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JOURNAL OF Economic Theory

Journal of Economic Theory 176 (2018) 255–292

www.elsevier.com/locate/jet

Information design and capital formation *

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Received 16 November 2016; final version received 1 March 2018; accepted 6 March 2018 Available online 3 April 2018

Abstract

Could a firm benefit from not disclosing all of its private information before its stock is traded in public financial markets? So long as the investors' marginal utility function is convex and the investors differ only in their risk-sharing needs, three substantive results hold: (1) a full disclosure policy *minimizes* the value of the firm; (2) lifting a mandate of full disclosure does *not* imply that firms will necessarily choose to withhold information maximally; and (3) with many firms that strategically choose disclosure policies, all Nash equilibria display only partial disclosure. Our insight is based on the role that the firm's equity can play as a risk-sharing device: if the firm chooses to keep some information private, its stock can be used by investors to hedge against risk.

The problem that we study is of theoretical interest, but it is also topical: the *Jumpstart Our Business Startups (JOBS) Act*, which was signed into law in April 2012, substantially eases securities regulations for small companies going public. The declared intent of this change in regulation was to promote capital formation in new and small companies. Our results indicate that the provisions of this new legislation are in line with its intentions. Less stringent requirements on information disclosure for smaller firms may also benefit investors.

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JEL classification: G18; G32; G38; D80

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https://doi.org/10.1016/j.jet.2018.03.004

^{*} The authors thank seminar participants at EUI, EPGE-FGV, Minnesota, Rice, UC Davis, and Western Ontario, and conference participants at SITE (Paris) for comments and suggestions. Special thanks are also due to Gilles Chemla, Gonçalo Farias, Burkhard Schipper, Pierre-Olivier Weill, and Nathan Yoder.

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Keywords: Information disclosure; Information design; Value of information; Financial regulation; Crowdfunding; Initial public offerings

0. Introduction

Could a firm benefit from not disclosing all of its private information before its stock is traded in public financial markets? The literature on asymmetric information has produced a number of important results concerning this question. In particular, the literature on persuasion games examines how in an attempt to benefit from privately known facts, the informed party manages the disclosure of these facts (e.g., Grossman and Hart, 1980; Milgrom and Roberts, 1986; Shin, 2003; Milgrom, 2008; and Che et al., 2013). But these "unraveling" mechanisms would play no role if the disclosure policy is chosen by the firm before it acquires the private information in question (e.g., Kamenica and Gentzkow, 2011).

In this paper we consider the case of a firm that needs to fund the R&D phase of a project, and can borrow for this purpose the expected value of the project's market price. This price is to be determined after the R&D stage has concluded. Now, suppose that at the time of subscribing the loan, the firm can commit to a rule determining how detailed the disclosure of the R&D results will be when the price is determined. Would the firm ever choose to commit to disclose this private information in full detail?

Broadly speaking, the problem we study is one of information design when the information to be made available in the market is chosen *ex ante* (i.e., when information is symmetric across agents). While the recent and growing literature on information design (referenced in Gentzkow and Kamenica, 2014; Bergemann and Morris, 2017) focuses on the strategic effects of information through beliefs, this paper develops market-based (risk sharing) implications of information through allocations. We argue that if the potential investors of the firm have a convex marginal utility, then it would never be in the firm's best interest to commit to a policy of full information disclosure.

Our insight is based on the role that the firm's equity can play as a risk-sharing device. If the firm chooses to disclose all of its private information, then its equity will become a riskless bond at the time of the price determination, which implies that it will lose its utility as risk-sharing device. If the firm chooses to keep some information private, the stock can still be used by investors to hedge against some risks, and their marginal valuation of the firm will change.

We present three sets of substantive results. We first show that, as long as the investors differ in their risk-sharing needs and have the same utility over consumption, with convex marginal utility a full disclosure policy *minimizes* the amount of capital that the issuer can raise. The key reason is that full disclosure minimizes the average marginal utility of income across investors, state by state. With less disclosure, investors typically cannot share their risks perfectly, which increases the average marginal utility across investors in *all* states of the world, at least weakly, so that the firm's price in the market increases. We show that any partial disclosure is therefore preferred over full disclosure by a firm seeking financing, for a large class of firm objectives encompassing all non-risk-loving behavior.

Second, we show that lifting the mandate of full disclosure does *not* imply that firms will necessarily choose to withhold information maximally. In general, firms will benefit from providing investors with some information — the value of information is not monotone in disclosure. Additionally, we show that *ex ante*, firms benefit more from a policy that only requires disclosure of detailed information about losses than from one that requires disclosure of detailed infor-

mation about gains. This, too, follows from the investors' convex marginal utility: with higher average marginal utility, the firm's potential losses are more detrimental to the equilibrium price (and, hence, to the firm's funding). To highlight one implication, if investors are not protected by limited liability, when firms choose to disclose some information they will tend to commit to disclosing information about losses rather than about high profits. This contrasts sharply with the classic "good news/bad news" result in asymmetric information environments.

Finally, we consider a market with many firms that strategically choose disclosure policies, and ask whether the strategic interaction among firms would lead to full disclosure. When the returns of the firms are not perfectly correlated, we show that all Nash equilibria have partial disclosure generically, even with a large number of firms.¹

In summary, we show that limited disclosure is beneficial for firms, so long as the investors' marginal utility function is convex (essentially, absolute risk aversion is decreasing in wealth).² The results demand little knowledge from the firm and hold for any distributions of firm returns and distributions of investors' wealth, and regardless of other assets traded by the investors, even if investors can insure against some of the shocks of the firm, so long as they cannot insure against all of them — if they could, disclosure would be irrelevant.³

Related literature We model an entrepreneur's incentive for disclosure at the early stage of financing as a problem of *information design*, as in Kamenica and Gentzkow (2011). We emphasize the asset pricing implications of information design by studying the effect that information disclosure has on risk sharing and hence on the pricing kernel. The main results in Kamenica and Gentzkow (2011) suggest that if the value function of the sender is convex in the belief of the receiver, then information disclosure creates dispersion in posterior beliefs, which benefits the sender since the value function lies strictly below its concavification at the prior. This paper argues that with convex marginal utility, the value function of the sender is convex in the allocation; full information disclosure minimizes the dispersion in the allocation, which minimizes the benefit of the sender.

This paper contributes to the literature on games of persuasion, cf. Milgrom (2008). While our focus on the *ex ante* incentive for disclosure gives strong conclusions by muting channels due to information asymmetry, the mechanism we highlight operates through the impact that information has on asset prices *ex-post*. Hence, the mechanism through which information affects asset prices, which is key to our results on the *ex ante* incentive for disclosure, is also relevant to the incentives to disclose information once the informational asymmetry has arisen. After the sender has acquired superior information, in the absence of commitment to a disclosure rule, the

 $^{^{1}}$ If the returns are perfectly correlated, there is a Nash equilibrium where all firms disclose all the information (which is now common across them), just because all other firms are doing the same and in spite of the fact that they are all worse-off because of it. This equilibrium disappears, though, if one assumes that information disclosure is costly, no matter how small this cost is, in which case there is no Nash equilibrium that results in full information disclosure.

² The utility functions most common in economic models (e.g., CARA, CRRA, logarithmic) have globally convex marginal utility functions. If the marginal utility function were linear, imperfect risk sharing would have no impact on the equilibrium price of the asset, which would depend on the *average consumption* alone; disclosure of any events would have no effect. Let us also note that the convexity of marginal utility matters for the implications of information disclosure through heterogeneity in marginal utility functions *across* investors and in *every* state. This differs from the precautionary saving motive, which operates through the *individual* optimization effects of marginal utility differences *across* states.

 $^{^{3}}$ While we refer to full disclosure throughout, all our results require only that the stricter disclosure policy is sufficiently close to full disclosure.

incentive for disclosure is the result of two forces that have opposite effects on the price of the asset: i) disclosing precise information about "good news" (i.e., states in which the firm's return is high) tends to increase the price of the asset, whereas ii) withholding information promotes the role that the firm's equity play as a risk-sharing device, which also tends to increase the price. The first force is the source of the unravelling mechanism (Milgrom, 1981). The second force is the economic mechanism that this paper highlights.

In the past decade, some authors have discussed the potential welfare-reducing effects of disclosing public information about fundamentals when agents learn from public (price) and private signals (e.g., Morris and Shin, 2002; Angeletos and Pavan, 2007; Amador and Weill, 2010; and Kurlat and Veldkamp, 2013). These arguments, too, explore inference and coordination externalities among investors when information is asymmetric. In any case, the economic mechanisms deriving from the information dispersed among investors, as proposed by these authors, would reinforce our conclusions once the effects of differential disclosure based on the scale of the firm are taken into account.

In the context of the recent economic recession, several works have put forward new arguments according to which less transparency ensures more market liquidity. Pagano and Volpin (2012) and Dang et al. (2009) suggest that security design itself may give rise to adverse selection and shut down trade. Morris and Shin (2012) argue how market confidence, defined as approximate common knowledge, can shut down trade in the presence of adverse selection. The closely related discussion of regulatory reforms regarding transparency has pointed to trade-offs between accuracy and commonality of beliefs (Morris and Shin, 2007; Holmstrom, 2009). Again, all these effects of asymmetric information, either direct or through higher-order beliefs, are absent in our analysis.

Importantly, the incentive effects due to asymmetric information between the investors and the firm do not bind for early-stage financing, given that information about returns is limited for entrepreneurs seeking funding and investors alike. Likewise, the effect that disclosure has on incentives for moral hazard by tying stock prices to managerial actions, thereby enhancing investment efficiency at the firm level (e.g., Fishman and Hagerty, 1989), is less likely to play a role at the stage where the information about returns needed for structuring incentives is unavailable.

To be sure, the idea that less information can make agents better off has been recognized at least since Hirshleifer (1971). The economic mechanism we present differs from the Hirshleifer effect: in our setting, investors would benefit unambiguously if information were to be disclosed fully.

JOBS v. SOX The problem that we study is of theoretical interest, but it is also topical. During the recession of 2009–2011, with a decline in IPOs and mergers and acquisitions, concerns arose about the inability of small companies to raise the equity needed to fund the start of their economic activities.⁴ In response to this concern, the U.S. Congress approved the *Jumpstart Our Business Startups (JOBS) Act*, which was signed into law in April 2012. This act substantially eases securities regulations for small companies going public: it lightens reporting requirements, reduces the amount of time required between the planning and the actual occurrence of the IPO, and permits confidentiality of communications between the company and the SEC, including the

⁴ After peaking in 1997 with 8,823 exchange-listed companies, public company listings in the United States declined for 15 consecutive years to only 4,916 companies at the end of 2012 (Weild et al., 2013).

company's intention to issue the IPO and information about its finances, until just before shares are sold.⁵

The intent of this change in the regulatory framework is not only to make it easier for companies to go public in an IPO process,⁶ but also to enhance the ability of smaller business to raise capital in private markets through *crowdfunding*, subject to even lighter regulation. The rationale cited was that forcing small firms to engage in extensive information disclosure was detrimental to their funding, particularly for young, high-growth companies seeking capital from the public market, and for small startups looking for financing in the private market in a rapid manner.

At the same time, the JOBS Act has been criticized by state regulators and investor advocate groups,⁷ who argue that the practice of lesser information disclosure will not only decrease investors' welfare, but also reduce their willingness to invest and hence the value of equity ultimately raised by the issuing firms.

To a large extent, these criticisms are consistent with the existing literature. The arguments based on the classic information unraveling suggest that the lighter provisions of the JOBS Act would not be beneficial for firms raising equity: a potential buyer would interpret the limited disclosure of information as an indication that the firm is of low value, which would in turn lower his willingness to pay. A message of this paper, on the other hand, is that the criticisms levied on the JOBS Act are excessively pessimistic and that, in fact, the provisions of this new legislation are in line with its intentions. To begin, the very nature of the innovations and start-ups explicitly targeted by the JOBS Act is that the seeking of financing occurs at an early, often experimental stage of project development, when uncertainty about the return is faced by both investors and entrepreneurs. Information gathering by entrepreneurs itself requires a certain advancement in project development, and the unraveling it causes is not of the first order: the canonical information asymmetry in which the 'quality' of innovation (or the state of the world) is known to one of the parties is largely absent at the phase in which firms seek funding.

As for efficiency and investor welfare, major concerns of the JOBS Act's critics, our results suggest that less stringent requirements on information disclosure for smaller firms may also benefit investors. Indeed, the financing options introduced by the JOBS Act have created a regulatory

⁵ Since 2002, the *Sarbanes-Oxley (SOX)* Act has been the general regulatory framework governing the informational, financial, accounting, and remuneration practices of firms whose shares are traded on U.S. securities exchanges. It was introduced in response to the scandals that affected corporations as large as Enron and WorldCom in 2000 and 2001. Among other restrictions, the SOX Act imposes rules requiring the disclosure of all internal information that may be of relevance to potential investors in *any* publicly traded firm.

In particular, strict disclosure rules apply during the period in which a company prepares to release its stock on the market for the first time in an IPO. The Act requires that all communications between the company and the Securities and Exchange Commission (SEC) regarding the IPO be made available to the public; it also establishes minimum time periods between different phases of the IPO, which are meant to give potential buyers ample opportunity to scrutinize the information made available by the company. Analogous legislation has been enacted in Australia, Canada, France, Germany, Holland, India, Italy, Japan, South Africa, Turkey, and the United Kingdom.

 $^{^{6}}$ For the past 80 years, any public announcement by a startup that they were seeking investment — be it at speaking engagements, through videos, or via a post on their website or social networks — was deemed illegal by Rule 506 of Regulation D and Rule 144A of the Securities Act of 1933. Title IV of the JOBS Act, which was approved on March 25, 2015, lifted the solicitation ban with respect to unaccredited (as well as accredited) investors, subject to limitations on their investment.

