Lecture 12
Asset Pricing
Ricardian Equivalence & Social Security

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Asset Pricing

- We have thought about Euler equation as determining consumption given interest rates. But we can also use it to determine rates of return and so **asset prices** given consumption.

$$u'(c_t) = \beta E_t \left[u'(c_{t+1})(1 + r_{t+1}) \right]$$

- Generalization of Euler equation is the pricing relation for an asset with price p_t stochastic payoff x_{t+1} next period:

$$\begin{aligned} p_t &= E_t \left[\frac{\beta u'(c_{t+1})}{u'(c_t)} x_{t+1} \right] \\ &= E_t(m_{t+1} x_{t+1}) \end{aligned}$$

- A return has price 1, payoff $R_{t+1} = 1 + r_{t+1}$, i.e. $R_{t+1} = \frac{p_{t+1}+d_{t+1}}{p_t}$.

Power Utility and Risk-Free Rates

Now assume $u(c) = c^{1-\gamma}/(1-\gamma)$
Risk-free rate when c_{t+1} known:

$$R = \frac{1}{E_t\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\right]} = \frac{1}{\beta}\left(\frac{c_{t+1}}{c_t}\right)^{\gamma}$$

$$\begin{aligned} \log R &= -\log\beta + \gamma\log(c_{t+1}/c_t), \text{ or approximately:} \\ r^f &= \theta + \gamma\Delta c_{t+1} \end{aligned}$$

where Δc_{t+1} is the growth rate of consumption, r^f is the (net) risk free rate, $\beta = \frac{1}{1+\theta}$

Power Utility and Returns: A Characterization

Now for a general, stochastic return r_{t+1}

$$1 = E_t\left[\frac{1}{1+\theta}(1+\Delta c_{t+1})^{-\gamma}(1+r_{t+1})\right]$$

Take 2nd order Taylor approximation of right side so approximately we have:

$$\begin{aligned} 1+\theta &= E_t\left[(1+\Delta c_{t+1})^{-\gamma}(1+r_{t+1})\right] \\ &\approx E_t\left[1+r_{t+1}-\gamma\Delta c_{t+1}-\gamma\Delta c_{t+1}\cdot r_{t+1}\right. \\ &\quad \left.+\frac{1}{2}\gamma(\gamma+1)\left(\Delta c_{t+1}\right)^2\right] \end{aligned}$$

Take unconditional expectations of right side and rearrange:

$$E(r) = \theta + \gamma E(\Delta c_t) + \gamma cov(r_t, \Delta c_t) - \frac{1}{2}\gamma(\gamma+1)\sigma^2(\Delta c_t)$$

Power Utility and the Equity Premium

$$E(r) = \theta + \gamma E(\Delta c_t) + \gamma cov(r_t, \Delta c_t) - \frac{1}{2}\gamma(\gamma+1)\sigma^2(\Delta c_t)$$

For risk free rate $cov(r_t, \Delta c_t) = 0$ so:

$$r^f = \theta + \gamma E(\Delta c_t) - \frac{1}{2}\gamma(\gamma+1)\sigma^2(\Delta c_t)$$

So excess return on risk assets can be written:

$$\frac{E(r_t) - r^f}{\sigma(r)} = \gamma\sigma(\Delta c_t) corr(\Delta c_t, r_t)$$

Left side known as Sharpe ratio

Attempted Resolutions of Equity Premium

- Consumption based model fails empirically in explaining premium on stocks vs. bonds.
- Change preferences: recursive preferences, ambiguity/robustness, habit persistence
- Change constraints: Limited participation, transaction costs, incomplete markets
- Change shocks: disaster models, long-run risk, learning

Application of Consumption-Savings I: Social Security in the Life-cycle model

- Retirement saving is an important component of household saving decisions.
- In US, main government program to support retired is Social Security which is a “pay-as-you-go” system, as opposed to a fully-funded one.
- Use simple two-period life-cycle model to analyze the impact of Social Security on saving, welfare.
- Assume $y' = 0$ and $A = 0$, for simplicity. So $y^{PV} = y$.
- Consider the parametric example from before $u(c) = \log c$.

