SOLOW MODEL WITHOUT TECH. CHANGE

ANALYSIS OF THE MODIFIED SYSTEM

(PER WORKER / CAPITA VARIABLES)

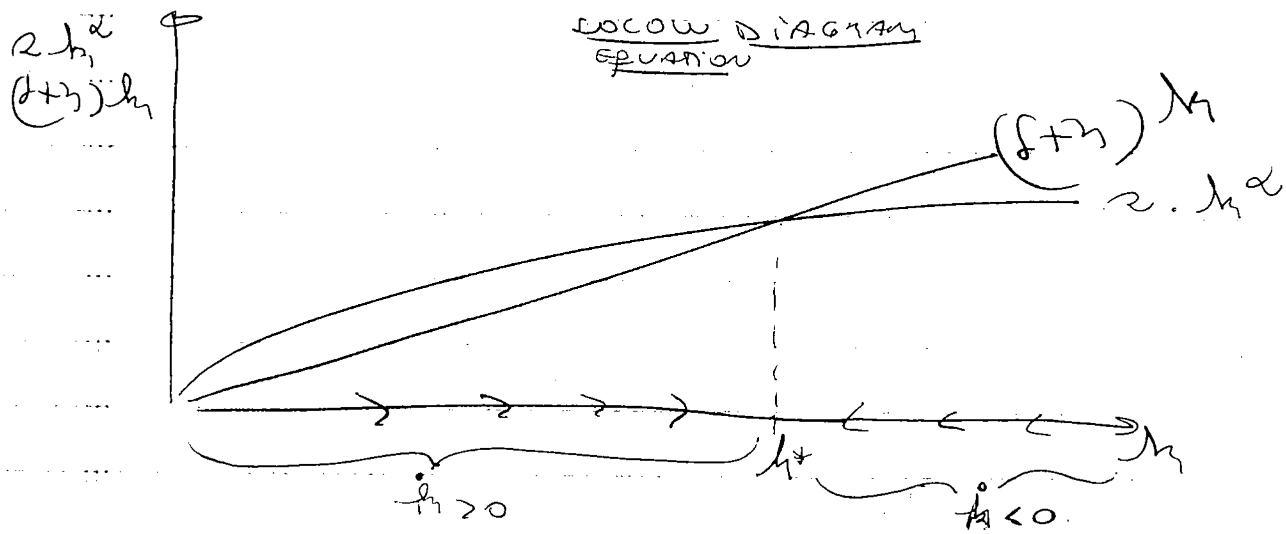
$$y = Y/L \qquad k = K/L \qquad c = \frac{C}{L} = (1-s)\frac{Y}{L}$$

