Lecture 4: Equity Premium, Complete and Incomplete Markets

Economics 714, Spring 2018

1 Equity Premium

We can write excess returns on stocks over the risk free rate, or the equity premium as:

$$E(r_t) - r^f = \gamma cov(r_t, \Delta c_t)$$

Which in turn can be re-written to give the risk-return tradeoff:

$$\frac{E(r_t) - r^f}{\sigma(r)} = \gamma \sigma(\Delta c_t) corr(\Delta c_t, r_t)$$

Left side known as Sharpe ratio

Moments of aggregate consumption growth and level of risk aversion then determine equity premium. The equity premium puzzle is that to rationalize observed asset prices requires implausibly high levels of risk aversion.

1.1 Attempted Resolutions

There has been a very large literature in macro asset pricing to try to explain the equity premium puzzle. These theories can be grouped into three broad classes:

- Change preferences: Get away from time additively separable power utility function. Examples: recursive preferences, robustness, habit persistence

- Change constraints: Get away from representative agent complete markets with no frictions. Examples:Limited participation, transaction costs, incomplete markets
- Change shocks: Get away from typical log-normal assumption for consumption growth. Examples: disaster models, long-run risk, learning

2 Implications of Complete Markets

2.1 A finite setting

Suppose now for simplicity that $s_t \in \{s_1, \ldots, s_N\}$ finite, transitions then are governed by the Markov transition matrix $Q(s, s') = P(s_{t+1} = s'|s_t = s)$ which is the finite counterpart of the transition function we've been working with.

The pricing kernels q and q^j have the obvious counterparts

We will use the shorthand notation for histories $s^t = \{s_0, s_1, \ldots, s_t\}$, and summations like $\sum_{s^j} q^j(s, s^j)$ mean summing over all possible histories j periods ahead.

Then with the full set of contingent securities, we can write the budget constraints like we had before:

$$\begin{aligned} w &= c + p(s)a' + \sum_{j=1}^{\infty} \sum_{s^j} q^j(s, s^j)\theta^j(s, s^j) \\ w'(s') &= a'(p(s') + s') + \theta^1(s', s) + \sum_{j=2}^{\infty} \sum_{s^j|s'} q^{j-1}(s', s^j)\theta^j(s^j, s) \end{aligned}$$

The Bellman equation for the agent's problem can be written:

$$V(w, s) = \max_{c, a', \theta^j} \left\{ u(c) + \sum_{s'} \beta V(w'(s'), s')Q(s, s') \right\}$$

subject to the law of motion for w.

Then the first order condition for any $\theta^1(s, s_i)$ is:

$$u'(c)q(s, s_i) = \beta \sum_{s'} \beta V_w(w'(s'), s') 1_{\{s'=s_i\}} Q(s, s') = \beta V_w(w'(s_i), s_i) Q(s, s_i)$$

With the envelope condition:

$$V_w(w, s) = u'(c)$$

Thus we have the Euler equation for the asset:

$$u'(c)q(s, s_i) = \beta u'(c(s_i)) Q(s, s_i)$$

Along the same lines, we can derive the more general expression for the agent's optimality conditions for any future date and state:

$$u'(c_t) q^j(s_t, s^j) = \beta^j u'(c_{t+j}(s^j)) Q^j(s_t, s^j)$$

2.2 Allowing heterogeneity

We have focused on a representative agent setting, but now suppose we allow for heterogeneity in preferences and endowments. There are $i = 1, \ldots, I$ consumers who have different preferences $u_i(c), \beta_i$ and endowments $e_t^i = e^i(s_t)$ where $\sum_i e^i(s_t) = s_t$.

Note then that the same optimality condition holds for any consumer. Let's look from date 0:

$$u_i'(c_0^i) q^t(s_0, s^t) = \beta_i^t u_i'(c_t^i(s^t)) Q^t(s_0, s^t)$$

Since a similar expression holds for any consumer j as well, we have:

$$\frac{u_i'(c_0^i) q^t(s_0, s^t)}{u_j'(c_0^j) q^t(s_0, s^t)} = \frac{\beta_i^t u'(c_t^i(s^t)) Q^t(s_0, s^t)}{\beta_j^t u_j'(c_t^j(s^t)) Q^t(s_0, s^t)}$$

or

$$\frac{u_i'(c_0^i)}{u_j'(c_0^j)} = \left(\frac{\beta_i}{\beta_j}\right)^t \frac{u'(c_t^i(s^t))}{u_j'(c_t^j(s^t))}$$

We can use this to derive a few simple implications:

- Therefore if $\beta_i > \beta_j$, as $t \to \infty$ we must have $\frac{u'(c_t^i(s^t))}{u_j'(c_t^j(s^t))} \to 0$ and thus $c_t^j \to 0$. The same holds for any pairwise comparison, so in the long run the most patient agent will consume the entire aggregate endowment.

- Now suppose that we have preference homogeneity $\beta_i = \beta$ and $u_i(c) = \frac{c^{1-\gamma}}{1-\gamma}$. Then we have:

$$\frac{(c_0^i)^{-\gamma}}{(c_0^j)^{-\gamma}} = \frac{(c_t^i)^{-\gamma}}{(c_t^j)^{-\gamma}}$$

  That is, each agent's consumption is a constant fraction of any other's. Equivalently, each agent consumes a constant fraction of the aggregate endowment. Thus we have **perfect risk sharing**: no agent bears any idiosyncratic risk .