MATH NEEDED

VARIABLES ARE FUNCTIONS OF TIME.

EX: $y(t)$, $x(t)$

GROWTH RATES OF $y(t)$ $= \dfrac{\Delta y(t)}{y(t)} = \hat{y} = \dfrac{\frac{d\,y(t)}{dt}}{y(t)}$

$= \dfrac{d \log y(t)}{dt}$

$\hat{y} = 0$ IF $\Delta y(t) = 0$

SO CONSTANT GROWTH EQUAL TO ZERO $\Rightarrow$ $y(t)$ DOES NOT CHANGE

HAT ALGEBRA

$\widehat{(X\,Y)} = \hat{X} + \hat{Y}$

$\widehat{(X/Y)} = \hat{X} - \hat{Y}$

$\widehat{(X^{\alpha})} = \alpha\,\hat{X}$

STEADY STATE OF A VARIABLE (OR SYSTEM): SITUATION WHERE THE VALUE OF THE VARIABLE DOES NOT CHANGE (i.e. is STEADY)

EX: $\hat{Y} = 0$

BALANCED GROWTH PATH:
(OF SYSTEM OR VARIABLE)

SITUATION WHERE ALL VARIABLES GROW THAT CONSTANT RATES

i.e:

$\hat{Y}$ = CONSTANT
$\hat{X}$ = "

) NOT NECESSARILY THE SAME CONSTANT

DIAGRAMS – HELP WITH PATHS OF VARIABLES AND GROWTH RATES

EX: IF $\Delta Y > 0 \Rightarrow Y \uparrow$
$(\text{i.e. } \hat{Y} > 0)$

IF $\Delta Y = 0 \Rightarrow Y$ CONSTANT

IF $\Delta Y < 0 \Rightarrow Y \downarrow$

TECHNOLOGY / PROD FUNCTION REVIEW

- PROD. FUNCTION
  - MARG. PRODUCTS
  - RETURNS TO SCALE

PROD. FUNCTION : 2 INPUTS (K, L)

$$Y = F(K, L) = A K^{\alpha} L^{1-\alpha}$$

$$A > 0 \quad , \quad 0 < \alpha < 1$$

MPL = MARGINAL PRODUCT LABOR = $\left.\frac{\Delta Y}{\Delta L}\right|_{\text{SMALL CHANGES}}$

MPK = " " CAPITAL = $\left.\frac{\Delta Y}{\Delta K}\right|_{\text{SMALL CHANGES}}$

IN THIS CASE MPL ↓ AS L ↑] i.e.
MPK ↓ AS K ↑] ↓ MARGINAL PRODUCTS

RETURNS TO SCALE

THIS FUNCTION HAS CONSTANT RETURNS TO SCALE (CRS) i.e.
IF K & L CHANGE IN THE SAME PROPORTION, Y CHANGES IN THE SAME PROPORTION

EXAMPLE. ASSUME K & L ↑ BY 30%

$$\Rightarrow K_1 = 1.30\, K_0$$
$$L_1 = 1.30\, L_0$$

THEN $Y_1 = F(K_1, L_1) = A \cdot (1.30\,K_0)^{\alpha} (1.30\,L_0)^{1-\alpha}$

$= A\,(1.30)^{\alpha + 1 - \alpha}\, K_0^{\alpha} L_0^{1-\alpha}$

$= 1.30\ \underbrace{A\,K_0^{\alpha} L_0^{1-\alpha}}_{Y_0} = 1.30\,Y_0$

CHANGE OF VARIABLE

$Y = F(K, L)$ (3 VARIABLES)
$= A K^{\alpha} L^{1-\alpha}$

DIFFICULT TO GRAPH, ETC

WE WILL MAKE A CHANGE OF VARIABLE TO HAVE ONLY TWO
$\Rightarrow$ EASY DIAGRAM.

NEW VARIABLES: PER PERSON/WORKER

WE CAN DO THIS
BECAUSE OUR FUNCTION HAS CRS

STEPS

DIVIDE BOTH SIDES BY L

$$\frac{Y}{L} = \frac{A K^{\alpha} L^{1-\alpha}}{L}$$

WORK WITH RIGHT HAND SIDE

$$\frac{Y}{L} = A K^{\alpha} L^{1-\alpha} \cancel{L^{-1}} = A \frac{K^{\alpha}}{L^{\alpha}} = A\left(\frac{K}{L}\right)^{\alpha}$$

$$\boxed{y = A k^{\alpha}}$$

SO OUR NEW VARIABLES ARE

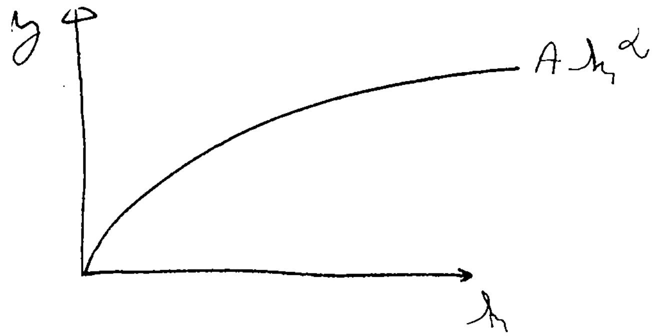
$y = Y/L$, $k = K/L$

OUTPUT PER PERSON/WORKER — CAPITAL PER PERSON/WORKER

SINCE $0 < \alpha < 1 \Rightarrow$

$\uparrow k \Rightarrow \uparrow y$

BUT AT $\downarrow$ RATES


y
$A h^{\alpha}$
h