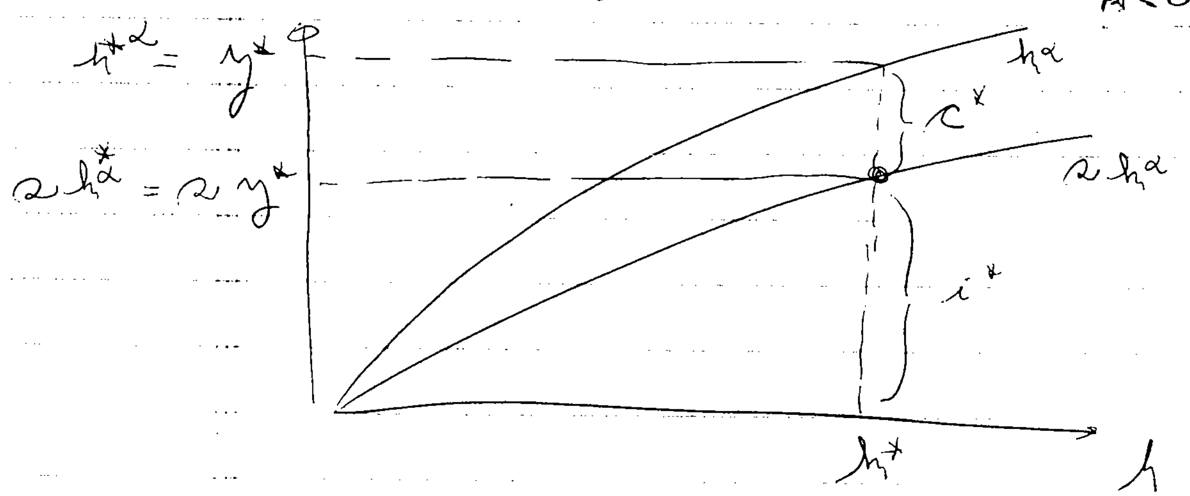
(10) $y = k^{\alpha}$

(14) $\Delta k = s k^{\alpha} - (\delta + n) k$ SOLOW EQUATION

(ALSO

$$\hat{k} = \frac{\Delta k}{k} = s k^{\alpha - 1} - (\delta + n)$$
)





NOTICE FROM SOLOW EQUATION / DIAGRAM:

CAPITAL PER WORKER

IF $k < k^*$, THEN $\Delta k > 0 \Rightarrow k \uparrow$
(ALSO $\hat{k} > 0$)

IF $k > k^*$, THEN $\Delta k < 0 \Rightarrow k \downarrow$
(ALSO $\hat{k} < 0$)

IF $k = k^*$, THEN $\Delta k = 0 \Rightarrow k$ CONSTANT
(ALSO $\hat{k} = 0$)

SO AT $k = k^*$ WE HAVE A STEADY STATE

INCOME PER WORKER

SINCE $y = k^\alpha$

$\hat{y} = \alpha \hat{k}$ AT THE S.S. $y^* = k^{*\alpha}$

$\Rightarrow \hat{y} = 0$ IF $\hat{k} = 0$ $\Rightarrow$

$\Rightarrow$ IF $y < y^* \Rightarrow \Delta y > 0$, $y \uparrow$

IF $y > y^* \Rightarrow \Delta y < 0$, $y \downarrow$

IF $y = y^* \Rightarrow \Delta y = 0$, y CONSTANT
i.e. WE HAVE A S.S.

CONSUMPTION PER WORKER

SINCE $c = (1-s)\,y = (1-s)\,k^\alpha$

THEN $\hat{c} = \underbrace{\widehat{(1-s)}}_{=0} + \alpha \hat{k} = \alpha \hat{k}$

$\Rightarrow \hat{c} = 0$ IF $\hat{k} = 0$

THEN $c < c^*$, $\Delta c > 0 \Rightarrow c \uparrow$

$c > c^*$, $\Delta c < 0 \Rightarrow c \downarrow$

$c = c^*$, $\Delta c = 0 \Rightarrow c$ CONSTANT (S

WHAT IS k^*, y^*? i.e. steady state values

CAN BE CALCULATED BY SETTING (14) TO 0

i.e.

$$\Delta k = 0 = s k^{\alpha} - (\delta + n) k$$

SOLVING THE EQUATION FOR k

$$k^* = \left[\frac{s}{\delta + n}\right]^{\frac{1}{1-\alpha}}$$

& SINCE $y = k^{\alpha}$ $\Rightarrow$

$$y^* = k^{*\alpha} = \left[\frac{s}{\delta + n}\right]^{\frac{1}{1-\alpha}}$$

REMARKS / RESULTS OF SOLOW WITHOUT TECH. CHANGE

$\boxed{1}$ • OUTPUT PER WORKER AT THE S.S.

INCREASES WITH $\uparrow s$ (SAVINGS RATE)

DECREASES WITH $\uparrow \delta$ (depreciation)

$\uparrow n$ (rate of population growth)

$\boxed{2}$ GROWTH RATES

• GROWTH RATE OF OUTPUT PER WORKER AT THE S.S. IS ZERO

• IF CAPITAL PER WORKER IS BELOW S.S. THEN IT GROWS & SO DOES OUTPUT PER WORKER

- IF CAPITAL PER WORKER IS ABOVE THE S.S. LEVEL THEN IT SHRINKS & SO DOES OUTPUT PER WORKER.

- THE FURTHER THE CAPITAL PER WORKER IS FROM THE S.S. LEVEL IN ABSOLUTE VALUE THE HIGHER THE GROWTH RATE OF OUTPUT PER WORKER IN ABSOLUTE VALUE.
(i.e. GROWTH RATE ↓ AS WE APPROACH THE S.S.)

