Lecture 2
Labor Demand

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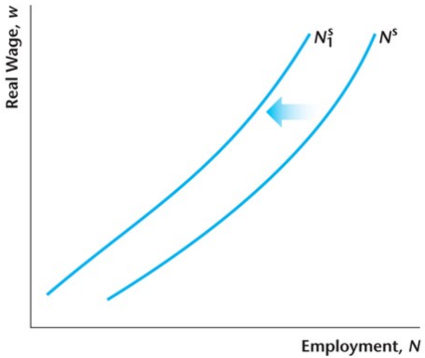
Aggregate Labor Supply

- In the aggregate, labor supply supply curve embodies both intensive and extensive margins, and is is upward sloping.
- Intensive margin: for those *already working*, increase in wage has income and substitution effects.
- Extensive margin: increases in wages may induce some *who were not in labor force* to enter and supply labor. Always increasing in w.
- Aggregate labor supply curve also smooths out kinks in individual supply, for example due to fixed costs of work.

Figure 4.10 Effect of an Increase in Dividend Income or a Decrease in Taxes



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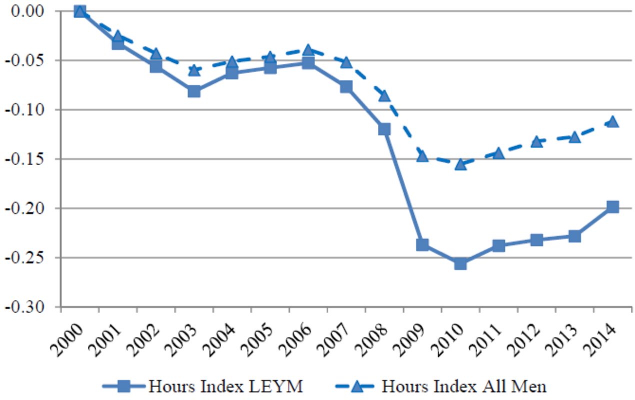
Application: Decline in Employment of Young Men

- In recent paper Aguiar, Bils, Charles, and Hurst (2018) document decline in employment and hours worked of young men (21-30) who did not attend college.
- Also document increased leisure time of this same group, and increase in computer and videogame use.
- Argue that improvements in leisure activities (productivity of videogames, taste for leisure) have made non-participation and less work more prevalent.
- Other factors as well: changing job prospects, living with parents

Figure 3: Annual Hours Index for Less Educated Young Men and All Prime Age Men, March CPS



Notes: Figure shows annual hours index for lower educated young men (squares) and all prime age men (triangles). Annual hours are calculated by multiplying self-reported weeks worked last year by self-reported usual hours worked per week last year. We convert the series to an index by setting year 2000 values to 0. All other years are log deviations from year 2000 values. Data from the March supplement of the Current Population Survey.

Table 3: Leisure Activities for Men 21-30 (Hours per Week): By Employment Status

Activity	Employed 2004-2007	Employed 2012-2015	Employed Change	Non-Employed 2004-2007	Non-Employed 2012-2015	Non-Employed Change
Total Leisure	57.6	59.6	2.0	87.0	82.1	-4.9
Recreational Computer	3.0	4.3	1.3	5.4	9.6	4.2
Video Game	1.8	2.9	1.0	3.5	5.9	2.4
ESP	23.6	23.9	0.3	30.2	29.9	-0.2
TV/Movies/Netflix	15.9	15.5	-0.4	27.8	25.0	-2.8
Socializing	7.4	7.8	0.3	10.6	8.9	-1.7
Other Leisure	7.7	8.1	0.5	13.0	8.6	-4.4
Job Search and Education	2.0	1.9	-0.1	9.2	14.1	4.9

Note: Components sum to total leisure time. Video gaming is a subcomponent of total computer time. ESP refers to eating, sleeping and personal care net of 49 hours per week.

Improvements in Leisure Activities

- Model as an increase in leisure productivity θ:

$$\max_{c,l} u\left(c, \theta l\right)$$
$$s.t.\ c = (h - l)w + \pi$$

- Optimality condition:

$$\theta \frac{u_l}{u_c} = w$$

- Increase in productivity equivalent to increase in preference for leisure $\Rightarrow$ decrease in labor supply.

What is a firm?

- A firm uses capital K and labor N to produce output Y via a production function F:

$$Y = zF(K, N)$$

  z is the level of technology or total factor productivity (TFP).

- The main example we'll use is Cobb-Douglas production function, with $\alpha \in (0,1)$:

$$Y = zK^{\alpha}N^{1-\alpha}$$

- We will start with a static model: K (supply) is constant.

