Economics 702
Macroeconomics
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Problem Set 2
Due in class on February 19.

1. In recent years there has been an increase in capital's share of income α. Consider the optimal growth model with inelastic labor supply, with $F(K, N) = F(K, 1) = K^\alpha$, and for simplicity assume that productivity $z = 1$. Household preferences are:

$$\sum_{t=0}^{\infty} \beta^t u(C_t)$$

The capital evolution equation is:

$$K_{t+1} = (1 - \delta)K_t + F(K_t) - C_t.$$

Suppose the economy is initially in the steady state and then there is an unexpected and permanent increase in the capital share parameter α.

(a) What are the long-run (steady state) effects of this change on consumption, capital, and output? What would be the impact on labor income? Show these explicitly.

(b) What happens to consumption and capital at the time of the change and following periods? Illustrate and discuss your answer.

2. Consider the dynamic optimal allocation model as in class, but instead of a Cobb-Douglas production function, suppose that production is linear $F(k) = zK$.

(a) Write down the planner's Lagrangian and find the conditions characterizing the optimal allocation.

(b) Does this model have a steady state? If so, find an expression for it. If not, describe the dynamics of the economy.

(c) Suppose that there is a once and for all unforeseen increase in the level of productivity z. Describe as fully as you can what happens to the economy upon impact and over time.

3. This question uses the Solow growth model to analyze the effects of immigration. Suppose that productivity grows at rate g, that the population grows at a constant rate n, and depreciation is the constant δ. Suppose that the economy is initially in the steady state.

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(a) Now suppose that there is a one-time increase in the labor force from immigration ($N' > N$), but the population growth rate n remains constant. Analyze the short-run and long-run effects of this change for the levels of per-capita capital, consumption and output, and the growth rates of (total) output and per-capita output.

(b) Now suppose instead that there is an increase in immigration as a continuing process, so that n increases to a higher value n'. Analyze the short-run and long-run effects of this change for the levels of per-capita capital, consumption and output, and the growth rates of (total) output and per-capita output.

(c) Now suppose that immigration increases productivity growth. That is, suppose that there is a one-time increase in the labor force as in part (a) to $N' > N$. But at the same time, there is an increase in innovation so the rate of productivity growth increases to $g' > g$. (There is no change in the initial level of productivity at the time of the population increase.). Now describe what happens in the short-run and long-run to the levels of per-capita capital, consumption and output, and the growth rates of (total) output and per-capita output.

(d) If wages are equal to the marginal product of labor (as they would be in a competitive equilibrium), how are they affected in by immigration? How do your answers differ in parts (a)-(c)?

4. This problem considers a variation on the basic Solow model with no productivity growth. Suppose that consumers spend more income when capital is higher, so consumption is given by $C = (1 - s)Y + hK$, where $s, h > 0$ are both constants. Output is produced via a Cobb-Douglas production function $Y = K^\alpha N^{1-\alpha}$ and the population grows at the constant rate n.

(a) Determine the steady state per-worker quantities of capital, output, and consumption.

(b) Suppose that the economy is in the steady state and there is an increase in h. What are the effects on the per-worker levels of capital, output, and consumption, both at the time of the change and in the long-run?