Social Security in the Life-cycle model

- Without social security. Euler Equation:

$$\frac{1}{c} = \beta (1 + r) \frac{1}{c'} \Rightarrow c' = \beta (1 + r) c$$

- Note that

$$c = y - \frac{c'}{1 + r} = y - \beta c$$

So that:

$$\begin{aligned} c &= \frac{y}{1 + \beta} \\ c' &= \frac{\beta (1 + r) y}{1 + \beta} \\ s &= \frac{\beta y}{1 + \beta} \end{aligned}$$

Pay As-You-Go Social Security System

- Introduce a pay as-you-go social security system: currently working generation pays payroll taxes, whose proceeds are used to pay the pensions of the currently retired generation
- Payroll taxes at rate τ in first period. After tax wage is $(1-\tau)y$. Currently in US $\tau = 15.3\%$
- Social security payments b in second period: assume that population grows at rate n and pre-tax-income grows at rate g.
- Social security system balances its budget:

$$b = (1+g)(1+n)\tau y$$

- Household's budget constraints

$$\begin{aligned} c + s &= (1-\tau)y \\ c' &= (1+r)s + b \end{aligned}$$

Figure 10.8 Pay-As-You-Go Social Security for Consumers Who Are Old in Period *T*

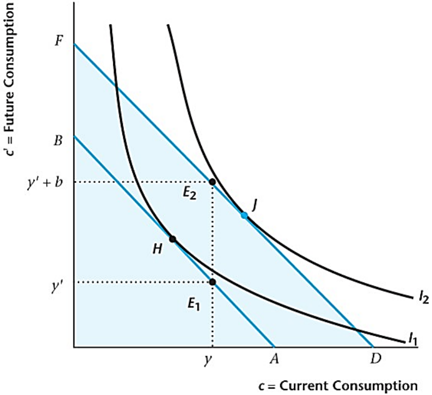
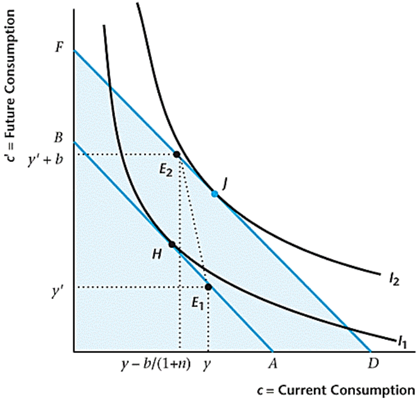


Figure 10.9 Pay-As-You-Go Social Security for Consumers Born in Period *T* and Later



- Present value budget constraint

$$c + \frac{c'}{1+r} = (1-\tau)y + \frac{b}{1+r} \equiv y^{PV}$$

- Maximizing utility subject to the budget constraint again yields

$$\begin{aligned} c &= \frac{y^{PV}}{1+\beta} \\ c' &= \frac{\beta(1+r)y^{PV}}{1+\beta} \end{aligned}$$

But now for new y^{PV}.

- Since $b = (1+g)(1+n)\tau y$:

$$\begin{aligned} y^{PV} &= (1-\tau)y + \frac{b}{1+r} \\ &= (1-\tau)y + \frac{(1+g)(1+n)\tau y}{1+r} \\ &= y - \left(1 - \frac{(1+g)(1+n)}{1+r}\right)\tau y \\ &= y + \left(\frac{(1+g)(1+n)}{1+r} - 1\right)\tau y \equiv \tilde{y} \end{aligned}$$

- Hence consumption in both periods is higher with social security than without if and only if $\tilde{y} > y$. So people are better off with social security if:

$$(1+g)(1+n) > 1+r$$

- Intuition: If people save by themselves for retirement, return on their savings equals $1 + r$. If they save via a social security system, return equals $(1 + n)(1 + g)$
- Rough US numbers: $n = 1\%$, $g = 2\%$, $r = 7\%$ (avg. stock returns). Suggests reform of the social security system desired.
- In 2005, Pres. Bush proposed a transition to a fully funded private system. Went nowhere in Congress.
- Proposal was controversial, to say the least. Social security is also a redistributive program. That role would be lessened or eliminated with private accounts.
- Private retirement accounts also subject to greater risk due to market fluctuations.

Transition to Full Funding

- Even in this basic model there is one main problem: Costly transition from pay-as-you-go to full funding.
- Problem: one missing generation: at the introduction of the system there was one generation that received social security but never paid taxes.
- Dilemma:
  1. Currently young pay double, or
  2. Default on the promises for the old, or
  3. Increase government debt, financed by higher taxes in the future, i.e. by currently young and future generations. (Tax those who would benefit from switch.)

Application of the Theory II: Ricardian Equivalence

- What are the effects of government deficits in the economy?
- A first answer: none (Ricardo (1817) and Barro (1974)).
- How can this be?
- All that matters is present value of government expenditures and taxation. Timing does not matter. Deficits today imply higher taxes in future.
- The answer outside our simple model is not as clear.

- Lump-sum taxes. Government debt B, borrows at rate r.
- Government budget constraints:

$$G = T + B$$
$$G' + (1 + r)\, B = T'$$

- Consolidating to present value govt. BC:

$$G + \frac{G'}{1+r} = T + \frac{T'}{1+r}$$

- Now suppose that the government changes timing of taxes but (PV of) spending unchanged. Example: Cuts taxes today by Δ, runs a deficit $B = \Delta$, pays back next period. So current taxes now $T - \Delta$, future taxes $T' + (1 + r)\Delta$.