⁷ Including the American Association of Retired Persons, or AARP, the Consumer Federation of America, the Council of Institutional Investors, the North American Securities Administrators Association, and Americans for Financial Reform.

model that allows for differential disclosure that "scales" with business size.⁸ Our argument has two parts. We first show that by affecting the investors' ex-ante risk-sharing needs, and hence their willingness to pay, any limitation on information to investors also alters the marginal expected revenue function of the entrepreneurs. This marginal revenue effect differs at small and large business scales. Limited disclosure raises more capital than stricter disclosure requirements for small-scale firms, whereas disclosing *more* may be preferred by larger firms. By recognizing that the disclosure policy transforms firms' marginal revenue functions and indirectly impacts their choices of business scale, our analysis suggests that the new financing framework exploits the *differential* impact of disclosure between small and large firms in a way that can be Pareto-improving. This qualifies the negative conclusion about strict disclosure — which continues to apply to large firms in the new financing framework — that one might draw from the analysis for a fixed business size.⁹

The economic mechanism that this paper highlights as relevant in the design of the financing framework is that disclosure policy can be an effective instrument to impact firm financing, incentives and welfare *even absent inference effects associated with asymmetric information*. The mechanism through which limited disclosure can increase capital raised operates by introducing less than full risk sharing among investors. While this conflicts with efficiency when the magnitude of innovation is held fixed, it may represent a Pareto improvement when the effect of information disclosure on the size of innovations is recognized. Incidentally, much of the funding collected via crowdfunding platforms comes from investors within the same economic and business communities as the entrepreneur, in particular during the early stages of the project's development. We therefore allow the firm to affect the pricing kernel through its information disclosure and the scale of its production.¹⁰

1. The setting

Consider a market in which a firm, competitive investors, and an investment bank interact over three periods, t = 0, 1, 2. In period 0, the firm wishes to raise capital to fund the research and development (R&D) phase of a project. The outcome of this project, namely the firm's payoff (or gross profit) in period 2, is *ex ante* uncertain. Denote by S the finite set of states and for each $s \in S$, let π_s be the profit in that state. There is a commonly held prior belief, Pr(s) > 0, that state s will occur.

There are $I \ge 2$ classes of investors, indexed by i = 1, ..., I. We assume for simplicity that each of these classes is of unit mass, but this is immaterial for our results. Investors differ in their

⁸ We review these options (crowdfunding, IPO On-Ramp and Regulation A+) and how their business size contingency scales in Appendix B.

⁹ Let us remark that the Act also scales the investments allowed in firms of certain size, imposing a limit on investments in small firms and accreditation requirements for investors in large firms. An implication of decreasing absolute risk aversion is that the differential-with-scale disclosure framework matches the investments of small investors, whose capacity to bear risk is lower, with small firms; and those of larger investors, whose capacity to bear risk is greater, with large firms.

¹⁰ Despite transactions occurring online, the average distance between the lead venture capitalist and the project seeking funding is approximately 70 miles. Similarly, investors who provide capital for a business start-up in exchange for convertible debt or ownership equity, often known as *angel investors*, are typically located less than half a day of travel from the business they fund (Sohl, 1999; Stuart and Sorenson, 2005; Wong et al., 2009; and Agrawal et al., 2011). When categories of funding are considered independently, geographic concentration is higher in *every* category than it is in the aggregate (Mollick, 2014). In addition, since they are often potential users, investors are attracted by having early access to the product.

future wealth: those of type *i* have an income w_s^i in period 2 when the state is s.¹¹ Investors are risk averse: given wealth *x* in period 1 and uncertain wealth $(x_s)_{s \in S}$ in period 2, their *ex ante* utility is

$$x + \sum_{s \in \mathcal{S}} \Pr(s) \cdot u(x_s).$$

We assume that function u is twice continuously differentiable, strictly increasing and strictly concave, and satisfies the standard Inada conditions. Throughout, we also assume that the investors' marginal utility is strictly convex, so that they exhibit *prudence*. Quasi-linear preferences mute the wealth (Hirshleifer) effect to isolate the effect of information on period 2 wealth heterogeneity. The assumption that all investors have common preferences means that their motive for trading is inter-temporal smoothing and/or risk sharing, and not any kind of betting.

In period 0, the firm receives liquidity from a risk-neutral investment bank. These funds represent the amount that the bank and the firm agree upon in the underwriting contract, discounted at a risk-free interest rate, \bar{r} .¹² Once the R&D stage is completed, at date 1, the firm privately learns the realized state of nature, after which the underwriting investment bank sells the firm's stock to the investors.

1.1. Information disclosure and liquidity

The firm chooses how much of its private information to disclose before the selling of its asset takes place. While the firm need not disclose all its information, its statements about the payoff of its investment project are verifiable: we assume that, after realizing state *s*, the firm announces an event $E \subset S$ such that $s \in E$ and that each agent places prior probability $Pr(E) = \sum_{s \in E} Pr(s)$ on the event *E*.

The firm's asset is traded by investors in a competitive market, and the firm uses the investment bank as a commitment device.¹³ We assume that when the bank and the firm sign the underwriting contract, the firm commits to the partition \mathcal{P} of the state space which determines

$$\sum_{s \in \mathcal{S}} \Pr(s) \cdot u'(w_s^i) = \sum_{s \in \mathcal{S}} \Pr(s) \cdot u'(w_s^{i'}).$$
(*)

Our results remain unaffected by this assumption.

¹¹ In general, after trading in all other existing assets, the investors' wealth and the project's payoff may still be correlated. We will assume that no other assets are traded until Section 5. The reader may want to impose further structure on the distribution of investors' incomes. Of particular interest is the assumption that investors may have traded a risk-less asset, in which case one would like to impose that for all investors *i* and *i'*,

¹² E.g., in the most commonly used types of contracts, the *firm commitment contract* and the *best effort contract*, the bank guarantees the sale of the entire or feasible amount at this agreed upon price. Implicitly, we assume that there is a set of competitive risk-neutral investment banks, with free entry to the market, who share a common prior with the firm and the investors. This is why we assume that the underwriting bank does not extract any surplus, and discount the price at rate \bar{r} .

¹³ At the risk of being repetitive, note for instance that under the SOX Act, the firm would have no legal choice but to reveal all its private information. Under JOBS this is no longer the case. For instance, the firm can sign a contract with the investment bank under which the bank will impose a penalty if the information disclosure does not agree with some partition. If the stipulated penalty is high enough, the bank will have an incentive to enforce this contract, which is now legal.

how it will disclose information to be discovered.¹⁴ In period 1, once the firm realizes what state occurred, it makes public the event of the partition containing that state. Thus, prices will be event-contingent. The *ex ante* commitment of the firm to a partition implies that the investors cannot discern any of the firm's private information beyond the event that is revealed: the "unraveling" argument of Milgrom and Roberts (1986) does not operate, and the posterior belief of investors for state *s* is simply Pr(s | E).¹⁵ Note that the mechanism by which information disclosure affects the pricing kernel, which is the key to our results, is equally relevant to the incentive for disclosure after the firm acquires superior information (cf. discussion at the end of Section 2).

We will refer to the case when the firm chooses the finest partition, $\mathcal{P}^* = \{\{s\} \mid s \in S\}$, as *full information disclosure*. The opposite case, when the firm chooses $\mathcal{P}_* = \{S\}$, amounts to *no information disclosure*. Any other partition will be referred to as a case of *partial information disclosure*.

The partition chosen by the firm at date 0 will determine the information with which investors trade in period 1. Denote by p(E) the price, to be determined endogenously, when the investors are informed of event *E*. Foreseeing these event-contingent prices, the liquidity provided by the investment bank to the firm at date 0 is

$$L(\mathcal{P}) \equiv \frac{1}{1+\bar{r}} \cdot \sum_{E \in \mathcal{P}} \Pr(E) \cdot p(E).$$
⁽¹⁾

Formally, once the firm chooses a partition \mathcal{P} , the price of its stock becomes a random variable p over the state space S, measurable with respect to partition \mathcal{P} . This variable is the mapping $s \mapsto p(E_s)$, where E_s denotes the partition cell that contains state s.

1.2. The price of the firm

After event *E* has been announced, if the price of the stock is \bar{p} , investor *i* chooses a quantity $y_E^i(\bar{p})$ of the stock trade in order to solve the following optimization problem

$$\max_{y\in\mathbb{R}}\left\{-\bar{p}\cdot y+\sum_{s\in E}\Pr(s\mid E)\cdot u(w_s^i+y\cdot\pi_s)\right\}.$$

The stock price after the announcement of event E, p(E), is such that the total equity of the firm is absorbed by the public: $\sum_i y_E^i(p(E)) = 1$. This price is uniquely determined for each event in the partition; hence, the objective function for the firm is well-defined.

Denote by X(E) the set of all period-2 investor wealth levels that may result from the trade of the stock after event *E* has been announced. That is, X(E) contains all profiles

$$[(x_s^i)_{s\in E}]_{i=1}^I \in \mathbb{R}^{||E|| \times I}$$

$$\tag{2}$$

 $^{^{14}}$ Alternatively, the firm's choice of partition can be understood without requiring this *ex ante* commitment: suppose that at date 0 the firm chooses how much information it will *gather* to be published in the future IPO prospectus; the R&D phase will then reveal which cell of the partition the state lies in. All our conclusions carry over even if not all partitions are feasible.

¹⁵ In line with Ft. 14, one may simply assume that the firm only observes an event that contains the actual state of nature, and then announces an event in the partition that contains its private observation, which is consistent with verifiability. Relatedly, it has become common for underwriters in IPOs under the "emerging growth status" to voluntarily impose a contractual research-quiet period. For IPOs that will be listed on a national securities exchange registered with the SEC, this period typically lasts for 25 calendar days following the IPO effective date.

that satisfy the following conditions: (1) for each $s \in E$, $\sum_i (x_s^i - w_s^i) = \pi_s$; and, (2) for each *i*, there exists some y^i such that $x_s^i = w_s^i + y^i \cdot \pi_s$ for all $s \in E$. Let x(E) be the unique maximizer of

$$\max\left\{\sum_{i}\sum_{s\in E} \Pr(s \mid E) \cdot u(x_{s}^{i}) : [(x_{s}^{i})_{s\in E}]_{i=1}^{I} \in X(E)\right\},\tag{3}$$

and define, for each $s \in E$,

$$\kappa(E,s) \equiv \frac{1}{I} \cdot \sum_{i} u'\left(x_s^i(E)\right). \tag{4}$$

Eq. (4) defines a random variable, $s \mapsto \kappa(E_s, s)$, where, as before, E_s denotes the cell in \mathcal{P} that contains *s*, which will be key in the analysis. We refer to this variable as the *pricing kernel*. This is so, because the equilibrium price of the stock satisfies

$$p(E) = \sum_{s \in E} \Pr(s \mid E) \cdot \kappa(E, s) \cdot \pi_s.$$
(5)

This result follows from the equivalence of the first-order conditions that characterize the solution to problem (3) and the ones that characterize the competitive equilibrium allocation of the firm's equity.¹⁶ Thus, the kernel relates the investors' willingness to pay to the price of the firm's asset and is a measure of the asset's relative scarcity after the realization of each event in the partition of the state space created by the firm.

1.3. The firm's objective

We first assume that the firm ranks information partitions according to the liquidity they generate. That is, the firm prefers \mathcal{P} over \mathcal{P}' if, and only if, the expected liquidity generated under partition \mathcal{P} is higher than that under \mathcal{P}' : $L(\mathcal{P}) \ge L(\mathcal{P}')$.¹⁷ In Section 7.1, we show that our results hold for a general class of firm objectives in which the firm prefers partitions which induce larger pricing kernels.

Lemma 1 (Monotonicity of the firm's preferences). If the payoff of the firm is positive in all states of the world, then its preferences are strictly monotonic in the following sense: whenever a partition induces a first-order stochastic improvement in the pricing kernel relative to that of another partition, the firm strictly prefers the former to the latter.

2. Suboptimality of full disclosure

Our first main claim is that *any* partial disclosure partition will be preferred to full disclosure. The key insight for this result is that full information disclosure minimizes the pricing kernel in the first-order stochastic dominance sense.

¹⁶ The reader may find the result counter-intuitive, as it seems to imply that the equilibrium allocation is first best even though the investors are effectively trading on an incomplete financial market. But this is not so, since the domain of problem (3), namely set X(E), already considers the incompleteness of that market. What the characterization exploits is the fact that, since there is only one commodity in each state of the world, the competitive equilibrium allocation is constrained efficient, or second best.

 $^{1^{7}}$ With risk-neutral investment banks, it is immaterial whether the firm gets the loan at date 0 or instead wants to maximize its expected value at time 1. This is why we later refer to this class of preferences as *expected* liquidity preferences.

In the statement of the proposition and in what follows, a property is said to hold generically if it holds on an open subset whose complement has null Lebesgue measure.¹⁸ Also, we will say that the investors' wealth is *heterogeneous* if there exist at least two investors *i* and *j* for whom $w_s^i \neq w_s^j$ at some state *s*.

Proposition 1 (Suboptimality of full disclosure). Suppose that the payoff of the firm is positive in all states of the world. Then:

- 1. Any partition that discloses no or only partial information is strictly preferred to the full disclosure partition, generically in the investors' wealth profiles and the firm's payoffs.
- 2. Given the firm's payoffs, the latter is true generically in the investors' wealth profiles.
- 3. If the investors' wealth is heterogeneous, there exists at least one partition that is strictly preferred to the full disclosure partition, generically in the firm's payoffs.
- 4. For all wealth profiles and firm payoffs, all partitions are at least as good as the full disclosure partition.

The intuition behind this result is the following. In the full information partition, the firm eliminates all risk present during asset issuance, so the investors trade the stock only because it provides risk-less savings, and all investors have the same wealth *ex post* (i.e., for all *s*, $x_s^i = x_s^j$, for all *i* and all *j*). If a partition instead discloses less information, some risk remains at issuance. Generically, the investors will be unable to use the firm's equity to trade away the remaining risk, and consequently they will not all have the same *ex post* wealth in at least one state. Since the marginal utility of wealth is convex, this dispersion in wealth will increase the average marginal utility of investors, namely the pricing kernel, in that state.

The second statement in the proposition includes as a particular case the situation where the only risks in the economy pertain to the investors' future wealth, as in Example 2 below; the third one allows for the case where the investors themselves face no risk, but have heterogeneous wealth, as in Example 1 below. Notably, the issuer does not need to know the wealth profiles of the investors in order to determine that full disclosure is suboptimal: the firm prefers any disclosure scheme which yields only partial information over full disclosure. This holds for all distributions of investor wealth and not just in expected terms. We will show below that the result extends to a general class of firm preferences that includes essentially any form of non-risk loving behavior.

Proposition 1 asserts that the full disclosure partition is the least preferred one, but it does not imply that disclosing no information is the optimal decision of the firm. Furthermore, the proof of the proposition shows that in its optimal partition the firm will fully disclose *at most one* state. Otherwise, when combining any two singleton states in the same event, the pricing kernel would increase in first-order stochastic dominance. Intuitively, whether a contingency will be disclosed depends on the heterogeneity in the distribution of investors' wealth, i.e., how they correlate with the payoffs of the firm.¹⁹ With two states, it follows that no information will be disclosed.

¹⁸ Genericity in investors wealth means, then, that the property holds for all $(w_s^i)_{i,s}$ on an open subset of \mathbb{R}^{SI}_{++} with full Lebesgue measure. Genericity in the firm's payoffs refers to $(\pi_s)_s$ on a generic subset of \mathbb{R}^{S}_{++} . Genericity in both investors' wealth and firm's profit refers to a generic subset of $\mathbb{R}^{SI}_{++} \times \mathbb{R}^{S}_{++}$.