OTHER REMARKS

AT S.S. $\hat{y} = 0 \Rightarrow \widehat{\left(\frac{Y}{L}\right)} = 0$

$\Rightarrow \boxed{\hat{Y} = \hat{L} = n}$

AND $\hat{k} = 0 \Rightarrow \widehat{\left(\frac{K}{L}\right)} = 0$

$\Rightarrow \boxed{\hat{K} = \hat{L} = n}$

THEN $\hat{Y} = \hat{K} = \hat{L} = n$

i.e. OUTPUT, CAPITAL & LABOR GROW AT THE SAME RATE (RATE OF POPULATION GROWTH)

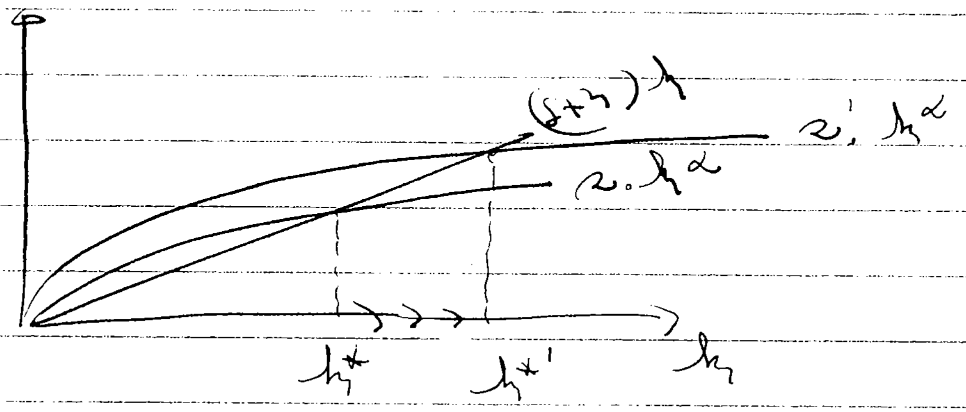
SO :

THE STEADY STATE OF
THE MODIFIED SYSTEM
IS THE BALANCED
GROWTH PATH OF
THE ORIGINAL SYSTEM —

COMPARATIVE STATICS (USE SOLOW EQ. / DIAGRAM)

$$\Delta k = s k^{\alpha} - (\delta + n) k$$

1) ↑ SAVINGS RATE (FROM s TO s')



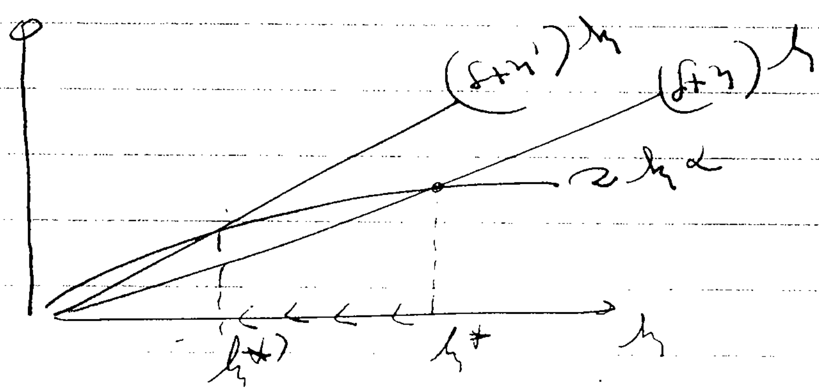
LEVEL EFFECTS: k^* ↑ to $k^{*'}$
AT S.S. $\Rightarrow y^* = k^{*\alpha}$ ↑

] i.e. output & capital per worker ~~at S.S. ↑~~

GROWTH EFFECTS:

- S.S. rate of growth of y DOES NOT CHANGE (it is still zero)
- TEMPORARY EFFECTS:
  y GROWS UNTIL NEW S.S. IS REACHED (so $\hat{k}>0$, $\hat{y}>0$ UNTIL WE GET TO $k^{*'}$)

