K_t$$

## 1.2 Characterization

First order conditions for  $t \geq 1$ :

$$W_C(t) = \mu_t$$

$$W_N(t) = -\mu_t F_N(t)$$

$$\mu_t = \beta \mu_{t+1} (F_K(t+1) + 1 - \delta)$$

Special optimality conditions for date 0.

These imply modified Euler equation and intra-temporal optimality condition:

$$\begin{aligned} W_C(C_t, N_t) &= \beta W_C(C_{t+1}, N_{t+1}) [1 + F_K(K_{t+1}, N_{t+1} - \delta)] \\ -\frac{W_N(C_t, N_t)}{W_C(C_t, N_t)} &= F_N(K_t, N_t) \end{aligned}$$

Implications:

- zero long-run capital tax: if  $G_t \rightarrow \bar{G}$ , allocation converges to steady state, then  $\tau_t^K \rightarrow 0$ . Capital income tax implies growing distortion. Equivalent to increasing consumption tax.

- smoothing of tax rates: allocation smoothes tax response to changes in  $G_t$ . Implicitly use government debt as a buffer to smooth tax distortions.

Example:  $U(C, N) = \frac{C^{1-\gamma}}{1-\gamma} - \frac{N^{1+\theta}}{1+\theta}$

Then  $W_C = U_C(1 + \lambda - \lambda\gamma)$ ,  $W_N = U_N(1 + \lambda - \lambda\phi)$ .

Implies  $\tau_t^K = 0$  for  $t \geq 2$ , and  $\tau_t^N$  constant for  $t \geq 1$ .

So even when expenditure and allocation is time varying, the use of debt allows the smoothing of distortions. General point holds beyond this simple case.