¹⁹ Precisely, suppose that the optimal choice of the entrepreneur is $\mathcal{P} = \{E_1, \dots, E_N, \{S\}\}$, so that state *S* is the contingency that is fully disclosed, when realized. A necessary condition for optimality is that, for all $n \leq N$,

Corollary 1 (Optimality of no disclosure with two states). Suppose that there are only two states of nature, and that the payoff of the firm is positive in both of them. Disclosure of no information is optimal for the firm. Generically in the investors' wealth profiles, this strategy is strictly preferred to full disclosure.

To examine the role of firm vs. investors' risk, we now introduce two examples that we will analyze throughout the paper. For simplicity, the examples feature only two potential investors. To reiterate, consider the following question: suppose that before the equity of the firm is allocated to investors, the firm learns the state of the world; is it in its best interest to reveal this information to the investors? For clarity of exposition, we answer this question in two parts. First, we argue that full disclosure of a state minimizes the dispersion in wealth among investors in that state. Second, we argue that the pricing kernel is higher, in a given state, when investors' wealth is dispersed in that state.

If the firm reveals the state, no uncertainty remains and the allocation will be made such that the marginal utilities of income are the same for both investors: upon revelation of state s,

$$u'(w_s^1 + \pi_s y^1) \cdot \pi_s = u'(w_s^2 + \pi_s y^2) \cdot \pi_s,$$
(6)

where y^i is the share of the stock of the firm allocated to investor *i*, so that $y^1 + y^2 = 1$. Assuming that the marginal utility of the investors is decreasing and profits are positive in both states, Eq. (6) implies that both investors will consume the same amount, which is necessarily

$$x_s^i = \bar{x}_s = \frac{1}{2} \cdot (w_s^1 + w_s^2 + \pi_s).$$
⁽⁷⁾

The average marginal utility is, hence, $\kappa(\{s\}, s) = u'(\bar{x}_s)$.

In the previous case the allocation of equity may differ across states of the world. This cannot happen if the entrepreneur does not disclose the state, in which case the allocation will be such that

$$\sum_{s} \Pr(s) \cdot u'(w_{s}^{1} + \pi_{s} \cdot y^{1}) \cdot \pi_{s} = \sum_{s} \Pr(s) \cdot u'(w_{s}^{2} + \pi_{s} \cdot y^{2}) \cdot \pi_{s}.$$
(8)

The average marginal utility would then be, $\kappa(S, s) = \frac{1}{I} \cdot \sum_{i} u' (w_s^i + \pi_s \cdot y^i)$.

Whether the allocation characterized by Eq. (8) raises more capital than the one resulting from Eq. (6) depends on the effect that the revelation of information has on the average marginal utilities, $(\kappa_s)_s$. Imagine for a moment that the same allocation is induced in both cases. For this to be true, it must be that

$$\frac{1}{2} \cdot (w_s^1 + w_s^2 + \pi_s) = w_s^i + \pi_s \cdot y^i$$

$$\sum_{s\in E_n} \Pr(s) \cdot [\kappa(E_n, s) - \kappa(E_n \cup \{S\}, s)] \cdot \pi_s \ge \Pr(S) \cdot [\kappa(E_k \cup \{S\}, S) - \kappa(\{S\}, S)] \cdot \pi_S,$$

which can be expressed as

$$\sum_{s \in E_n} \frac{\Pr(s)}{\Pr(S)} \cdot \left[\sum_i u' \left(x_s^i(E_n) \right) - \sum_i u' \left(x_s^i(E_n \cup \{S\}) \right) \right] \cdot \frac{\pi_s}{\pi_S} \ge \sum_i u' \left(x_s^i(E_n \cup \{S\}) \right) - \sum_i u' \left(x_s^i(\{S\}) \right).$$

The right-hand side of this inequality is strictly positive, generically in investors' wealth.

for both investors, at all states. These equations reduce to the requirement that for some ϑ that does *not* vary with *s*,

$$\frac{1}{2} \cdot (w_s^1 - w_s^2) = \pi_s \cdot \left(\vartheta - \frac{1}{2}\right) \tag{9}$$

for all states. This is a linear system that has as many equations as there are states of the world, and only one variable. Generically in the values of the investors' wealth and the profits of the firm, the system has no solution. If follows that the allocation of consumption that results when the firm reveals no information differs from the one resulting upon revelation of the state of the world, generically.

Let us consider two extreme environments. In Example 1, investors face no risk, the firm does, and the information it obtains privately refers to its own, idiosyncratic risk. In Example 2, the firm itself faces no risk and its information pertains to the risks of its investors. For instance, imagine that the two investors are the producers of some differentiated technology that the firm will use in its project. The firm itself only needs to test which input better suits its own technology, and the question is whether it should reveal the chosen technology before releasing its own equity.²⁰

Example 1 (*Pure firm risk*). Suppose that there are two states of the world, in both of which investor 1 has a wealth $w^1 = 1$ and investor 2 has wealth $w^2 = w$. In state 1, the firm has profits of 1, while in state 2 these are π . Then, Eq. (9) becomes

$$\begin{pmatrix} 1-w\\ 1-w \end{pmatrix} = 2 \cdot \begin{pmatrix} 1\\ \pi \end{pmatrix} \cdot \left(\vartheta - \frac{1}{2}\right).$$

If $w \neq 1$, so that the investors' wealth displays heterogeneity, the system has a solution if, and only if, $\pi = 1$. In words, this means that for the disclosure policy to be neutral in terms of income allocation across investors, it would have to be true that the firm itself faces no risk. \Box

Example 2 (*Pure investor risk*). Again, there are two states of the world. Investor 1 has a future wealth $w_1^1 = w$ in state 1 and $w_2^1 = 1$ in state 2, while investor 2 has the opposite: $w_1^2 = 1$ and $w_2^2 = w$. The firm has profits of π in both states of the world.

In this case, Eq. (9) becomes

$$\binom{w-1}{1-w} = 2 \cdot \binom{\pi}{\pi} \cdot \left(\vartheta - \frac{1}{2}\right).$$

Assuming that $\pi \neq 0$, this system has a solution if, and only if, w = 1. That is, in this case the policy would be neutral only if the investors faced no risk to begin with. \Box

The previous analysis illustrates the fact that the revelation of information will generically affect the allocation of wealth among investors. In order to see how this affects the value of the firm, suppose now that the investors have logarithmic utility indexes, namely that $u(c) = \ln(c)$.

Suppose initially that we are in the (negligible) case where Eq. (9) has a solution. Then, in each state both investors consume \bar{x}_s , and the average marginal utility is $\kappa(\{s\}, s) = 1/\bar{x}_s$. Alternatively, in the generic set where (9) has no solution, it must be true that in at least one state one

 $^{^{20}}$ A notable example of this situation is the significant effect that engine choices made by airlines and airplane manufacturers have on the profits of engine manufacturers. It is perhaps not a coincidence that cross shareholding and joint ventures have increased between the engine divisions of companies that remain competitors in other market sectors.

of the agents will consume more than \bar{x}_s and the other less. For the former, the marginal utility would increase, and for the latter it would decrease. Since u'(x) = 1/x is a convex function, this dispersion in ex-post wealth across investors will increase the average marginal utility,

$$\kappa(S,s) = \frac{1}{2} \cdot \left(\frac{1}{x_s^1} + \frac{1}{x_s^2}\right) > \frac{1}{\bar{x}_s} = \kappa(\{s\}, s).$$

This will in turn result in an increase in capital raised.

In Example 1, the equity of the firm serves as an instrument to transfer income across the two time periods. In Example 2 it serves that same purpose, but it also becomes a risk-sharing device. In general, we allow for situations in which both the firm and the investors are subject to some risk, and our results hold generically in such setting.

We just illustrated that full disclosure minimizes the pricing kernel, state-by-state. Proposition 1 concludes that, *ex ante*, limited disclosure raises more liquidity than full disclosure does; that is, generically,

$$\max_{\mathcal{P}} \sum_{E \in \mathcal{P}} \sum_{s \in E} \Pr(s) \cdot \kappa(E, s) \cdot \pi_s > \sum_{s \in S} \Pr(s) \cdot \kappa(s, s) \cdot \pi_s.$$

Hence, the firm would like to commit to a disclosure rule that withholds some information.

It is worth noting that the mechanism behind Proposition 1 is also relevant to the incentive to disclose information *ex post* (i.e., after the firm acquires information but before investors do). Once the firm has superior information, however, the disclosure of information also leads to inference by investors. The firm's incentive to disclose *ex post* is thus determined by the balance of two opposing forces. On the one hand, inference by investors provides an incentive for precise disclosure of good news and, by the unravelling argument, of all states. On the other hand, the effect of limited risk sharing on the pricing kernel provides an incentive to withhold information. If the effect of limited risk sharing on the pricing kernel dominates the good news associated with each state, then partial disclosure dominates full disclosure, even *ex post*; this is the case if

$$\max_{\mathcal{P}} \sum_{E \in \mathcal{P}} \sum_{s \in E} \Pr(s) \cdot \kappa(E, s) \cdot \pi_s \ge \max_s \kappa(s, s) \cdot \pi_s.$$

This condition is satisfied for Example 2. If the risk pertains solely to the investors, partial disclosure dominates full disclosure not only *ex ante*, but also *ex post*. For Example 1, however, the incentive to disclose good news of high profits may dominate the effect of limited risk sharing on the pricing kernel. If there is no heterogeneity among investors (i.e., w = 1), then although information disclosure has no effect on liquidity *ex ante*, the unravelling argument applies and the firm would benefit from full disclosure *ex post*.

In the remainder of the paper we demonstrate results in these two examples as well as the general setting. To highlight the key structure underlying the relationship between disclosure and firm financing, we will consider competition among firms, allow investors to trade other assets, allow for the scale of the firm to be determined endogenously, and study the case of negative firm profits. We will also consider alternative assumptions concerning the firm's objective and the investors' preferences.

3. Unlimited liability

If the investors are protected by limited liability regulations, the assumption that payoffs are positive is not very restrictive. We now examine how the possibility of a negative payoff interacts

with the firm's preferred disclosure strategy. In the setting of Section 1 with a single firm and asset, we now allow the payoffs to be negative in some states and positive in others. Suppose that

$$\pi_1 \ge \pi_2 \ge \ldots \ge \pi_{\bar{s}} > 0 > \pi_{\bar{s}+1} \ge \ldots \ge \pi_S,\tag{10}$$

and consider the following two partitions

$$\mathcal{P}^{\bullet} \equiv \{\{1, 2, \dots, \bar{s}\}, \{\bar{s}+1\}, \{\bar{s}+2\}, \dots, \{S\}\}\}$$

and

$$\mathcal{P}_{\bullet} \equiv \{\{1\}, \{2\}, \dots, \{\bar{s}\}, \{\bar{s}+1, \bar{s}+2, \dots, S\}\}.$$

Partition \mathcal{P}^{\bullet} discloses detailed information in states where the firm loses money and only the fact it is not losing money in states where it makes positive profits. We call this the *candid* partition. Partition \mathcal{P}_{\bullet} does the opposite: in states where the firm makes positive profits, it reveals all information; but if the firm is to lose money, this partition only reveals that profits will not be positive. We refer to \mathcal{P}_{\bullet} as the *braggart* partition.

Proposition 2 (Optimality of detailed disclosure of losses). Suppose that the firm generates positive and negative payoffs, as in Eq. (10). Generically in the investors' endowments and the firm's payoffs, the firm strictly prefers the candid partition to the braggart partition. For all endowments and payoffs, the former is at least as good as the latter.

In fact, for any payoff distribution, the issuer prefers informing investors of negative payoffs in detail to informing them coarsely:

$$\kappa(\{1,\ldots,\bar{s}\},s) \ge \kappa(\{s\},s) \text{ for all } s \le \bar{s};$$

$$(11)$$

while for positive payoffs, the opposite is true:

$$\kappa(\{s\}, s) \ge \kappa(\{\bar{s}+1, \dots, S\}, s) \text{ for all } s > \bar{s}.$$

$$(12)$$

This is because the pricing kernel weighs the payoff of the firm in the determination of its price. Hence, in states where it loses money, the firm would prefer to have a low pricing kernel, which it induces by informing the investors of the state of the world so that they can insure better. Convexity of the marginal utility function of the investors, in turn, guarantees that better insurance leads to a lower average marginal utility in the relevant states.

This contrasts with the classic "good news/bad news" prediction (cf. Milgrom, 1981) of asymmetric information models, where it is optimal for the seller of a product to test it and reveal "good news," and to withhold "bad news" by not testing the product. In that literature, when missing detailed information, the uninformed buyers reduce their purchases, though to a lesser extent than if they learned actual bad news about the good: the seller can thus benefit from not conducting and reporting verifiable tests.²¹

In our problem, as in the "good news/bad news" problem, the "positive payoff/negative payoff" news translates into more or less trade. Here, however, payoff distributions matter through

²¹ Similarly, the issuers would not choose to pander (strategically bias disclosed information as conditionally *bet*-*ter*-looking, Che et al., 2013): Proposition 2 holds for any distribution of firm payoffs and any distribution of the investors' endowments, and if the firms were to exploit the correlation between the payoffs and wealth distributions to tailor the particulars of disclosure, they would aim for a presentation that *enhances* riskiness from the investors' perspective.

how they impact risk sharing, and not through the payoffs' intrinsic value or 'quality.' Unlike the case of value that derives from information, the payoffs' effect on the average valuation is *ex ante* favorable for the issuer if the negative payoffs are to be announced in detail.

4. Scale of the firm

More lenient information disclosure requirements may impact the choice of business scale. We examine this impact from the perspective of capital raised by the firm, and show that limited disclosure may also induce larger investments.²²

4.1. Motivating examples

Example 3 (*Pure investor risk*). Consider a market with two equally likely states of the world and two (classes of) investors, who have cardinal utility $u(x) = \ln(x)$ and incomes $w^1 = (2, 1)$ and $w^2 = (1, 2)$, respectively. Suppose that the firm's project is riskless, but can be undertaken at different scales: if the business scale chosen is K, the payoff is $\pi = (1, 1) \cdot K$.²³ Under full disclosure, the state of the world is reported to investors before trade. In equilibrium, investors will have the same income *ex post*: the investor who receives bad news will invest one more unit. This implies that, regardless of the state, both investors consume an amount (3 + K)/2, and, therefore, $\kappa(\{s\}, s; K) = 2/(3 + K)$. The firm's expected revenue from asset issuance as a function of scale is given by $R^*(K) = 4K/(3 + K)$.