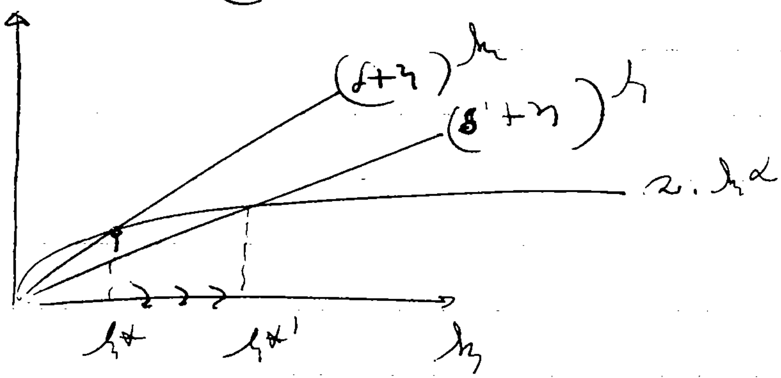
2) ↑ n (↑ RATE OF POP. GROWTH FROM n TO n')



LEVEL EFFECTS : $k^* \downarrow$ to $k^{*'}$
AT S.S. $\Rightarrow y^* = k^{*\alpha} \downarrow$

GROWTH EFFECTS
- S.S. rate of growth of y DOES NOT CHANGE (it is still zero)
- TEMPORARY EFFECTS
  y decreases UNTIL NEW S.S. IS REACHED (so $\hat{k} < 0$, $\hat{y} < 0$ UNTIL WE GET to $k^{*'}$)

3) $\downarrow \delta$ (DEPRECIATION RATE $\downarrow$ FROM δ to δ')



LEVEL EFFECTS : $k^* \uparrow$ to $k^{*'}$
AT S.S. $y^* = k^{*\alpha} \uparrow$

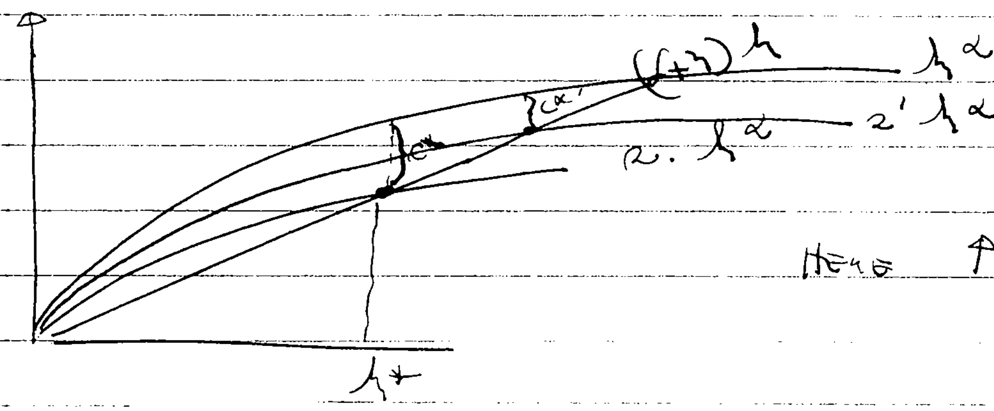
GROWTH EFFECTS:
- S.S. rate of growth of y does not change (it is still zero)
- temporary effects
  y grows until a new S.S. is reached (so $\hat{k} > 0$, $\hat{y} > 0$ until we get to $k^{*'}$)

GOLDEN RULE SAVINGS RATE (s^G)

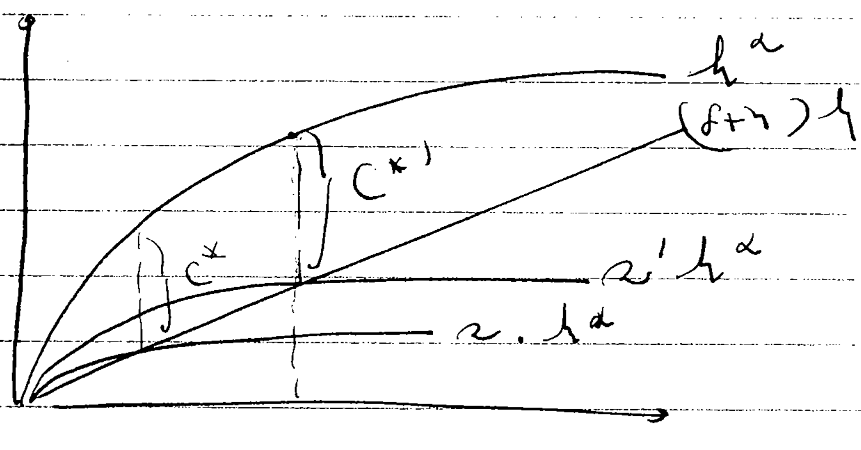
AGENTS CARE ABOUT CONSUMPTION ⇒ POLICYMAKERS MAY WANT TO FOCUS ON CONSUMPTION PER WORKER/CAPITA AT THE STEADY STATE:

