ECON. GROWTH

- FOCUS ON GROWTH OF GROSS DOMESTIC PRODUCT (GDP) AND GDP PER CAPITA
- WANT TO HAVE MODEL TO UNDERSTAND:
  1. DIFFERENT LEVELS OF GDP P. CAPITA ACROSS COUNTRIES
  2. DIFFERENT GROWTH RATES OF GDP & GDP p.c. ACROSS COUNTRIES & ACROSS TIME
  3. EFFECT OF GOVERNMENT POLICIES ON LEVELS OF GDP p.c. & GROWTH RATES

SOLOW MODEL:

FOCUS ON: CAPITAL, LABOR, TECHNOLOGY

MAIN CHARACTERISTICS:

- DYNAMICS
- SINGLE GOOD → CONSUMPTION / INVESTMENT
- VERY SIMPLE DEMAND/CONSUMER SIDE
- SUPPLY OF GOODS (Demand of Goods approach
- CLOSED ECONOMY
- TECHNOLOGY EVOLVES EXOGENOUSLY
- LABOR FORCE PARTICIPATION CONSTANT (MANY TIMES POPULATION = WORK FOR

A SOLOW MODEL WITHOUT TECHNOLOGICAL CHANGE & WITH POPULATION GROWTH

OBJECTIVE: CHARACTERIZE MODEL BEHAVIOR TO GET IMPLICATIONS FOR POLICY AT:

(I) BGP (BALANCED GROWTH PATH)

(i) LOOK AT GROWTH RATES OF VARIABLES AND PER WORKER/CAPITA VARIABLES

$\left(\hat{K}, \hat{Y}, \hat{C} \text{ \& } \widehat{(Y/L)} = \hat{y}, \widehat{(K/L)} = \hat{k} \right), \widehat{(C/L)} = \hat{c}$

(ii) LOOK AT LEVELS OF PER WORKER/CAPITA VARIABLES

$y, k, c,$

(II) OUTSIDE BGP

(i) LOOK AT GROWTH RATES OF VARIABLES & PER WORKER/CAPITA VARIABLES

(ii) LOOK AT ADJUSTMENT OF LEVELS OF PER WORKER/CAPITA VARIABLES.

(2) BASICS

PRODUCTION FUNCTION

(1) $Y = F(K, L) = K^{\alpha} L^{1-\alpha}$ $\qquad 0 < \alpha < 1$

CONSUMPTION / SAVINGS DECISION

(2) $S = s \cdot Y$ $\qquad 0 < s < 1$

s: SAVINGS RATE

SINCE $Y = C + I \Rightarrow C = (1-s)Y$ (3)

POPULATION = # WORKERS

(4) $\hat{L} = n$

LAW OF MOTION OF K

(5) $\Delta K = I - \delta K$ $\qquad 0 < \delta < 1$

δ: DEPRECIATION RATE

EQUILIBRIUM: SAVINGS = INVESTMENT

(6) $S = I$

USING (2) (5) & (6) WE GET

(7) $\boxed{\Delta K} = S - \delta K = \boxed{s \cdot Y - \delta K}$

SINCE $\hat{K} = \Delta K / K$, USING (7)

(8) $\boxed{\hat{K}} = \dfrac{\Delta K}{K} = s\dfrac{Y}{K} - \dfrac{\delta K}{K} = \boxed{s\dfrac{Y}{K} - \delta}$

③ CHANGE OF VARIABLE (NEW VARIABLES $y = \frac{Y}{L}$, $k = \frac{K}{L}$, $c = \frac{C}{L}$)

LOOK AT MODEL IN THESE NEW VARIABLES

NEED TO CALCULATE LAW OF MOTION OF k.

DIVIDE BOTH SIDES OF (1) BY L

$$\frac{Y}{L} = \frac{K^{\alpha} L^{1-\alpha}}{L} = K^{\alpha} \cancel{L^{1-\alpha-1}} = \left(\frac{K}{L}\right)^{\alpha}$$

i.e.

(10) $$y = \underbrace{k^{\alpha}}_{\text{WE CALL THIS IN GENERAL } f(k)}$$

LAW OF MOTION OF k

SINCE $k = K/L$ USING HAT ALGEBRA

(11) $$\hat{k} = \hat{K} - \underset{n \text{ BY } (4)}{\underset{\parallel}{\hat{L}}} = \hat{K} - n$$

USING (8) & (11)

$$\hat{k} = s\,(Y/K) - \delta - n = s\left(\frac{Y}{K}\right) - (\delta + n)$$

DIVIDING & MULTIP. THE 1st TERM BY L

$$= s\left[\frac{Y/L}{K/L}\right] - (\delta + n)$$

THEN

(12) $$\boxed{\hat{k}} = \boxed{s\,\frac{y}{k} - (\delta + n)}$$

USING (10)

(12') $$\hat{k} = s\,(k^{\alpha}/k) - (\delta + n) = s\,k^{\alpha-1} - (\delta + n)$$

④ SINCE $\hat{k} = \Delta k / k$, WE HAVE $\Delta k = \hat{k} \cdot k$

WE MULTIPLY (12') BY k AND GET:

$$(14) \qquad \boxed{\Delta k} = \hat{k} \cdot k = s \cdot k^{\alpha-1} k - (\delta+n) k$$

$$= \boxed{s k^{\alpha} - (\delta+n) k}$$

REMARK:

EQ. (14) IS ALWAYS TRUE
(i.e. HOLDS AT BGP & OUTSIDE)

$$(14) \; \Delta k = s f(k) - (\delta+n) k$$

$$= s k^{\alpha} - (\delta+n) k$$

DIAGRAM