Suppose instead that trade occurs under no information disclosure. By symmetry, each investor will buy one half of the asset issued, and their *ex post* incomes will differ, with one of the investors having 2 + K/2, and the other having 1 + K/2. The pricing kernel will thus be

$$\kappa(\{1,2\},s;K) = \frac{1}{2}\left(\frac{2}{4+K} + \frac{2}{2+K}\right)$$

and the revenue brought by the issuance is then

$$R_*(K) = \frac{4K(3+K)}{(4+K)(2+K)}.$$

It is immediate that for any given scale of the project, the revenue under no disclosure is higher. Nevertheless, how *marginal* revenue changes is *not* independent of the scale. Fig. 4.1 depicts the entrepreneur's marginal revenue functions under both disclosure policies. Marginal revenue under no disclosure dominates that under full disclosure for all project scales below a certain threshold, \bar{K} . For either policy, the optimal scale is determined by equalization of the induced marginal revenue with the entrepreneur's primitive marginal cost, which is nondecreasing. To illustrate, when the cost function is linear with marginal cost c > 0, if $c > \bar{\delta}$, the optimal scale will be larger under no or partial information disclosure than under full disclosure.²⁴ In turn, a firm with sufficiently small marginal costs, $c < \bar{\delta}$, would choose a smaller business scale when

 $^{^{22}}$ Gale and Stiglitz (1989) emphasize how potential investors may change their perception of the conditions of the firm as a result of the size of the intended IPO. This change in perception does not take place in our framework.

²³ This particular specification is borrowed from Example 4 in Carvajal et al. (2011).

 $^{^{24}}$ The larger investment scale induced by the higher profitability of asset issuance under no disclosure can potentially offset the detrimental welfare effect of the distortion in efficient risk sharing due to uncertainty. We come back to this point in Section 8.



Fig. 4.1. Marginal expected revenue as a function of the scale of the project, under full disclosure and under no disclosure.

issuing under no or partial disclosure than under full disclosure. Stricter disclosure requirements therefore enhance efficiency for large businesses.

Example 4 (*Pure firm risk*). There are two states of the world. Investor 1 has wealth of 1 and investor 2 has wealth of w, both regardless of the state of the world. Their preferences are given by

$$x + \frac{1}{2}(\ln x_1 + \ln x_2)$$

Per unit of scale, the profits of the firm are 1 in state 1 and π in state 2. Taking into account the chosen scale, they are the random variable $(K, \pi K)$.

If the firm discloses the state of the world, its unit price can be computed explicitly:

$$p_1(K) = \frac{2}{w+K+1}$$
 and $p_2(K) = \frac{2\pi}{w+\pi K+1}$.

Its overall revenue is

$$R^*(K) = \frac{1}{2} \left[p_1(K) + p_2(K) \right] \cdot K,$$

which can be computed explicitly.

If, alternatively, the firm does not inform investors of the state of the world, its unit price, $p_*(K)$, results from solving the equilibrium system

$$p = \frac{1}{2} \left(\frac{1}{1+y^1} + \frac{\pi}{1+\pi y^1} \right)$$
$$p = \frac{1}{2} \left(\frac{1}{w+y^2} + \frac{\pi}{w+\pi y^2} \right)$$
$$K = y^1 + y^2.$$

The resulting revenue is $R_*(K) = p_*(K) \cdot K$.

We cannot compute the solution for $\partial R_*(K)$ explicitly, so we approximate it numerically. For comparability, we also approximate $\partial R^*(K)$. We do this for a range of combinations of the parameters w and π , and for different scale levels. Table 4.1 reports the approximate marginal effect of scale on the firm's revenue,

				1						
Κ	w = 1/2	and $\pi = 19$	9/10		w = 10 a	$w = 10 \text{ and } \pi = 1/10$				
	R_*	R^*	ΔR_*	ΔR^*	R_*	R^*	ΔR_*	ΔR^*		
0.1	0.1762	0.1749	1.5787	1.5691	0.0111	0.0099	0.1100	0.0983		
0.2	0.3218	0.3198	1.3201	1.3143	0.0220	0.0197	0.1083	0.0967		
0.3	0.4445	0.4420	1.1229	1.1195	0.0328	0.0293	0.1067	0.0951		
0.4	0.5496	0.5468	0.9685	0.9667	0.0434	0.0387	0.1050	0.0936		
0.5	0.6407	0.6378	0.8450	0.8443	0.0538	0.0480	0.1034	0.0921		
0.6	0.7205	0.7175	0.7446	0.7445	0.0641	0.0571	0.1019	0.0907		
0.7	0.7911	0.7881	0.6616	0.6620	0.0742	0.0662	0.1004	0.0893		
0.8	0.8540	0.8511	0.5922	0.5929	0.0842	0.0750	0.0989	0.0879		
0.9	0.9105	0.9077	0.5334	0.5343	0.0940	0.0837	0.0974	0.0866		
1	0.9615	0.9588	0.4833	0.4842	0.1037	0.0923	0.0960	0.0853		
1.1	1.0079	1.0052	0.4400	0.4410	0.1132	0.1008	0.0946	0.0840		
1.2	1.0501	1.0476	0.4024	0.4035	0.1226	0.1092	0.0933	0.0827		
1.3	1.0889	1.0865	0.3696	0.3706	0.1319	0.1174	0.0920	0.0815		
1.4	1.1245	1.1222	0.3407	0.3417	0.1410	0.1255	0.0907	0.0803		
1.5	1.1574	1.1552	0.3151	0.3161	0.1500	0.1335	0.0894	0.0792		
1.6	1.1878	1.1857	0.2923	0.2933	0.1589	0.1413	0.0882	0.0781		
1.7	1.2161	1.2141	0.2720	0.2729	0.1677	0.1491	0.0870	0.0770		
1.8	1.2425	1.2406	0.2538	0.2546	0.1763	0.1567	0.0858	0.0759		
1.9	1.2671	1.2653	0.2373	0.2381	0.1849	0.1643	0.0847	0.0748		
2	1.2902	1.2884	0.2224	0.2232	0.1933	0.1717	0.0836	0.0738		
2.1	1.3118	1.3101	0.2089	0.2097	0.2016	0.1790	0.0825	0.0728		
2.2	1.3321	1.3305	0.1966	0.1973	0.2098	0.1863	0.0814	0.0718		
2.3	1.3513	1.3497	0.1854	0.1861	0.2179	0.1934	0.0803	0.0709		
2.4	1.3693	1.3679	0.1751	0.1757	0.2259	0.2005	0.0793	0.0699		
2.5	1.3864	1.3850	0.1657	0.1663	0.2338	0.2074	0.0783	0.0690		
2.6	1.4026	1.4012	0.1570	0.1575	0.2415	0.2143	0.0773	0.0681		
2.7	1.4179	1.4166	0.1490	0.1495	0.2492	0.2210	0.0763	0.0672		
2.8	1.4325	1.4312	0.1415	0.1420	0.2568	0.2277	0.0754	0.0664		
2.9	1.4463	1.4451	0.1347	0.1351	0.2643	0.2343	0.0744	0.0655		
3	1.4595	1.4583	0.1283	0.1287	0.2717	0.2408	0.0735	0.0647		

Table 4.1 The effect of information disclosure in Example 4: $\partial R_*(K)$ and $\partial R^*(K)$.

$$\Delta R(K) = \frac{R(K+0.01) - R(K)}{0.01}$$

for both revenue functions and for different firm scales. We report a configuration where there is moderate heterogeneity between investors and moderate firm risk, and one where both of these features are much larger. In both cases, the revenue is larger under no disclosure than under disclosure, for all firm scales. This we already knew, from our previous analysis. What is new is the effect of disclosure on the marginal revenue. For the case of moderate heterogeneity and risk, the results are qualitatively similar to the ones in Example 3. For low scale, the marginal revenue is higher under no disclosure, as in Fig. 4.1. As the scale increases, both marginal revenues decrease and there is a scale level where the difference reverts: again as in Example 3, for higher levels of firm scale, the marginal revenue is higher when the firm chooses not to disclose the state of the world. For a configuration with high heterogeneity and risk, the marginal revenue is higher row disclosure and low scale, and both marginal revenues are decreasing. In our computations, the marginal revenue under no disclosure tends to a value higher than the asymptotic level of the marginal revenue under disclosure.

4.2. Disclosure and the marginal revenue of the firm

In general, suppose that, in the same setting as Section 1, the payoff of the firm in state *s* is $\pi_s \cdot K$, where *K* is the scale chosen by the firm. At date 0, the firm chooses both the scale of the project and the information partition it will follow. The cost of undertaking a project of scale *K* is c(K), which is increasing and convex.

While the cost of the investment undertaken by the firm is unaffected by the partition it chooses, the revenue from the asset issuance depends on both of those decisions. We can treat the price p as corresponding to each "unit of scale" of the project, so that we can write the revenue as $R = p \cdot K$. The scale will affect the firm's revenue via p, and not just directly.

In keeping with our previous notation, for an event *E* and a scale *K*, re-define the set X(E; K) as the set of arrays $(x_s^1, x_s^2)_{s \in E}$ such that: (i) for each state $s \in E$, $(x_s^1 - w_s^1) + (x_s^2 - w_s^2) = \pi_s K$; and (ii) for both investor classes *i*, there exists y^i such that $x_s^i = w_s^i + y^i$ at all $s \in E$. Here, y^i is the number of units of investment purchased by *i*, each paying 1 unit of revenue in each state of the world. The first condition simply says, then, that the whole project is sold: $y^1 + y^2 = K$. Given an event and a scale, the investors' allocation continues to be characterized by the solution to (3) and the pricing kernel of Eq. (4) holds; the pricing equation is now written as

$$p(E; K) = \sum_{s \in E} \Pr(s \mid E) \cdot \kappa(E, s; K) \cdot \pi_s.$$
(13)

The revenue function is

$$R(K; \mathcal{P}) = \sum_{s \in S} \Pr(s) \cdot \kappa(\mathcal{P}, s, K) \cdot \pi_s \cdot K;$$

and marginal revenue, $\partial R(K; \mathcal{P})$, is therefore

$$\sum_{s \in S} \Pr(s) \cdot \left[\frac{\partial \kappa(\mathcal{P}, s, K)}{\partial K} \cdot \pi_s \cdot K + \kappa(\mathcal{P}, s, K) \cdot \pi_s \right].$$
(14)

Increasing the scale has two effects on marginal revenue: a direct *scale effect* and an indirect *pecuniary effect*; respectively,

$$\sum_{s\in S} \Pr(s) \cdot \kappa(\mathcal{P}, s, K) \cdot \pi_s$$

and

$$\sum_{s\in S} \Pr(s) \cdot \frac{\partial \kappa(\mathcal{P}, s, K)}{\partial K} \cdot \pi_s.$$

The scale effect is positive and decreases with scale while the pecuniary effect is negative and increases (i.e., becomes less negative) with scale. Marginal revenue also depends on disclosure requirements: the direct effect of scale is larger under partial or no disclosure than under full disclosure (cf. Proposition 1), whereas, as Proposition 3 below shows, if investors also dislike fat tails, the pecuniary effect is larger (less negative) under full disclosure than under no or partial disclosure.

To characterize the pecuniary effect of an increase in scale, recall that

$$\frac{\partial \kappa(\mathcal{P}, s, K)}{\partial K} = \frac{1}{I} \cdot \sum_{i} u'' \left(x_s^i(\mathcal{P}, K) \right) \cdot \frac{\partial x_s^i(\mathcal{P}, K)}{\partial K}.$$

This motivates restricting attention to the case of pure investor risk, namely a two-state, symmetric market for which $\partial x_s^i(\mathcal{P}, K)/\partial K = 1/I$ for all *i*, all *s*, and all \mathcal{P} . We thus maintain the assumptions of Example 3 that there is a market with two equally likely states of the world and two classes of investors of equal mass, with incomes $w^1 = (2, 1)$ and $w^2 = (1, 2)$. In this market, if u^4 is strictly negative, Jensen's inequality implies that for a generic set of investors' endowments (defined in the proof of Proposition 1),

$$\frac{1}{I}\sum_{i}u''\left(x_{s}^{i}(\mathcal{P}_{*},K)\right) < u''\left(\frac{1}{I}\sum_{i}x_{s}^{i}(\mathcal{P}_{*},K)\right) = \frac{1}{I}\sum_{i}u''\left(x_{s}^{i}(\mathcal{P}^{*},K)\right).$$

That is, the indirect pecuniary effect is strictly smaller under no disclosure, \mathcal{P}_* , than under full disclosure, \mathcal{P}^* , generically in endowments.

Proposition 3 provides sufficient conditions under which differential regulation for small and large firms improves efficiency: for small firms (i.e., those with high marginal cost), the lighter information disclosure requirements induce a larger scale. This holds robustly, so long as the investors' marginal utility is convex. If investors dislike fat tails sufficiently, i.e., $u^4 < 0$, it is full disclosure that induces a larger scale for large firms.²⁵ Namely, when u^4 is sufficiently negative, the effect of Jensen's inequality on u'' dominates its effect on u', and hence the indirect pecuniary effect outweighs the direct scale effect in Eq. (14). This is intuitive: just as a condition on the convexity of marginal utility determines the optimality of full information disclosure given the firm's scale (cf. Proposition 1), a condition on the convexity of risk preferences (u^4) determines the effect of disclosure on marginal revenue.

Proposition 3 (Marginal revenue and scale). In the above symmetric market:

- 1. There exists a threshold \underline{K} for the scale of the firm such that for all $K < \underline{K}$, marginal revenue is higher under no disclosure of information than under full disclosure, and strictly so for a generic set of investors' endowments.
- 2. Suppose that u^4 is strictly negative, and sufficiently so in the sense that there exists a strictly convex, twice continuously differentiable function f with f' > 1 such that $-u'' = f \circ u'$. Then there exists a threshold \hat{K} for the scale of the firm such that for all $K > \hat{K}$, marginal revenue is higher under full disclosure than under no disclosure, and strictly so for a generic set of investors' endowments.

We can now formalize the claim that disclosure requirements that are more stringent at larger scale induce *both* small and large firms to choose a larger scale.²⁶

Corollary 2 (Differential disclosure and firm scale). In the above symmetric market, if the condition stated in Proposition 3 (ii) holds, full disclosure induces large firms to choose a larger scale than no disclosure does, whereas no disclosure induces small firms to choose a larger scale than full disclosure does.

²⁵ Investors with $u^4 < 0$ tend to exhibit aversion to kurtosis, a risk attitude called *temperance* — cf. Kimball (1993).

²⁶ Denote by δ^h (respectively, δ^l), the marginal revenue under full disclosure (respectively, full disclosure) associated with scale \underline{K} (respectively, \overline{K}) from Proposition 3. We say that a firm is *small* if its marginal cost is above the threshold δ^h for some K smaller than \underline{K} . We say that a firm is *large* if its marginal cost is below the threshold δ^l for some K larger than \overline{K} . In other words, to equalize marginal revenue and marginal cost, small firms choose a scale smaller than \underline{K} and large firms choose a scale larger than \overline{K} , where $\underline{K} \leq \overline{K}$.