$$c^* = (1-s)\, y^* = (1-s)\, k^{*\alpha}$$

BUT k^* DEPENDS ON s (i.e. $k^* \uparrow$ AS $s \uparrow$) SO c^* CAN $\uparrow$ OR $\downarrow$ WITH $\uparrow s$ — SEE EXAMPLES:



HERE $\uparrow s \Rightarrow c^* \downarrow$



HERE $\uparrow s \Rightarrow c^* \uparrow$

SAVINGS RATE THAT MAXIMIZES c^* (cons. per worker at s.s.) IS s^G (GOLDEN RULE SAVINGS RATE

NUMERICAL EXAMPLE

$Y = K^{1/3} L^{2/3}$ (i.e $\alpha = 1/3$)

& thus $y = k^{1/3}$ $s = 0.3$ $\delta = 0.1$ $n = 0$

1) what is k^*? y^*?

AT k^*, $\Delta k = 0$ $\Rightarrow$ $\underset{=0.3}{s} k^{\alpha} = (\underset{=0.1}{\delta} + \underset{=0}{n}) k$ ($\alpha = 1/3$)

$$\Rightarrow \frac{0.3}{0.1} = k^{1-1/3} = k^{2/3}$$

$$\Rightarrow \boxed{k^*} = \left(\frac{0.3}{0.1}\right)^{3/2} = 3^{3/2} = \boxed{5.196}$$

$$y^* = k^{*1/3} = (5.196)^{1/3}$$

2) Suppose $k_0 = 2$, is k ↑ or ↓?

since $k^* = 5.196$ & $k_0 < k^* \Rightarrow k$ ↑

3) Calculate $\hat{k}$ & $\hat{y}$ at $k_0 = 2$

$$\Delta k = s k_0^{\alpha} - (\delta + \cancel{n}) k_0$$

$$= 0.3 \times 2^{1/3} - 0.1 \times 2 = 0.3777 - 0.2 = 0.1778$$

$$\Rightarrow \boxed{\hat{k}_0} = \frac{\Delta k_0}{k_0} = \frac{0.1779}{2} = \boxed{0.088} = \boxed{8.8\%}$$

$$\boxed{\hat{y}_0} = \alpha \hat{k}_0 = \frac{1}{3} \times 0.088 = 0.0296 = \boxed{2.96\%}$$

4) PATH

$Y = A_t K^{\alpha} L^{1-\alpha}$ $\hat{L} = n$ $S = s \cdot Y$ $C = (1-s)Y$

$\Delta K = sY - \delta$

$\hat{K} = s\frac{Y}{K} - \delta$ since $\hat{K}_t = \frac{K_{t+1} - K_t}{K_t}$

$\Rightarrow \boxed{K_{t+1}} = K_t \hat{K}_t + K_t$

$= \boxed{(1+\hat{K}_t) K_t}$

since $\hat{L} = \frac{L_{t+1} - L_t}{L_t}$

$\Rightarrow \boxed{L_{t+1} = (1+n) L_t}$

	L_t	$K_t = (1+\hat{K}_t)K_t$	A_t	$Y_t = A_t K^{\alpha} L^{1-\alpha}$	$\Delta K = sY - \delta K$	$\hat{K} = \frac{\Delta K}{K}$
$t=0$	100	200	1	$200^{1/3} \cdot 100^{2/3}$		
$t=1$						
$t=2$						

LET $K_0 = 200$ $L_0 = 100$ $\alpha = 1/3$ $n = 0$ $s = 0.3$

$A_t = 1$ $\delta = 0.10$

OR USE PER WORKER EQUATIONS