Properties of the Technology

We'll make several assumptions on the technology F, all of which are satisfied by Cobb-Douglas.

1. Inputs are essential.

$$F(0, N) = F(K, 0) = 0$$

2. Constant returns to scale:

$$F(\lambda K, \lambda N) = \lambda F(K, N)$$

   Doubling inputs doubles output. Compared to decreasing (increasing) returns to scale where doubling inputs leads to less (more) than double output.

$$\begin{aligned} F(K, N) &= K^{\alpha} N^{1-\alpha} \\ F(\lambda K, \lambda N) &= (\lambda K)^{\alpha} (\lambda N)^{1-\alpha} \\ &= \lambda K^{\alpha} N^{1-\alpha} \end{aligned}$$

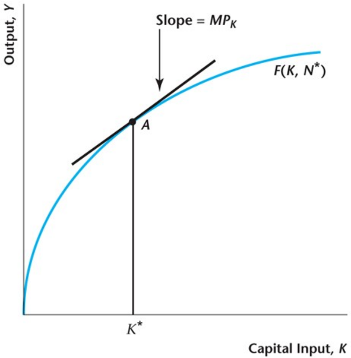
3. Marginal productivities of capital and labor are positive and decreasing.

$$\begin{aligned} MPK &= F_K > 0,\ F_{KK} < 0 \\ MPN &= F_N > 0,\ F_{NN} < 0 \end{aligned}$$

Increasing each factor gives more output, but at a decreasing rate.

$$\begin{aligned} F(K,N) &= K^{\alpha}N^{1-\alpha} \\ F_K &= \alpha K^{\alpha-1}N^{1-\alpha} > 0 \\ F_{KK} &= \alpha(\alpha-1)K^{\alpha-2}N^{1-\alpha} < 0 \\ F_N &= (1-\alpha)K^{\alpha}N^{-\alpha} > 0 \\ F_{NN} &= -\alpha(1-\alpha)K^{\alpha}N^{-\alpha-1} < 0 \end{aligned}$$

Figure 4.13 Production Function, Fixing the Quantity of Labor and Varying the Quantity of Capital



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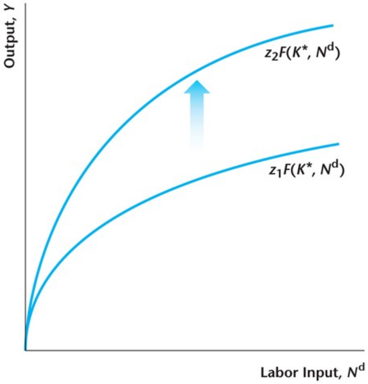
4. Marginal productivity of each factor increases in the other.

$$\frac{\partial MPK}{\partial N} = \frac{\partial}{\partial N} F_K = F_{KN} > 0$$
$$\frac{\partial MPN}{\partial K} = \frac{\partial}{\partial K} F_N = F_{KN} > 0$$

Note one implies other since $F_{KN} = F_{NK}$.
Additional capital makes workers more productive: spread workers among more machines (and vice versa).

$$\begin{aligned} F_K &= \alpha K^{\alpha-1} N^{1-\alpha} \\ F_{KN} &= \alpha(1-\alpha) K^{\alpha-1} N^{-\alpha} > 0 \\ F_N &= (1-\alpha) K^{\alpha} N^{-\alpha} \\ F_{NK} &= \alpha(1-\alpha) K^{\alpha-1} N^{-\alpha} > 0 \end{aligned}$$

Figure 4.16 Total Factor Productivity Increases



Problem of the Firm I

- Competitive firm rents capital at rate r, hires labor at wage w.
- Profits: output minus costs

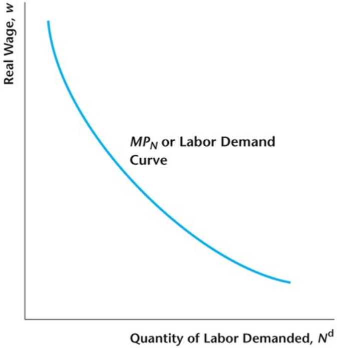
$$\pi = zF(K,N) - rK - wN$$

- Note everything in real terms – same as setting price of output to 1.
- Firm maximizes profits by hiring labor and capital. We take first order conditions for choice of K and N:

$$\begin{aligned} zF_K(K,N) &= r \\ zF_N(K,N) &= w \end{aligned} \qquad (1)$$

- Factors are paid their marginal products.
- (1) can be solved to give the labor demand: $N^d(w)$

Figure 4.20 The Marginal Product of Labor Curve Is the Labor Demand Curve of the Profit-Maximizing Firm