5. Trading in other assets

So far, the analysis has assumed that the investors only trade the asset issued to finance the firm. Trade of other assets would generally affect incentives. In this section, we introduce multiple assets. Proposition 4 shows that our conclusions continue to apply as long as investors cannot insure against all of the firm's shocks, in which case disclosure would be irrelevant.

Suppose that other assets can be traded in addition to the equity of the firm, and that their payoffs are random variables over the same state space. Let there be A such assets, indexed by a = 1, ..., A, and denote their payoffs by the random variable ρ^a , so that the payoff of the a-th asset in state of the world s is ρ_s^a . For simplicity, we assume that these assets are available in zero net supply.

The following notation will be useful. Taking all assets as column vectors, define $\rho = (\rho^1, \dots, \rho^A)$, which we interpret as an $S \times A$ matrix, and let $\Pi = (\pi, \rho)$ be the $S \times (A + 1)$ matrix where the first column is the payoff of the firm. Also, for any random variable *x* defined over S and any event $E \subseteq S$, let x_E denote the restriction of *x* to E.²⁷

Let $q \in \mathbb{R}^{K}$ denote the prices of the new assets; p continues to represent the price of the firm's equity. Taking q as a row vector, let P = (p, q) represent the complete vector of prices in the market. Similarly, let v^{i} denote investor i's demand for the extra assets; y^{i} continues to represent his demand for the firm's stock. With v^{i} taken as a column vector, the individual's portfolio will be

$$Y^i = \begin{pmatrix} y^i \\ v^i \end{pmatrix}.$$

With the new assets, after event *E* is announced, if the vector of asset prices is \overline{P} , investor *i* trades a portfolio $Y_E^i(\overline{P})$ of the stock to solve

$$\max_{Y \in \mathbb{R}^{K+1}} \left\{ -\bar{P} \cdot Y + \sum_{s \in E} \Pr(s \mid E) \cdot u(w_s^i + \Pi_s \cdot Y) \right\}.$$
(15)

Asset prices after the announcement of event E, P(E) = [p(E), q(E)], are such that the total equity of the firm is absorbed by the public, while all demands for the other assets are met by corresponding short supply in the aggregate: $\sum_i y_E^i(P(E)) = 1$ and $\sum_i v_E^i(P(E)) = 0$. For Eq. (4) to continue to apply, we weaken the second part of the definition of the set X(E),

For Eq. (4) to continue to apply, we weaken the second part of the definition of the set X(E), from Subsection 1.2, as follows: (2') for each *i*, there exist some y^i and some v^i such that $x_s^i = w_s^i + \pi_s \cdot y^i + \rho_s \cdot v^i$, for all $s \in S$. Now,

$$q(E) = \sum_{s \in E} \Pr(s \mid E) \cdot \kappa(E, s) \cdot \rho_s.$$
(16)

Proposition 4 (Suboptimality of full disclosure). Suppose that the payoff of the firm is positive in all states of the world. Then, any partition \mathcal{P} such that for some event $E \in \mathcal{P}$ the rank of matrix ρ_E is less than ||E|| - 1 raises more liquidity than the full disclosure partition, generically in the firm's profits. For all wealth profiles, all partitions raise at least as much liquidity as the full disclosure partition.

²⁷ In particular, Π_E denotes the $||E|| \times (A + 1)$ matrix that includes only the rows of Π that correspond to states in event *E*. For any singleton event, we will still use x_s for $x_{\{s\}}$.

6. Multiple entrepreneurs

When many entrepreneurs present in a market are choosing their disclosure policies strategically, would competition result in full disclosure? We show that strategic interaction among entrepreneurs still results in partial disclosure. Suppose that there are J firms engaged in the R&D phases of their projects. These firms are indexed by j = 1, ..., J, and their state spaces are denoted by S^j . The overall state space is $S = \times_j S^j$, the product of the individual spaces, so that a state is now a profile $s = (s^1, ..., s^J)$.

The profits of firm *j* in state *s* are π_s^j , and we denote by $\pi_s = (\pi_s^1, \ldots, \pi_s^J)$ the profile of firms' payoffs, which for algebraic purposes we will treat as a row. Replacing the notation of the previous section, we now use Π to denote the $S \times J$ matrix constructed using all the vectors π_s . As before, the restriction of Π to event $E \subseteq S$ is denoted by Π_E .

The price of firm j's equity is p^j , and we denote by $P = (p^1, ..., p^J)$ the vector of firm equity prices. We treat this vector as a row, while the portfolio of equity holdings of investor $i, Y^i \in \mathbb{R}^J$, is treated as a column. Upon revelation of event E, if prices are \bar{P} , such portfolio is again the solution to problem (15), and is denoted by $Y_E^i(\bar{P})$. The equilibrium prices in such event are P(E) such that $\sum_i Y^i(P(E)) = (1, ..., 1)$.

Eq. (4) continues to define the pricing kernel, so long as we again adapt the second part of the definition of the set X(E), from Subsection 1.2: for each *i*, there exist some Y^i such that $x_s^i = w_s^i + \pi_s \cdot Y^i$, for all $s \in S$. Given this, Eq. (16) continues to define the vector of equilibrium prices contingent on the disclosure of event *E*.

This setting describes a normal-form game as follows. At date 0, each entrepreneur chooses a partition \mathcal{P}^i of state space \mathcal{S}^i . At date 1, when state *s* realizes, firm *j* discloses $E^j \in \mathcal{P}^j$ such that $s^j \in E^j$, and the resulting prices are $P(\times_j E^j)$. Ex-ante, firm *j* cares about

$$L^{j}(\mathcal{P}) = \frac{1}{1+\bar{r}} \cdot \sum_{E \in \mathcal{P}} \Pr(E) \cdot P_{j}(E),$$

where product partition \mathcal{P} is defined as

$$\left\{E \subseteq \mathcal{S} \mid \exists (E^1, \dots, E^J) \in \times_j \mathcal{P}^j : E = \times_j E^j\right\}.$$

In a Nash equilibrium in pure strategies, each partition \mathcal{P}^j maximizes $L^j(\mathcal{P})$, given \mathcal{P}^k for all $k \neq j$.

Proposition 5 (Less-than-full disclosure in Nash equilibrium). Suppose that there is a firm j such that π^j is strictly positive and, for some sub-profile $s^{-j} = (s^k)_{k\neq j}$ of states of other firms, the set

$$\left\{s^j \in \mathcal{S}^j \mid \Pr(s^j, s^{\neg j}) > 0\right\}$$

contains more states than the number of firms in the game. Generically in investors wealth and in the firms' profits, there is no Nash equilibrium where all the firms choose to reveal all their private information.

Under the hypothesis of the proposition it cannot occur that there are two different entrepreneurs who can disclose essentially the same information to the market, as it rules out the possibility of perfectly correlated states across firms. If that were the case, there may be full revelation in a Nash equilibrium, with firms revealing their information just because another firm is revealing it too. But this Nash equilibrium is not strong, and even an arbitrarily small cost of information disclosure would suffice to eliminate it, irrespectively of the number of firms.

To see this, suppose now that the firms' profits are perfectly correlated. Still, each entrepreneur chooses how much information to disclose independently. For simplicity, treat the common state space simply as S, and as before suppose that each firm commits to a partition \mathcal{P}^j of S. After all firms choose their disclosure policies, the resulting partition is their coarsest common refinement, or *meet*, $\wedge_j \mathcal{P}^j$. The value of each of the firms depends on this resulting partition.

Proposition 6 (Less-than-full disclosure in Nash equilibrium). Suppose that information disclosure is costly, in the sense that the firms pay $\varepsilon \cdot ||\mathcal{P}^j||$ upon commitment to partition \mathcal{P}^j , where $\varepsilon > 0$. For all profiles of investors' wealth and firms' profits, there is no Nash equilibrium where the resulting partition is fully revealing.

7. More general preferences

7.1. Firms: general objective

The assumption that the goal of the firm is to maximize its expected liquidity is restrictive: it implies risk neutrality. In practice, the riskiness of the funding raised is a common consideration in the financing process. It is of concern not only to business innovators; often, investment banks are reluctant to bear all of an offering's risk, and a syndicate of underwriters is formed instead. Because of this, we now consider firm objectives that are not risk-neutral.

We treat the preferences of the firm as a binary relation \succeq over the set of partitions of the state space. We say that the firm's preferences *are monotonic* if whenever a partition \mathcal{P} induces a firstorder stochastic improvement in the pricing kernel relative to partition \mathcal{P}' , the firm strictly prefers the former partition, so $\mathcal{P} \succ \mathcal{P}'$. If, under the same premise, we only have that $\mathcal{P} \succeq \mathcal{P}'$, we say that the preferences are *weakly monotonic*. Similarly, we say that preferences are *monotonic over information coarsening* if whenever \mathcal{P} is a coarsening of \mathcal{P}' that induces a first-order stochastic improvement in the pricing kernel, we have that $\mathcal{P} \succ \mathcal{P}'$, with the weak version defined as above.

It follows from the proof of Proposition 1 that its result holds for any monotonic preference relation, and not just for the expected liquidity preferences we were considering there. The next result strengthens this insight and shows that the disclosure partition induced by the SOX Act is the least preferred one according to any preferences that are monotonic over information coarsening.

Proposition 7 (Optimality of partial disclosure). Suppose that the payoff of the firm is positive in all states of the world. If the firm's preferences are monotonic in information coarsening, then any partition that discloses no or only partial information is strictly preferred to the full disclosure partition, generically in the investors' wealth profiles and in the firm's payoffs. For all wealth profiles and payoffs, if the firm's preferences are weakly monotonic in information coarsening, all partitions are at least as good as the full disclosure partition.

It follows that with two states no contingency will be disclosed.

Definition. Over the set of partitions, the firms has *worst-case risk aversion* if its preferences are given by \succeq_1 , defined by

$$\mathcal{P} \succeq_1 \mathcal{P}' \Leftrightarrow \min_{E \in \mathcal{P}} p(E) \ge \min_{E \in \mathcal{P}'} p(E).$$

In general, for each number $\lambda \in [0, 1]$, define the relation \succeq_{λ} by saying that $\mathcal{P} \succeq_{\lambda} \mathcal{P}'$ if and only if

$$\lambda \min_{E \in \mathcal{P}} p(E) + (1 - \lambda) \sum_{E \in \mathcal{P}} \Pr(E) \cdot p(E)$$

is at least as large as

$$\lambda \min_{E \in \mathcal{P}'} p(E) + (1 - \lambda) \sum_{E \in \mathcal{P}'} \Pr(E) \cdot p(E).$$

A firm seeking financing exhibits *worst-case risk aversion* if it is concerned only about the lowest possible price it could attain in its issuance. In order to capture preferences in between the two extremes of risk-neutrality and worst-case risk aversion, we consider also the family of their convex combinations.²⁸ In this family, expected liquidity preferences are nested by $\lambda = 0$ and worst-case scenario preferences by $\lambda = 1$. We denote by \gtrsim_0 the expected-liquidity preference relation and use \succeq to denote arbitrary preference relations. The higher the value of λ , the more weight is given to the worst-case price in the issuance, so we interpret λ as a measure of risk aversion. Denote by Λ the class of all preferences \gtrsim_{λ} , for $\lambda \in [0, 1)$, and let $\overline{\Lambda} = \Lambda \cup \{\succeq_1\}$.

Of the preferences in Λ , only the expected liquidity relation satisfies the Independence Axiom and can be given a von Neumann–Morgenstern representation. We now consider the class of all relations that have such a representation, restricting attention to risk-averse preferences (using the usual definition of risk aversion). As usual, we say that \succeq has an expected utility representation if there exists a function $v : \mathbb{R} \to \mathbb{R}$ such that $\mathcal{P} \succeq \mathcal{P}'$ if and only if

$$\sum_{E \in \mathcal{P}} \Pr(E) \cdot v\left(p(E)\right) \ge \sum_{E \in \mathcal{P}'} \Pr(E) \cdot v\left(p(E)\right).$$

We denote by \mathcal{V} the class of all relations (over partitions) that are representable with a concave and strictly increasing cardinal utility function. Note that $\overline{\Lambda} \cap \mathcal{V} = \{\succeq_0\}$.

Proposition 7 is important, as it extends our results to these latter classes of preference relations. In general, the results extend to preferences satisfying monotonicity in information coarsening.

Lemma 2. Suppose that the payoff of the firm is positive in all states of the world. Then:

- 1. The worst-case scenario preferences are weakly monotonic over information coarsening;
- 2. All risk averse preferences in class Λ are monotonic over information coarsening; and
- 3. All preferences that admit an expected utility representation with a concave and strictly increasing utility function are monotonic over information coarsening.

The prediction about suboptimality of full disclosure from Proposition 7 follows.

Proposition 8 (General optimality of partial disclosure). Suppose that the payoff of the firm is strictly positive in all states of the world. For any preferences in classes Λ and V, any partition

 $^{^{28}}$ This also allows us to deal with the difficulty of directly applying the usual definition of risk aversion — there need not be a partition that delivers as value of the firm the expectation of the random value induced by another partition.

that discloses no or only partial information is strictly preferred to the full disclosure partition, generically in the investors' wealth profiles and the firm's payoffs.

For all preference relations in classes $\overline{\Lambda}$ and \mathcal{V} , and for all wealth profiles and payoffs, any partition that discloses no or only partial information is at least as good as the full disclosure partition. With two states, disclosing no information is optimal for the firm, and strongly so generically in the investors' wealth profiles and firm's payoffs.

More generally, our prediction (for both fixed and endogenous business scale in Section 4) hold for firm objectives that are affected by ambiguity of payoffs. Intuitively, the ranking of full disclosure and partial disclosure we have established holds state by state — in particular, the Hirshleifer effect plays no role. Hence, small firm financing can benefit from limited disclosure even if not only the realization but the distribution of payoffs is unknown to the firm. The availability of greater financing for large firms under more complete disclosure provides incentives to acquire the knowledge necessary for reporting.

7.2. Investors: non-quasilinear preferences

We have assumed that the investors' preferences are quasilinear in their consumption at the time of financing. This simplifies the first-order conditions of their optimization problems and, therefore, the pricing of the firm's equity. It also allows us to distinguish our mechanism from the Hirshleifer effect. We now consider two extensions. In the next subsection we consider markets in which the asset only serves risk-sharing purposes. Here, we allow that the investors' utility function on future consumption applies also to their present consumption. Income effects complicate the mathematics of the problem but does not change its essence: the firms equity continues to be an instrument that the investors can use to transfer wealth across time periods and across future states of the world. While we do not give a general result, we present an example showing that the mechanism behind our previous result still plays a role in the new setting. In the example, we do not find any case where our predictions are overturned.