- Original problem:

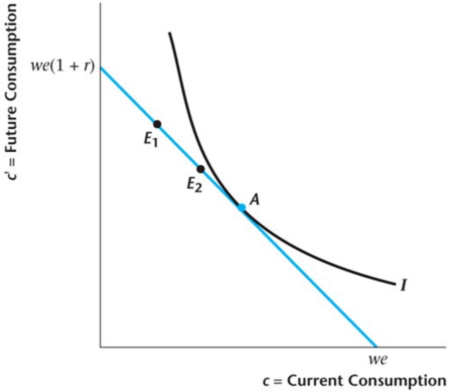
$$\max_{c,c'} u(c) + \beta u(c')$$
$$\text{s.t. } c + \frac{c'}{1+r} + T + \frac{T'}{1+r} = y^{PV}$$

- Tax cut changes budget constraint to:

$$\begin{aligned} c + \frac{c'}{1+r} + T - \Delta + \frac{T' + (1+r)\Delta}{1+r} &= y^{PV} \\ \Rightarrow c + \frac{c'}{1+r} + T + \frac{T'}{1+r} &= y^{PV} \end{aligned}$$

- Problem of the consumer is same as before.

Figure 9.16 Ricardian Equivalence with a Cut in Current Taxes for a Borrower



Pearson

Comments on Ricardian Equivalence

- Consumer spend same amount, but current income increases by Δ to savings increases by Δ. Individuals save their tax cut by buying government debt.

	Before Cut	After Cut
Period 1	$c + s = y + A - T$	$c + s = y + A - (T - \Delta)$
Period 2	$c' = y' + Rs - T'$	$c' = y' + Rs - (T' + R\Delta)$
Savings	s	$s + \Delta$

- Does not say fiscal policy is irrelevant. Here level of spending was held constant. (Compare to Lect. 5 on WWII.)
- Some argue that deficits "starve the beast": cut taxes today, run deficit, force reduction in future govt spending.

Deviations from Ricardian Equivalence

Exact Ricardian equivalence depends on some key assumptions:

1. Taxes are nondistortionary (lump-sum).
2. The tax change has no redistributive consequences.
3. Current taxpayers are alive to pay for future increases. (Or they care about their children.)
4. Credit markets are perfect. Consumers and government face same interest rate.

Empirical Evidence

- Many instances of temporary tax cuts.
- President George H. W. Bush (1992): withholding cut. Pure timing issue. Little effect on consumption.
- President George W. Bush (2001): tax rebate. Timing mixed with reduction in tax rates. Modest increases in consumption
- President George W. Bush (2007-08): Stimulus rebate. Seems to have mostly led to increased saving, modest consumption increase.
- In its exact form, Ricardian equivalence fails. Evidence that consumption does respond to temporary tax cuts, but effects not substantial.

Household Expenditure and the Income Tax Rebates of 2001, Johnson-Parker-Souleles

Using questions expressly added to the Consumer Expenditure Survey, we estimate the change in consumption expenditures caused by the 2001 federal income tax rebates and test the permanent income hypothesis. We exploit the unique, randomized timing of rebate receipt across households. Households spent 20 to 40 percent of their rebates on nondurable goods during the three-month period in which their rebates arrived, and roughly two-thirds of their rebates cumulatively during this period and the subsequent three-month period. The implied effects on aggregate consumption demand are substantial. Consistent with liquidity constraints, responses are larger for households with low liquid wealth or low income.

Impact of 2001 Rebate on Consumption

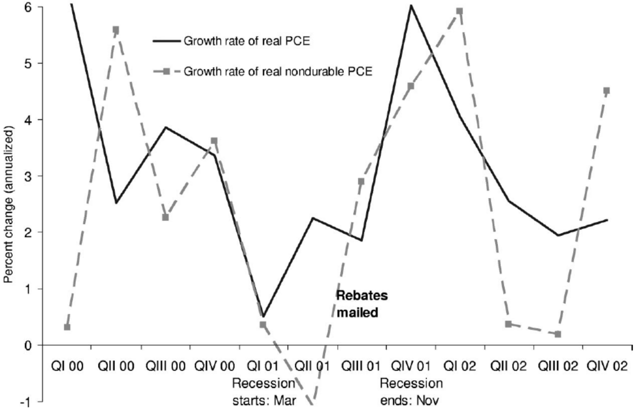


Figure 1. Growth Rates of Personal Consumption Expenditures from 2001:Q1 to 2002:Q4

Impact of 2007-08 Rebate on Savings