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Problem of the Firm I: Special Case

- With Cobb-Douglas production, we have:

$$\pi = zK^{\alpha}N^{1-\alpha} - rK - wN$$

- We take first order conditions for K and N:

$$zF_K = z\alpha K^{\alpha-1}N^{1-\alpha} = r \quad (2)$$

$$zF_N = z(1-\alpha)K^{\alpha}N^{-\alpha} = w \quad (3)$$

- (3) can be solved to give the labor demand:

$$N^d(w;K) = MPN^{-1}(w;K) = K\left(\frac{z(1-\alpha)}{w}\right)^{\frac{1}{\alpha}}$$

- N^d decreasing in w. Increases in z, K shift labor demand.

Problem of the Firm II

- To fully solve the example, we want to solve for K and N.
- We begin dividing (2) by (3):

$$\frac{\alpha z K^{\alpha-1} N^{1-\alpha}}{(1-\alpha)\, z K^{\alpha} N^{-\alpha}} = \frac{r}{w}$$

or

$$\frac{\alpha}{1-\alpha}\frac{N}{K} = \frac{r}{w}$$

or

$$K = \frac{w}{r}\frac{\alpha}{1-\alpha}N \tag{4}$$

This gives us the optimal capital/labor ratio of the firm.
Depends on relative prices w/r, relative productivities $\alpha, 1-\alpha$.

Problem of the Firm III

- But if we substitute (4) in (1), trying to solve for N, we get:

$$\begin{aligned} (1-\alpha)\, zK^{\alpha}N^{-\alpha} &= w \\ (1-\alpha)\, z\left(\frac{w}{r}\frac{\alpha}{1-\alpha}N\right)^{\alpha}N^{-\alpha} &= w \\ (1-\alpha)\, z\left(\frac{w}{r}\frac{\alpha}{1-\alpha}\right)^{\alpha} &= w \end{aligned}$$

  N disappears!

- You can check that the same happens with K if we substitute (4) in (2).
- What is wrong?

Problem of the Firm IV

- We have constant returns to scale.
- The size of the firm is indeterminate: we can have just one!
- Can show constant returns equivalent to $F(K,N) = F_K K + F_N N$.
- This implies profits are always zero if firm maximizes, since $r = MPK$, $w = MPN$.

$$\begin{aligned} \pi &= zK^{\alpha}N^{1-\alpha} - rK - wN \\ &= zK^{\alpha}N^{1-\alpha} - MPK \cdot K - MPN \cdot N \\ &= zK^{\alpha}N^{1-\alpha} - \alpha zK^{\alpha-1}N^{1-\alpha}K - (1-\alpha)\, zK^{\alpha}N^{-\alpha}N = 0 \end{aligned}$$

- Example of Euler's theorem for homegeneous functions
- So the firms really only pick the labor-capital ratio given relative prices:

$$\frac{N}{K} = \frac{1-\alpha}{\alpha}\frac{r}{w}$$

- In equilibrium, markets clear so:

$$\begin{aligned} N^d(w, K) &= N^s(w) \\ K^d(r, N) &= \bar{K} \quad \text{i.e., fixed} \\ r &= \alpha z K^{\alpha-1} N^{1-\alpha} \\ w &= (1-\alpha) z K^{\alpha} N^{-\alpha} \end{aligned}$$

- We have a system of four equations in four unknowns (K, N, r, w).
- In other words, firms choose K/N ratio, but by assumption K is fixed at $\bar{K}$ in short run, which gives labor demand N^d.
- We will return to discussing general equilibrium shortly.

A Simple Example

A representative worker has preferences:

$$u(C, l) = 2\sqrt{C} - a\frac{(1-l)^{1.5}}{1.5}$$

The budget constraint is:

$$C = w(1-l),$$

where 1 is the hours in the day, so $1 - l$ is labor supply. Capital is fixed at 1, and the representative firm technology is:

$$Y = zN^{0.5}$$

1. Find the household labor supply function.
2. Find expressions for the equilibrium values of the labor input and the wage.
3. Suppose that a increases but z is unchanged. What is the effect of this change on labor and the wage?

What are we missing?

- Wages are often different from marginal productivity. Labor market not completely a spot market.
- Some reasons:
  1. Long term contracting. Sticky wages: Keynes (1936), Taylor (1980).
  2. Search frictions in finding a job. Wages determined by bargaining: Nash (1950), McCall (1970), Mortensen-Pissarides (1994).
  3. Workers may exert effort which influences output, difficult to observe. Efficiency wages: Shapiro-Stiglitz (1984), moral hazard Holmstrom (1979).