Consider the monopolistic setting of Section 1, but suppose that the preferences of the investors are represented by

$$u(x_0) + \sum_{s} \Pr(s) \cdot u(x_s).$$

Eq. (4) is no longer valid and we need to use the following pricing kernel:

$$\tilde{K}(E,s) = \frac{1}{I} \sum_{s} \frac{u'(x_s^i(E))}{u'(x_0^i(E))}.$$

Moreover, the allocation can no longer be characterized as the solution to problem (3), and we need to explicitly solve for the competitive equilibrium equity holdings. This is a highly non-linear problem. The firm's decision will impact both the marginal utility of present and future consumption on the right-hand side of the pricing kernel. Moreover, the distribution of investors' present endowments appears in the resulting equations, and may also play a role in determining the effect. We provide an example.

Example 5 (*Nonquasilinear preferences and pure firm risk*). Consider the setting of Example 1. There are two investors and two future states of the world. The investors utility functions are

$$u(x) = \ln x_0 + \frac{1}{2} \left(\ln x_1 + \ln x_2 \right).$$
(17)

Investors endowments are

$$w^1 = \begin{pmatrix} 1\\1\\1 \end{pmatrix}$$
 and $w^2 = \begin{pmatrix} m\\w\\w \end{pmatrix}$

where the first entry corresponds to their wealth at the time of purchasing the firm's equity. We use *m* to parameterize the dispersion of present wealth, and *w* for the dispersion of future wealth. No investor faces idiosyncratic risk. The profits of the firm are 1 in state 1 and $\pi > 0$ in state 2, so that π parameterizes the firm's pure risk.

If the firm discloses state s, the resulting price is p_s such that $y_s^1 + y_s^2 = 1$, where y_s^1 and y_s^2 are, respectively, the solutions to

$$\max_{y} \{\ln(1 - p_{s}y) + \ln(1 + \pi_{s}y)\} \text{ and } \max_{y} \{\ln(m - p_{s}y) + \ln(w + \pi_{s}y)\},\$$

with $\pi_1 = 1$ and $\pi_2 = \pi$. These prices are, by direct computation,

$$p_1 = \frac{1+m}{1+w+2}$$
 and $p_2 = \frac{(1+m)\cdot\pi}{1+w+2\pi}$

The ex-ante liquidity of the firm would then be

$$L(\mathcal{P}^*) = \frac{1}{2} \left[\frac{1+m}{1+w+2} + \frac{(1+m)\cdot\pi}{1+w+2\pi} \right]$$

Alternatively, if the firm reveals no information, its equity is priced at p such that $y^1 + y^2 = 1$, for y^1 that solves

$$\max_{y} \left\{ \ln(1 - p_s y) + \frac{1}{2} \ln(1 + y) + \frac{1}{2} \ln(1 + \pi y) \right\}$$

and y^2 that solves

$$\max_{y} \left\{ \ln(m - p_{s}y) + \frac{1}{2}\ln(w + y) + \frac{1}{2}\ln(w + \pi y) \right\}.$$

This price is $L(\mathcal{P}_*)$ itself. It is the only positive solution to a quadratic system that is impractical to analyze directly, so instead we compute it numerically for different values of parameters m, w and π . We concentrate on a neighborhood of m = 1, w = 1 and $\pi = 1$, where the problem is trivial and information disclosure plays no role. Also, having both investors endowed with similar levels of wealth in the present and in the future allows us to think that we are not picking parameter values where one of the two effects that cause the ambiguity should clearly dominate. Table 7.1 presents the results: for different parameter values, the table presents (re-scaled) the difference $L(\mathcal{P}_*) - L(\mathcal{P}^*)$, namely the gain in liquidity that the firm experiences by choosing to disclose no information. Our results for the non-quasilinear case are that this difference is almost always strictly positive and never negative. For the functional form and parameter values used here, the result is the same: the firm cannot be worse off by choosing to reveal no information, and, except in the case where it faces no risk, it is strictly better off. \Box

In particular, if the firm's equity continues to serve as a vehicle to save, the effect of information disclosure on the first-period marginal utility of income does not overturn its effect on the second-period average marginal utility of income. We next extend the result to the case of pure investor risk.

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π	w = 1			m = 1						
	m = 0.5	m = 0.99	m = 1	m = 1.01	m = 1.5	w = 0.5	w = 0.99	w = 1	w = 1.01	w = 1.5
0.1	1.96	2.44	2.45	2.47	3.14	3.33	2.47	2.45	2.44	2.02
0.2	1.39	1.66	1.67	1.68	2.17	2.26	1.67	1.67	1.66	1.44
0.3	0.96	1.10	1.11	1.11	1.46	1.49	1.11	1.11	1.10	0.99
0.4	0.63	0.71	0.71	0.72	0.95	0.95	0.72	0.71	0.71	0.66
0.5	0.39	0.44	0.44	0.44	0.59	0.57	0.44	0.44	0.44	0.41
0.6	0.23	0.25	0.25	0.25	0.33	0.32	0.25	0.25	0.25	0.24
0.7	0.11	0.13	0.13	0.13	0.17	0.16	0.13	0.13	0.13	0.12
0.8	0.05	0.05	0.05	0.05	0.07	0.06	0.05	0.05	0.05	0.05
0.9	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.01	0.01	0.01
1	0	0	0	0	0	0	0	0	0	0
1.1	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
1.2	0.03	0.03	0.03	0.04	0.05	0.04	0.03	0.03	0.03	0.04
1.3	0.07	0.07	0.07	0.07	0.10	0.08	0.07	0.07	0.07	0.08
1.4	0.11	0.12	0.12	0.12	0.16	0.14	0.12	0.12	0.12	0.12
1.5	0.15	0.17	0.17	0.17	0.23	0.19	0.17	0.17	0.17	0.18
1.6	0.21	0.23	0.23	0.23	0.31	0.26	0.23	0.23	0.23	0.24
1.7	0.26	0.29	0.29	0.29	0.39	0.32	0.29	0.29	0.29	0.31
1.8	0.32	0.36	0.36	0.36	0.48	0.39	0.36	0.36	0.36	0.38
1.9	0.38	0.42	0.42	0.43	0.56	0.46	0.42	0.42	0.42	0.45
10	2.74	3.21	3.23	3.25	4.22	2.95	3.22	3.23	3.24	3.86
100	4.00	4.76	4.78	4.81	6.22	4.13	4.76	4.78	4.81	5.95
1000	4.15	4.95	4.98	5.00	6.47	4.27	4.96	4.98	5.00	6.22

Table 7.1 The effect of information disclosure in Example 5: $100 \times [L(\mathcal{P}_*) - L(\mathcal{P}^*)]$.

Example 6 (*Nonquasilinear preferences and pure investor risk*). Consider the setting of Example 5, where the investors' utility functions are given by Eq. (17), but suppose now that their endowments are

$$w^1 = \begin{pmatrix} m \\ 1 \\ w \end{pmatrix}$$
 and $w^2 = \begin{pmatrix} m \\ w \\ 1 \end{pmatrix}$,

while the payoff of the firm is $\pi > 0$ in both states.

If the firm discloses no information, by symmetry, the price of its equity is p such that y = 1/2 solves the following problem:

$$\max_{y} \left\{ \ln(m - py) + \frac{1}{2} \left[\ln(1 + \pi y) + \ln(w + \pi y) \right] \right\}.$$

This price equals the liquidity raised by the firm. By direct computation, it is

$$p_* = \frac{2m\pi(1+w+\pi)}{4(1+\pi/2)(w+\pi/2)+\pi(1+w+\pi)}.$$

If the firm discloses the state of the world, the price of its equity, in both states, is p such that $\hat{y} + \tilde{y} = 1$, where \hat{y} and \tilde{y} , respectively, solve the problems

$$\max_{y} \{\ln(m - py) + \ln(1 + \pi y)\} \text{ and } \max_{y} \{\ln(m - py) + \ln(w + \pi y)\}.$$

Again, by direct computation, the price of the firm is

$$p^* = \frac{2m\pi}{1+w+2\pi}.$$

After some algebra, one obtains that $p_* > p^*$ whenever $w \neq 1$, while the two prices are the same if w = 1. \Box

In both examples, the implication that full disclosure of information minimizes the value of the firm extends to non-quasilinear preferences. In the case of Example 6, the same argument that proves Corollary 3 in Carvajal et al. (2012) would yield that the result is not particular to the example: for any preferences, in settings displaying such symmetry across investors the result will hold. We do not know whether the same is true for Example 5, nor in general.

7.3. Investors: no present consumption

The robustness is lost, however, when the firm's equity loses any potential role as a savings device in the presence of an asset with exogenous return.²⁹ Suppose that the investors do not consume at the time of trading in the stock of the firm. Instead, they borrow the wealth necessary to afford their investment, at risk-free interest rate \bar{r} , and purchase the asset so as to solve

$$\max_{y} \left\{ \sum_{s} \Pr(s) \cdot u \left(w_{s}^{i} + (\pi_{s} - \bar{r} p) y \right) \right\}.$$
(18)

Pricing through Eq. (4) is, of course, no longer valid.

Suppose first that the firm chooses to disclose information fully. Upon revelation of the state, the latter problem becomes

$$\max_{y}\left\{u\left(w_{s}^{i}+(\pi_{s}-\bar{r}p)y\right)\right\}.$$

The equilibrium price has to be $p_s = \pi_s/\bar{r}$, where the investors are indifferent to the amount of stock they purchase. Ex ante, the value of the firm is

$$p^* = \frac{1}{\bar{r}} \sum_{s} \Pr(s) \pi_s.$$

Note that the stock is priced by non-arbitrage, independently of marginal utilities.

If, alternatively, the entrepreneur discloses no information, the (interior) solution to problem (18) is y^i such that

$$p \cdot \bar{r} \cdot \sum_{s} \Pr(s) u' \left(w_s^i + (\pi_s - \bar{r}p) y^i \right) = \sum_{s} \Pr(s) u' \left(w_s^i + (\pi_s - \bar{r}p) y^i \right) \cdot \pi_s.$$
(19)

The equilibrium prices, p_* , are such that $\sum_i y^i = 1$ and must be in the interval $(\min_s \pi_s/\bar{r}, \max_s \pi_s/\bar{r})$. As before, this equation is highly non-linear.

Suppose first that the situation is one of pure firm risk, so that $w_s^i = w^i$ for all investors. By direct substitution in Eq. (19), when $p = p^*$, the optimal investment is $y^i = 0$. Thus, if the firm discloses no information, at p^* there is excess supply of its equity. If preferences are such that the aggregate demand $\sum_i y^i$ is decreasing in the firm's price, it follows that the resulting equilibrium

 $^{^{29}}$ This finding contrasts with Section 5 which shows that our result are robust to trade in other *endogenously* priced assets.

The effect of information disclosure in Example 7: $L(\mathcal{P}_*) - L(\mathcal{P}^*)$.								
π	w = 0.5	w = 0.9	w = 1	w = 1.1	w = 1.5			
1/2	-0.1112	-0.1017	-0.1008	-0.1001	-0.0986			
1	0	0	0	0	0			
3/2	0.0500	0.0627	0.0639	0.0647	0.0666			
2	0.0284	0.0808	0.0858	0.0895	0.0972			
5/2	-0.0597	0.0525	0.0639	0.0723	0.0899			

is with $p_* < p^*$. In this case, information disclosure is unambiguously detrimental to the ex-ante value of the firm.

Consider next the case of pure investor risk, where $\pi_s = \pi$ in all states. Eq. (19) simplifies and the equilibrium price is $p_* = p^*$, where the investors are, again, indifferent to their holdings of the stock. Without firm risk, the stock is priced by non-arbitrage independently of marginal utilities, thereby overriding any effect of information disclosure on liquidity.

We now consider an example with both firm and investors' risk.

Example 7 (No present consumption). Suppose that there are four equally likely states of the world, and $u(x) = \ln x$. The investors state-contingent wealth is

$$w^1 = \begin{pmatrix} w \\ 1 \\ 2w \\ 2 \end{pmatrix}$$
 and $w^2 = \begin{pmatrix} 1 \\ w \\ 2 \\ 2w \end{pmatrix}$.

Table 7.2

The profits of the firm are $\pi_1 = \pi_2 = \pi$ and $\pi_3 = \pi_4 = 1$, and the risk-free interest rate is $\bar{r} = 1$. We will consider different values of w and π in our numerical solution, but note that the stock of the firm is particularly attractive for investors when $\pi > 1$: it delivers them more wealth in the first two states, where they marginally value it more. When $\pi = 1$, it follows from our previous analysis that information disclosure has no effect on the firm's ex-ante value.

If the firm were to disclose all its private information, then $p^* = (1 + \pi)/2$. By symmetry, if it discloses no information the resulting price is p_* for which Eq. (19) holds when $y^i = 1/2$. This is a fourth-degree polynomial so we compute this price numerically. We do this by finding the (only) value of p in the interval between π and 1, for which

$$p \cdot \left[\frac{\pi}{w + (\pi - p)/2} + \frac{\pi}{1 + (\pi - p)/2} + \frac{1}{2w + (1 - p)/2} + \frac{1}{2 + (1 - p)/2}\right]$$

equals

$$\frac{1}{w + (\pi - p)/2} + \frac{1}{1 + (\pi - p)/2} + \frac{1}{2w + (1 - p)/2} + \frac{1}{2 + (1 - p)/2}.$$

Table 7.2 reports the difference $p_* - p^*$ for various values of π and w about unity. This difference is positive only when $\pi > 1$, where the firm is a valuable insurance instrument for the investors. From the results it seems that $p_* - p^*$ is co-monotone with w for any given value of π , but there is no monotonicity in the dependence of the difference on π given w. In particular, when $\pi = 5/2$ and w = 1/2, the firm is better off disclosing all its private information.³⁰

This example shows that the economic mechanism at the heart of our results relies on the determination of asset prices by investors' marginal utility. If, for some event, the stock is priced by non-arbitrage alone, then full disclosure may be the firm's preferred choice, even conditionally on scale.

8. JOBS v. SOX: some welfare considerations

Information disclosure has the potential to affect investors' *ex ante* welfare in two ways: information reduces the risk that investors face (the risk effect), and as a by-product of this risk reduction, dampens the risk-sharing motive (the Hirshleifer effect). This paper focuses on the effect that information has through the risk effect: with quasi-linear preferences, the Hirshleifer effect is absent. With a muted Hirshleifer effect, the welfare of investors is monotonic in the fineness of the information partition for any scale of the firm.

The JOBS Act has been expected to transform the possibilities available to firms seeking to raise capital, and arguably, it already has, as evidenced by the considerable volume of firms seeking financing, in both public and private markets, that have opted for exemptions provided by the Act.³¹ It follows from the analysis of Section 4 that, insofar as the mechanism highlighted in this paper plays a role, partial disclosure of information promotes capital formation by the firm (cf. Proposition 3). In the absence of other considerations, we now point out that while in that situation investors face greater uncertainty, the larger scale of the firm is in fact beneficial for them.³²

The *ex ante* efficient scale, given an information partition \mathcal{P} , solves the following (information constrained) social planner's problem:

$$\max_{K, x_s^i} \left\{ -c(K) + \sum_i \sum_s \Pr(s) \cdot u(x_s^i) \mid [(x_s^i)_{s \in E}]_{i=1}^I \in X(E) \text{ for } E \in \mathcal{P} \right\}.$$

For a given information partition, the *ex ante* efficient scale, which we denote \hat{K} , equalizes marginal cost and marginal social benefit of scale (i.e., the average revenue):

$$c'(K) = \sum_{s} \Pr(s) \cdot \left[\frac{1}{I} \cdot \sum_{i} u'(x_{s}^{i}(K))\right] \cdot \pi_{s}.$$

³⁰ That the entries are the same when $(\pi, w) = (3/2, 1)$ and when $(\pi, w) = (5/2, 1)$ is not a typo: at the former configuration, $p_* = (11 - \sqrt{33})/4$ and $p^* = 5/4$; at the latter, $p_* = (13 - \sqrt{33})/4$ and $p^* = 7/4$.

³¹ Looking just at Title I provisions, over 90% of companies that publicly filed their first registration statement during the first year after April 5, 2012 chose at least one accommodation offered by the JOBS Act. Especially popular is confidential submission — approximately 65% of those confidentially submitted at least one draft of a registration statement prior to public filing (Latham & Watkins LLP, 2013).

 $^{3^{2}}$ This effect differs from the one underlying the result of Kurlat and Veldkamp (2013), who also examine the requirements to disclose payoff-relevant information as a measure of investor protection. There, the investors may benefit from higher uncertainty when this results in a higher risk premium and hence a less profitable issuance for the firm. Here, increasing the uncertainty of the investors by issuing equity under the JOBS Act is beneficial for the firm and detrimental for the investor, but the larger scale induced by the more profitable issuance can offset this detrimental effect. Moreover, the authors find that whether the welfare impact of mandatory disclosure is detrimental for investors depends on the extent of informational asymmetry and the costs of information, which our result is independent of.

In contrast, an entrepreneur would choose a scale to equate marginal cost and marginal revenue (cf. Eq. (14)), which is the sum of the direct and indirect effects. The direct effect is equal to the marginal social benefit of scale. The indirect pecuniary effect captures the entrepreneur's exercise of market power over the choice of scale. The pecuniary effect is negative, which together with the convexity of the cost function implies an inefficiently low choice of scale $K(\mathcal{P}) < \hat{K}$, generically, for any information disclosure \mathcal{P} . The argument goes as follows: for a given scale and information partition, the equilibrium allocations and the solution to the social planner's problem coincide. If there were no indirect effect, the choice of scale by the entrepreneur would thus be efficient. The equations characterizing equilibrium allocations and choice of scale as well as efficient scale and allocations are continuous in scale. It follows that the negative indirect effect, together with a non-decreasing marginal cost function, imply that the entrepreneur chooses an inefficient choice of scale. In particular, Corollary 2 then implies that $K(\mathcal{P}^*) < K(\mathcal{P}_*) < \hat{K}$ for small firms, whereas $K(\mathcal{P}_*) < K(\mathcal{P}^*) < \hat{K}$ for large firms. That is, full disclosure (respectively, no disclosure) encourages a more efficient choice of scale for small (respectively, large) firms.

We conclude that with endogenous project scale, while partial disclosure of information leaves risk for investors to face, it promotes a more efficient choice of scale for small firms and promotes capital formation. For large firms, full disclosure of information eliminates the risk investors face while still promoting a more efficient choice of scale, and possibly increasing total capital raised. In this sense, the investors' risk sharing motive itself offers support to the Act's (i) *weakening* of disclosure requirements for small companies and (ii) *differential* regulation of disclosure for small and large companies. While prices are determined in the competitive investor market, by altering the marginal revenue of investors, the choices of disclosure rule and business scale jointly determine the firms' ability to *de facto* affect the equilibrium price and, thus, capital raised. Our analysis suggests that the contingent-on-size disclosure framework of the JOBS Act encourages firms to use this form of market power (without allowing price setting) in the market for their equity in a way that is consistent with improving welfare relative to pre-JOBS regulation.

When applied to the specific provisions of the JOBS Act, our analysis thus suggests that lighter disclosure for small or high-growth, high-risk businesses and stricter disclosure for large projects appears consistent with the goals of the regulatory change with respect to efficiency, effectiveness in raising capital, and investor protection, the three statutory missions of the SEC. Additionally, the financing framework in which disclosure and investor protection scale with business size changes the market structure for financing by leveraging capital potential that may otherwise not be available or utilized. Namely, crowdfunding, along with the recent extension of business financing to unaccredited investors, gives rise to a *private* market with a new asset class and a new investor class for early stage investment.³³

9. Final remarks

The interaction between the issues of security design and information disclosure is important and interesting. Under our mechanism, the firm benefits from keeping imperfectly insured investors when its returns are positive, as is the case in Carvajal et al. (2012). It follows, then, that in the case of pure firm risk the firm has weak incentives to choose debt as its main funding instrument. Similarly, our results on the effect of information disclosure and firm scale suggest a

³³ A study by CrowdFund IQ (2013) reports that 58% of all adults in the United States are interested in participating in crowdfunding, with the average investment estimated at \$1,300, and the size of the market of unaccredited investors estimated as many times larger than that of accredited investors and venture capital combined.

latent effect on the optimal combination of debt and equity for the firm, if one interprets our variable K not as the scale of the investment project but as the portion of it that is financed through equity. Szydlowski (2016) considers the security design problem with Bayesian Persuasion in an investment game with security payoff that is monotonically increasing in the firm's fundamental. We leave the analysis of this interaction for future research.

Our model presents a class of games in which players' strategies are partitions.³⁴ Our analysis of efficiency contributes to other applications in this class of games. Additionally, our results demonstrate that economic implications of information changes that occur through the coarseness of the partition over the set of states are distinct from those based on asymmetric information about the likelihood of the states. In particular, the canonical results of unraveling and "good news/bad news" (the incentive to reveal good news and withhold bad news; Milgrom, 1981) do not play a role *ex ante*. Instead, the following general results hold: (i) there are incentives to disclose detailed information (i.e., a fine partition) on states with negative return, and coarse information on states with positive returns; (ii) full disclosure is suboptimal, holding firm size fixed; but (iii) disclosure that increases with quantity can be preferred by all parties. We assume that investors' wealth and project return are defined over the same set of states, so our results do not rely on informational asymmetries in this sense either.

Admittedly, asymmetric information considerations (adverse selection, moral hazard, persuasion, and pandering) continue to shape the new regulatory framework through the strict disclosure, audit and reporting requirements for financing of large businesses going public and for medium size firms beyond their exemption period. One insight from our results is that the financing benefits from more complete disclosure for larger businesses suggest that at the stage of company development when entrepreneurs will have gained knowledge about returns, taking measures against adverse selection or moral hazard can be self-enforcing. At the initial funding seeking stage instead, uncertainty itself rather than informational asymmetries are the main determinant. Indeed, the *ex ante* risk involved in the innovator's ability to generate equity value by building a company, and not just to deliver a product of certain quality, is seen as the main characteristic of equity-based crowdfunding (Agrawal et al., 2014).

For smaller firms the private market has developed ways to deal with asymmetric information. Strausz (2017) points out that common crowdfunding platforms use contingent payments to deal with moral hazard. Crowdfunding is used increasingly³⁵ as a pre-order scheme, i.e., transactions are made conditional on the aggregate volume of purchases being large enough. The recent literature has examined incentive-related issues of pre-order crowdfunding (Chang, 2016; Chemla and Tinn, 2016; Ellman and Hurkens, 2016; and Strausz, 2017).

Appendix A. Proofs

Proof of Lemma 1. Using Eq. (5),

$$\sum_{E \in \mathcal{P}} \left[\Pr(E) \cdot \sum_{s \in E} \kappa(E, s) \cdot \pi_s \right] = \sum_{s \in \mathcal{S}} \Pr(s) \cdot \kappa(E_s^{\mathcal{P}}, s) \cdot \pi_s,$$

where $E_s^{\mathcal{P}}$ denotes the event of \mathcal{P} that contains *s*.

 $^{^{34}}$ Carvajal et al. (2012) introduced games in spans instead and did not study information-related questions or welfare. To the best of our knowledge, it is an open question when games in spans and partitions are outcome equivalent.

³⁵ http://www.huffingtonpost.com/chris-shuptrine.

It is immediate that if $\pi_s > 0$ for all *s*, then an increase in κ for some *s*, without a decrease in it for any other *s'*, increases this value. \Box

Proof of Proposition 1. For each state *s*, let $\bar{x}_s = (\pi_s + \sum_i w_s^i)/I$ denote the average (per investor) wealth realized in future state *s*. As defined by Eq. (2), the set of period-2 incomes that can result from trade, $X(\{s\})$, is

$$\left\{x \in \mathbb{R}^{I} \mid \sum_{i} (x_{s}^{i} - w_{s}^{i}) = \pi_{s} \text{ and } \exists y \in \mathbb{R}^{I} : x^{i} = w_{s}^{i} + y^{i} \cdot \pi_{s} \text{ and } \sum_{i} y^{i} = 1\right\},\$$

and we have that $(\bar{x}_s, \ldots, \bar{x}_s) \in X(\{s\})$.³⁶

For the first claim, fix an non-singleton $E \subseteq S$, and consider $s, s' \in E$, $s \neq s'$. Suppose that $\{(\bar{x}_s, \ldots, \bar{x}_s), (\bar{x}_{s'}, \ldots, \bar{x}_{s'})\} \subseteq X(E)$. This means that, $w_s^1 - w_s^2 = \pi_s \cdot (y^2 - y^1)$ and $w_{s'}^1 - w_{s'}^2 = \pi_{s'} \cdot (y^2 - y^1)$ for some pair of scalars (y^1, y^2) . This in turn requires that

$$\frac{w_s^1 - w_s^2}{\pi_s} = \frac{w_{s'}^1 - w_{s'}^2}{\pi_{s'}},\tag{(*)}$$

a condition that fails on an open subset of $\mathbb{R}^{S \times I} \times \mathbb{R}^{S}_{++}$ with full Lebesgue measure.³⁷

Since S and I are finite, it follows that in a generic set of profiles of investors' wealth and firm payoffs, for any non-singleton event $E \subseteq S$, there exists at least one $s \in E$ such that $(\bar{x}_s, \ldots, \bar{x}_s) \notin X(E)$. Let us denote by \mathcal{G} such a generic set.

It is immediate from the strict concavity of *u* that the unique solution to maximization problem (3) for $E = \{s\}$ is $x(\{s\}) = (\bar{x}_s, ..., \bar{x}_s)$. By convexity of *u'*, we further have that \bar{x}_s also solves problem $\min_{x_s \in X(\{s\})} \{\sum_i u'(x_s^i)\}$. It follows that on \mathcal{G} , for any non-singleton event *E*, $\kappa(E, s) \ge \kappa(\{s\}, s)$ for all $s \in E$, with a strict inequality for some.

The argument for the second claim is immediate, once we note that, for any vector of firm payoffs, Eq. (*) fails generically in the investors' wealth profiles. Since S is finite, we can construct a generic set W of profiles of investors' wealth such that for any non-singleton event $E \subseteq S$, there exists at least one $s \in E$ such that $(\bar{x}_s, \ldots, \bar{x}_s) \notin X(E)$. The rest of the argument remains the same.

For the third claim, suppose with no loss of generality that $w_s^1 \neq w_s^2$. Then, pick any $s' \neq s$ and construct $E \supseteq \{s, s'\}$. Eq. (*) again fails generically in the firms payoffs, and the argument continues to hold true.

Finally, for the fourth claim just note that in the complement of the generic sets, if $(\bar{x}_s, \ldots, \bar{x}_s) \in X(E)$ for all $s \in E$ and all $E \in \mathcal{P}$, we have that, moreover, $\kappa(E, s) = \kappa(\{s\}, s)$, in which case the value of the firm is the same under \mathcal{P} and \mathcal{P}^* . \Box

Proof of Proposition 2. Recall Eqs. (11) and (12), noting that generically in endowments, each of them is strict for at least one state of nature. Using Eq. (5), generically,

³⁶ Since $\pi_s \neq 0$, simply let $y^i = (\bar{x}_s - w_s^i)/\pi_s$.

³⁷ Following the remarks in Ft. 11, notice that Eq. (*) in Ft. 11 does not disrupt this argument, so long as u' remains monotonically decreasing.

$$\begin{split} &\sum_{E \in \mathcal{P}^{\bullet}} \left[\Pr(E) \cdot \sum_{s \in E} \kappa(E, s) \cdot \pi_s \right] \\ &= \sum_{s \in \mathcal{S}} \Pr(s) \cdot \kappa(E_s^{\mathcal{P}^{\bullet}}, s) \cdot \pi_s \\ &= \sum_{s \leq \bar{s}} \Pr(s) \cdot \kappa(\{1, \dots, \bar{s}\}, s) \cdot \pi_s + \sum_{s > \bar{s}} \Pr(s) \cdot \kappa(\{s\}, s) \cdot \pi_s \\ &> \sum_{s \leq \bar{s}} \Pr(s) \cdot \kappa(\{s\}, s) \cdot \pi_s + \sum_{s > \bar{s}} \Pr(s) \cdot \kappa(\{\bar{s} + 1, \dots, S\}, s) \cdot \pi_s \\ &= \sum_{s \in \mathcal{S}} \Pr(s) \cdot \kappa(E_s^{\mathcal{P}^{\bullet}}, s) \cdot \pi_s \\ &= \sum_{E \in \mathcal{P}_{\bullet}} \left[\Pr(E) \cdot \sum_{s \in E} \kappa(E, s) \cdot \pi_s \right], \end{split}$$

where the inequality comes from Eqs. (10), (11) and (12). Then, by definition, $\mathcal{P}^{\bullet} \succ_0 \mathcal{P}_{\bullet}$.

On the complement of the generic set of endowments, the inequality above is weak, and, therefore $\mathcal{P}^{\bullet} \succeq \mathcal{P}_{\bullet}$. \Box

Proof of Proposition 3. Let *W* be the generic set constructed in the proof of Proposition 1.

For K = 0, the direct effect is strictly larger under \mathcal{P}_* than under \mathcal{P}^* in set W. Therefore, by continuity of u' and since for any i and s, $u''(x_s^i(\mathcal{P}^*, K))$ is bounded below for any $K \ge 0$, there exists a sufficiently small $\underline{K} > 0$ such that the difference in direct effects outweighs the difference in indirect effects for any $K \le \underline{K}$. This proves the first statement.