We will return to some of these frictions later in the class. For now continue with competitive, spot labor market.

Adding a Government

- Operational definition: takes in taxes T and spends G.
- We'll assume a balanced budget. No debt in this static model.
- Also assume household does not value government spending. Not crucial here.
- We'll also assume proportional labor income taxes on households. Later will consider lump sum taxes.

$$\begin{aligned} G &= T \\ T &= \tau w N \end{aligned}$$

New Problem of the Household

- Set unearned income equal to capital income rK. Incorporate labor income taxes.
- Problem for household is now:

$$\max_{c,N} u(c, l)$$

$$s.t.\ c = (1-\tau) wN + rK$$

- First order conditions:

$$\frac{u_l}{u_c} = (1-\tau) w$$

- Household now equates MRS to after-tax wage.
- Taxes affect labor supply!

Labor Supply Effects in an Example

- Reconsider the log example from earlier:

$$u(c, l) = \log c + \gamma \log l$$

- Then we get:

$$\gamma \frac{c^*}{h - N^*} = (1 - \tau) w$$
$$c^* = (1 - \tau) w N^* + rK$$

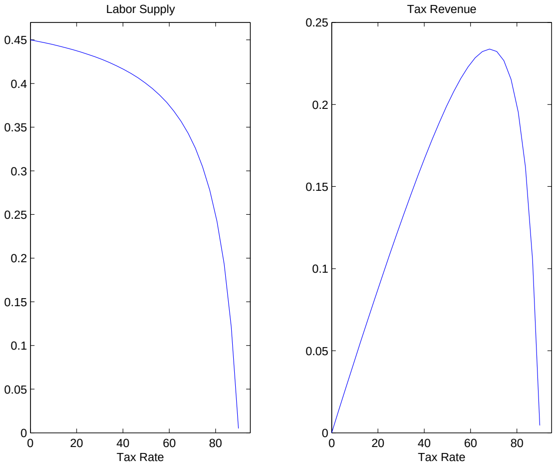
- Therefore we see that taxes affect labor supply:

$$N^* = \frac{(1 - \tau) wh - \gamma r K}{(1 + \gamma)(1 - \tau) w}$$

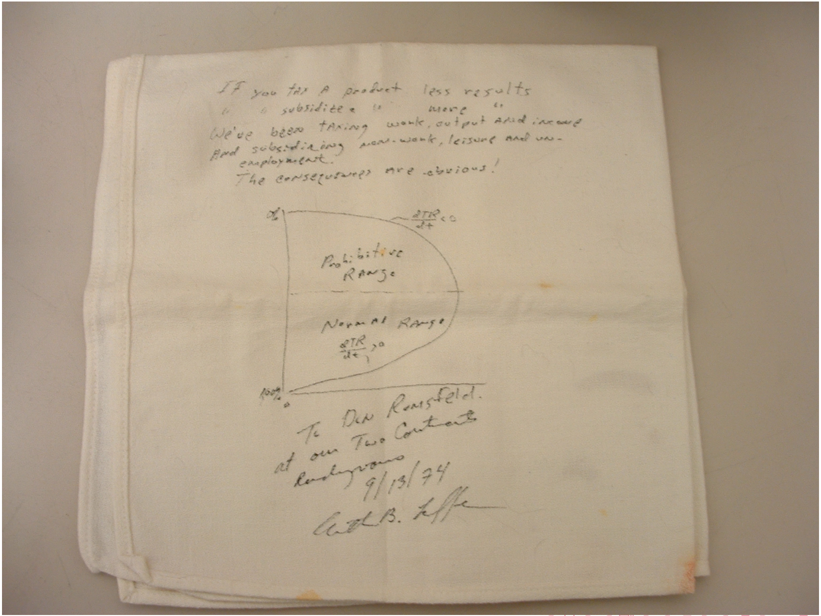
Laffer Curve

Tax revenue has a Laffer curve:

$$T = \tau w N = w\tau \frac{(1-\tau)\, wh - rK}{(1+\gamma)\,(1-\tau)\, w}$$



A Famous Cocktail Napkin



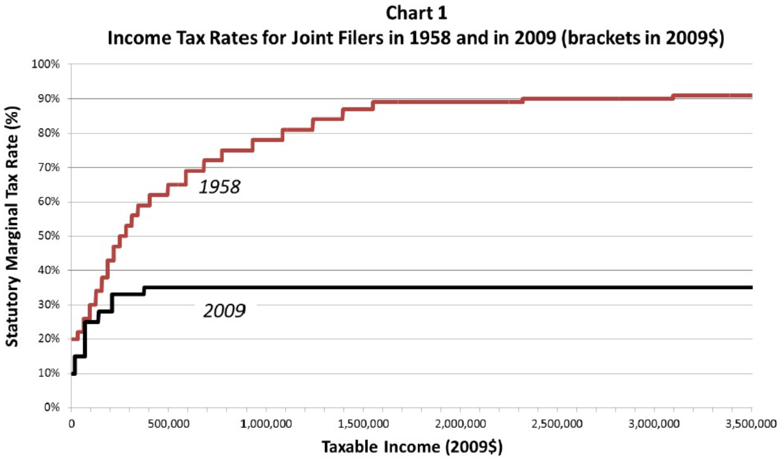
Laffer Curve

- There were arguments in 1981 and 2001 that US economy was on the bad side of the Laffer curve, and revenue could increase with tax cuts.
- Although these tax cuts may have increased economic growth, there is no evidence that revenue increased.
- Estimates of peak tax revenue in US are at 60% or greater federal income tax rate
- A 2010 ECB study found Sweden was on the bad side, with a top labor income tax of 57% and a payroll tax of 31%. Historically it had been even higher, up to 90%.
- Laffer curve arguments are more likely apply to narrower categories of goods which have higher elasticities of substitution, like luxury goods or possibly capital gains.

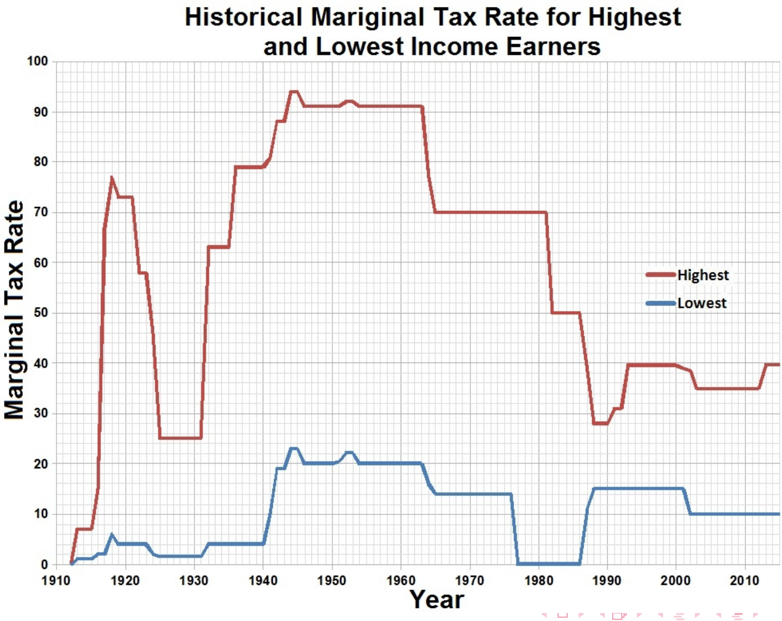
Progressive Taxation

- We have assumed a constant tax on labor income. In the US, the tax system is progressive: higher marginal tax rates apply to higher incomes.
- Over time the top marginal tax rates have fallen. There are current debates about whether the top tax rate should increase.
- In addition to the labor supply effects, or more broadly, the impact of taxes on taxable income, taxes affect other margins:
  - Form of income (tax shelters, incorporation)
  - Investment in physical and human capital
  - Location of people and businesses
  - Business formation and expansion decisions
  - Income distribution and political concerns

Progressive Tax Schedule



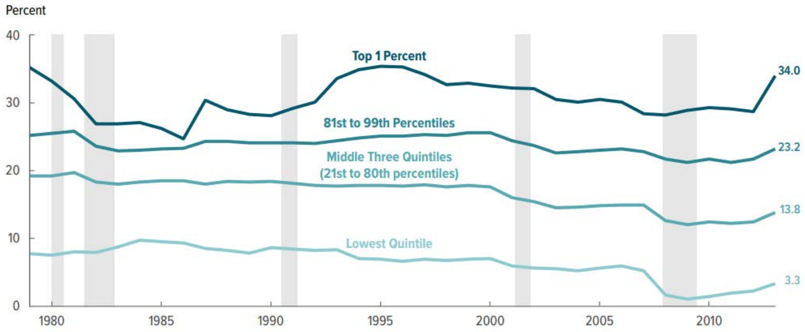
Historical Top and Bottom Marginal Tax Rates



Average Tax Rates by Income Group

Figure 2.

Average Federal Tax Rates, by Before-Tax Income Group, 1979 to 2013



Source: Congressional Budget Office.

Average federal tax rates are calculated by dividing federal taxes by before-tax income.