For the second claim, let f be the convex increasing function such that $-u'' = f \circ u'$. Since both u' and f are convex and f is increasing:

$$f\left(u'\left(x_s^1(\mathcal{P}^*), K\right)\right) < f\left(\frac{1}{I} \cdot \sum_i u'\left(x_s^i(\mathcal{P}_*, K)\right)\right) < \frac{1}{I} \cdot \sum_i f\left(u'\left(x_s^i(\mathcal{P}_*, K)\right)\right).$$

It follows that

$$\begin{split} &f\left(\frac{1}{I}\cdot\sum_{i}u'\left(x_{s}^{i}(\mathcal{P}_{*},K)\right)\right)-f\left(u'\left(x_{s}^{1}(\mathcal{P}^{*},K)\right)\right)\\ &<\frac{1}{I}\cdot\sum_{i}f\left(u'\left((x_{s}^{i}(\mathcal{P}_{*},K)\right)\right)-f\left(u'\left(x_{s}^{1}(\mathcal{P}^{*},K)\right)\right), \end{split}$$

and therefore,

$$f\left(\frac{1}{I} \cdot \sum_{i} u'\left(x_{s}^{i}(\mathcal{P}_{*}, K)\right)\right) - f\left(u'\left(x_{s}^{1}(\mathcal{P}^{*}, K)\right)\right)$$
$$< \left|\frac{1}{I} \cdot \sum_{i} u''\left(x_{s}^{i}(\mathcal{P}_{*}, K)\right) - u''\left(x_{s}^{1}(\mathcal{P}^{*}, K)\right)\right|.$$

Since f has slope larger than 1, we have

$$\begin{split} &\frac{1}{I} \cdot \sum_{i} u' \left(x_{s}^{i}(\mathcal{P}_{*}, K) \right) - u' \left(x_{s}^{1}(\mathcal{P}^{*}, K) \right) \\ &< f \left(\frac{1}{I} \cdot \sum_{i} u' \left(x_{s}^{i}(\mathcal{P}_{*}, K) \right) \right) - f \left(u' \left(x_{s}^{1}(\mathcal{P}^{*}, K) \right) \right). \end{split}$$

Combining the two inequalities above, it follows that for any K > I,

$$\frac{1}{I} \cdot \sum_{i} u'\left(x_{s}^{i}(\mathcal{P}_{*}, K)\right) - u'\left(x_{s}^{1}(\mathcal{P}^{*}, K)\right)$$
$$< \left|\frac{1}{I} \cdot \sum_{i} u''\left(x_{s}^{i}(\mathcal{P}_{*}, K)\right) - u''\left(x_{s}^{1}(\mathcal{P}^{*}, K)\right)\right| \cdot \frac{K}{I};$$

that is, for any $K \ge \overline{K} = I$, the difference in indirect effects outweight the difference in direct effects.

With u^4 strictly positive, Jensen's inequality implies that generically in W,

$$\frac{1}{I} \cdot \sum_{i} u'' \left(x_s^i(\mathcal{P}_*, K) \right) > u'' \left(\frac{1}{I} \cdot \sum_{i} x_s^i(\mathcal{P}_*, K) \right) = \frac{1}{I} \cdot \sum_{i} u'' \left(x_s^i(\mathcal{P}^*, K) \right).$$

It follows that both the direct and the indirect effects are larger under \mathcal{P}_* than under \mathcal{P}^* . \Box

Proof of Proposition 4. The argument generalizes the proof of Proposition 1. As in that proof, define for each *s* the average income \bar{x}_s . The set of period-2 incomes that can result from trade, $X(\{s\})$, is now

$$\left\{ x \in \mathbb{R}^{I} \mid \sum_{i} (x_{s}^{i} - w_{s}^{i}) = \pi_{s} \text{ and } \exists Y \in \mathbb{R}^{(1+A)I} : x^{i} = w_{s}^{i} + \Pi_{s} \cdot Y^{i} \text{ and } \sum_{i} Y^{i} = (1,0)^{\mathsf{T}} \right\},\$$

and we still have that $(\bar{x}_s, \ldots, \bar{x}_s) \in X(\{s\})$.

Consider $E \in \mathcal{P}$ such that ρ_E is of rank less than ||E|| - 1, and suppose that $(\bar{x}_E, \dots, \bar{x}_E) \in X(E)$. Then, denoting the column span of Π_E by $\langle \Pi_E \rangle$, for all investors it must be true that

$$\bar{x}_E - w_E^i = \frac{1}{I} \cdot \left(\sum_j w_E^j + \pi_E \right) - w_E^i \in \langle \Pi_E \rangle, \tag{20}$$

which is false, generically in (w_E^1, \ldots, w_E^I) and π , given that $\langle \Pi_E \rangle$ is a proper subspace of $\mathbb{R}^{\|E\|}$. It follows again that, generically, $\kappa(E, s) \ge \kappa(\{s\}, s)$ for all $s \in E$, with a strict inequality for some. This suffices to imply that $L(\mathcal{P})$ is higher than the liquidity raised by the full information partition. \Box

Proof of Proposition 5. For definiteness, suppose that $\|\{s^1 \in S^1 | \Pr(s^1, \bar{s}^{-1}) > 0\}\| > J$, for a given sub-profile $\bar{s}^{-1} = (\bar{s}^2, \dots, \bar{s}^J)$. It suffices for us to show that if $\mathcal{P}^j = \{\{s^j\} | s^j \in S^j\}$ for all $j \ge 2$, then firm 1 does *not* maximize L^1 by disclosing all the information in S^1 .

To see this consider the event $E = \{s^1 \in S^1 | \Pr(s^1, \overline{s}^{-1}) > 0\} \times \{\overline{s}^{-1}\}$ and suppose, as in the proof of Proposition 4, that $(\overline{x}_E, \ldots, \overline{x}_E) \in X(E)$. As before, Eq. (20) must hold. But since ||E|| < J, $\langle \Pi_E \rangle$ is a proper subspace of $\mathbb{R}^{||E||}$ and (20) is false generically in (w_E^1, \ldots, w_E^I) and π . Generically, then, $\kappa(E, s) \ge \kappa(\{s\}, s)$ for all $s \in E$, with a strict inequality for some.

This suffices to imply that for any \mathcal{P}^1 such that $E \in \mathcal{P}^1$, $L^1(\mathcal{P})$ is strictly higher than if firm 1 chooses $\{\{s^1\} | s^1 \in \mathcal{S}^1\}$, as all the states in *E* have positive probability. \Box

Proof of Proposition 6. Recall that \mathcal{P}_* and \mathcal{P}^* denote, respectively, the coarsest and finest partitions of \mathcal{S} .

By way of contradiction, suppose that there is a Nash equilibrium such that $\wedge_j \mathcal{P}^j = \mathcal{P}^*$. Since there are at least two states of the world, there must exist at least one firm for which $\|\mathcal{P}^j\| \ge 2$. For this firm,

$$L^{j}\left(\mathcal{P}_{*}\wedge(\wedge_{k\neq j}\mathcal{P}^{k})\right)=L^{j}(\wedge_{k\neq j}\mathcal{P}^{k})\geq L^{j}(\mathcal{P}^{*})=L^{j}(\wedge_{k}\mathcal{P}^{k}),$$

as in the proof of Propositions 4 and 5. Taking into account the cost of information disclosure, and given that $\varepsilon > 0$,

$$L^{j}\left(\mathcal{P}_{*}\wedge(\wedge_{k\neq j}\mathcal{P}^{k})\right)-\varepsilon>L^{j}(\wedge_{k}\mathcal{P}^{k})-2\varepsilon\geq L^{j}(\wedge_{k}\mathcal{P}^{k})-\varepsilon\cdot\|\mathcal{P}^{j}\|.$$

It follows that \mathcal{P}^j cannot be a best response to $\{\mathcal{P}^k\}_{k\neq j}$, contradicting the assumption that the profile of partitions is a Nash equilibrium. \Box

Proof of Proposition 7. This assertion follows from the proof of Proposition 1, which shows that a coarsening of information induces a first-order stochastic dominance increase in κ , generically in investors' endowments. \Box

Proof of Lemma 2. For the first statement, let \mathcal{P} be a coarsening of \mathcal{P}' that induces a first-order stochastic dominance increase in the pricing kernel, and fix $E' = \operatorname{argmin}_{\tilde{E} \in \mathcal{P}'} p(\tilde{E})$. Fix any $E \in \mathcal{P}$, and let $\mathcal{E} \subseteq \mathcal{P}'$ be such that $\bigcup_{\tilde{E} \in \mathcal{E}} \tilde{E} = E$. Using again Eq. (5),

$$p(E) = \sum_{s \in E} \Pr(s \mid E) \cdot \kappa(E, s) \cdot \pi_s$$

$$= \sum_{\tilde{E} \in \mathcal{E}} \sum_{s \in \tilde{E}} \Pr(s \mid E) \cdot \kappa(E, s) \cdot \pi_s$$

$$\geq \sum_{\tilde{E} \in \mathcal{E}} \sum_{s \in \tilde{E}} \Pr(s \mid \tilde{E}) \cdot \Pr(\tilde{E} \mid E) \cdot \kappa(\tilde{E}, s) \cdot \pi_s$$

$$= \sum_{\tilde{E} \in \mathcal{E}} \Pr(\tilde{E} \mid E) \cdot p(\tilde{E})$$

$$\geq \sum_{\tilde{E} \in \mathcal{E}} \Pr(\tilde{E} \mid E) \cdot p(E')$$

$$= p(E'),$$

where the first inequality comes from the improvement in the pricing kernel, and the second from the definition of event E'. Since the latter holds for any $E \in \mathcal{P}$, it follows that

$$\min_{E \in \mathcal{P}} p(E) \ge p(E') = \min_{E \in \mathcal{P}'} p(E).$$

For the second statement, fix \succeq_{λ} , $\lambda \in (0, 1)$, and as above, let \mathcal{P} coarsen \mathcal{P}' and induce a first-order stochastic dominance increase in the pricing kernel. By the previous argument,

$$\min_{E\in\mathcal{P}} p(E) \ge \min_{E\in\mathcal{P}'} p(E),$$

while

$$\sum_{E \in \mathcal{P}} \Pr(E) \cdot p(E) > \sum_{E \in \mathcal{P}'} \Pr(E) \cdot p(E)$$

by Lemma 1. Since $\lambda < 1$,

$$\lambda \min_{E \in \mathcal{P}} p(E) + (1 - \lambda) \sum_{E \in \mathcal{P}} \Pr(E) \cdot p(E) > \lambda \min_{E \in \mathcal{P}'} p(E) + (1 - \lambda) \sum_{E \in \mathcal{P}'} \Pr(E) \cdot p(E).$$

Finally, let \succeq be an expected utility preference relation with concave and strictly increasing utility function v. Once again, let \mathcal{P} coarsen \mathcal{P}' and induce a first-order stochastic dominance increase in κ . Let p and p' be, respectively, the (random) prices induced by the two partitions using Eq. (5), and let π be an auxiliary random variable constructed as follows: for each $E \in \mathcal{P}$, let $\{E'_1, \ldots, E'_N\} \subseteq \mathcal{P}'$ be such that $\bigcup_{n=1}^N E_n = E$, and let

$$\pi(E) = \sum_{n=1}^{N} \left[\Pr(E'_n \mid E) \cdot \sum_{s \in E'_n} \Pr(s \mid E'_n) \cdot \kappa(E'_n, s) \cdot \pi_s \right].$$

This variable gives us the counterfactual prices that would arise under the coarser partition \mathcal{P} , under the assumption that the pricing kernel is the one induced by the finer partition \mathcal{P}' .

Note that p' is a mean-preserving spread of π , so it follows that π is at least as large as p' in the sense of second-order stochastic dominance. Since $\pi_s > 0$ at all s, and the pricing kernel under \mathcal{P} first-order stochastically dominates that under \mathcal{P}' , it follows that p first-order stochastically dominates that under \mathcal{P}' , it follows that p first-order stochastically dominates the auxiliary variable π . By transitivity, then, p second-order stochastically dominates p', which suffices since v is concave and increasing. \Box

Proof of Proposition 8. This follows immediately from Proposition 7, given Lemma 2. \Box

Appendix B. Some details of the JOBS act and our results

The new financing options introduced by the JOBS Act are:

- *Crowdfunding*: This option enables financing through a large number of small investors in the private market, with light disclosure requirements for small businesses up to \$1 million in equity.
- *Regulation A+*: This option offers disclosure exemptions for financing through the public market that are *contingent* on the scale of business for a certain time period. It increases the equity dollar ceiling from \$5 million to \$50 million, and lowers the regulatory cost burden, at the cost of additional reporting and audit.³⁸
- *IPO On-Ramp*: This option makes it easier for young, high-growth firms to raise capital in the public market at an early stage by extending the period that temporarily lowers the cost of

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 $^{^{38}}$ Regulation A+ aims to improve upon Regulation A, which was designed to ease the process of going public for small firms (up to \$5 million in equity). Available for over 20 years, Regulation A was rarely used — a fact attributed to the small scale threshold and high regulatory costs. In addition, many states *de facto* prohibited utilizing Regulation A on the grounds of investor protection.

accessing the capital markets from 2 to up to 5 years. To "enter the ramp," a company must qualify as an *emerging growth company* — a newly defined class with annual gross revenue of less than \$1 billion in the prior fiscal year. A company "exits the ramp" when it has more than \$1 billion in gross revenue, completes the five-year transitionary period, issues more than \$1 billion in non-convertible debt within a three-year period, or becomes classified as a large accelerated filer (e.g., due to market capitalization starting at \$700 million).

Also scaled across the new financing options are investor restrictions (i.e., individual investment limits based on income, aggregate offering limits, and investor accreditation requirements).³⁹

Title I (IPO On-Ramp) of the JOBS Act entitles firms with annual gross revenues of up to \$1 billion to reduced regulatory and reporting requirements. Title II lifts the ban on general solicitation and general advertising. Under review by the SEC is Title III, or the *Capital Raising Online While Deferring Fraud and Unethical Non-Disclosure* Act (CROWDFUND Act), allowing small firms to raise capital from unaccredited investors through crowdfunding. Title IV (Regulation A+) is an exemption from the registration, auditing, and reporting requirements mandated by the Securities Act which is applicable to *small public* offerings. The SEC Staff Report issued in December 2013 (SEC, 2013) provides a summary of the studies and solicited comments on disclosure requirements, as mandated by the JOBS Act.

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³⁹ It is understood that many small businesses cannot be expected to meet the heightened audit and ongoing disclosure requirements of the new Regulation A+. The JOBS Act left unchanged the pre-Title IV Regulation A for small businesses seeking to raise up to \$5 million in the public market. At the same time, the SEC has solicited another round of comments on increasing the aggregate investment limit above \$1 million for crowdfunding investments.

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